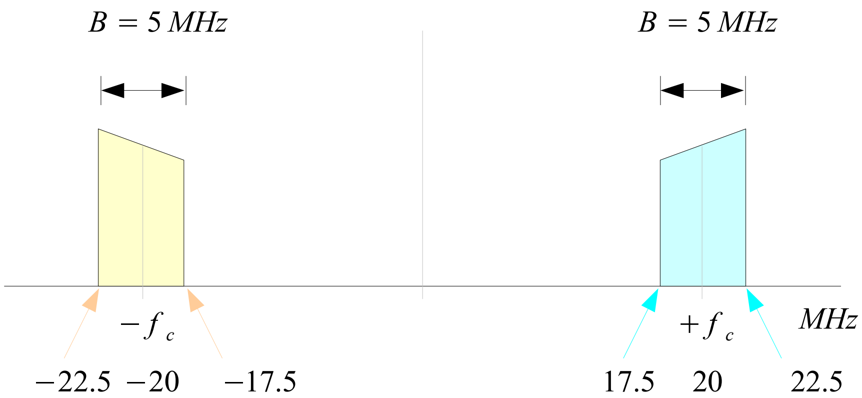
Bandpass Sampling (2B)

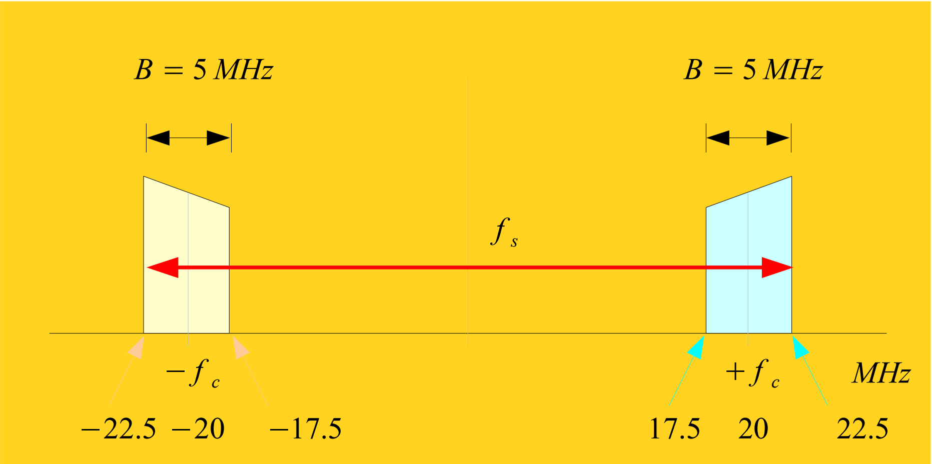
-
-

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice and Octave.

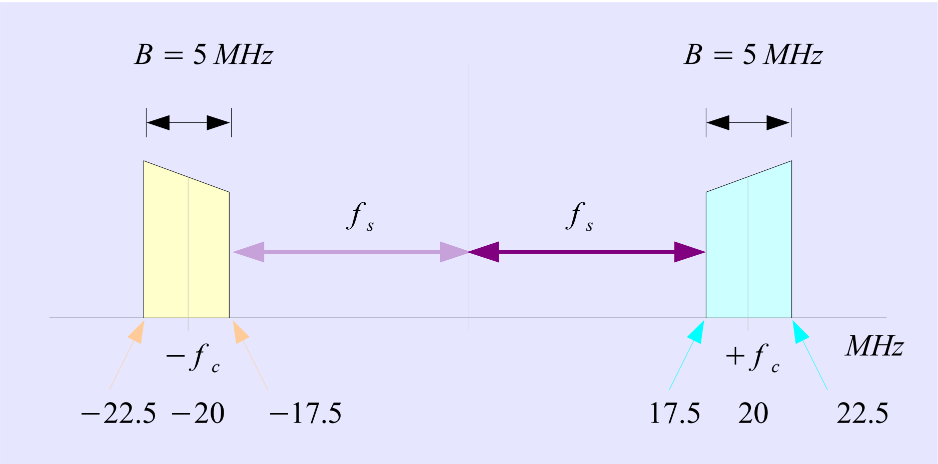
Band-limited Signal



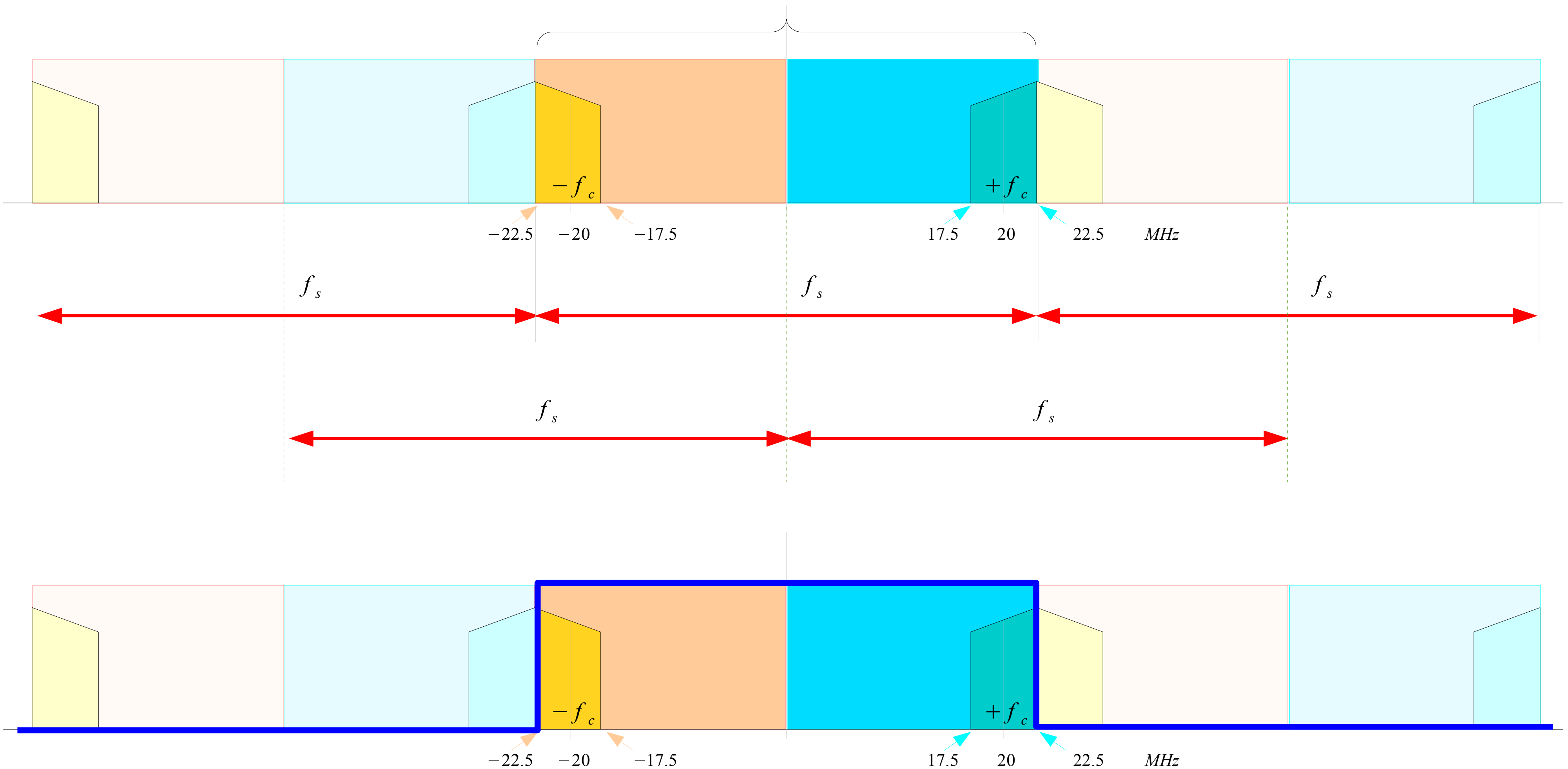


- **Lowpass Sampling**

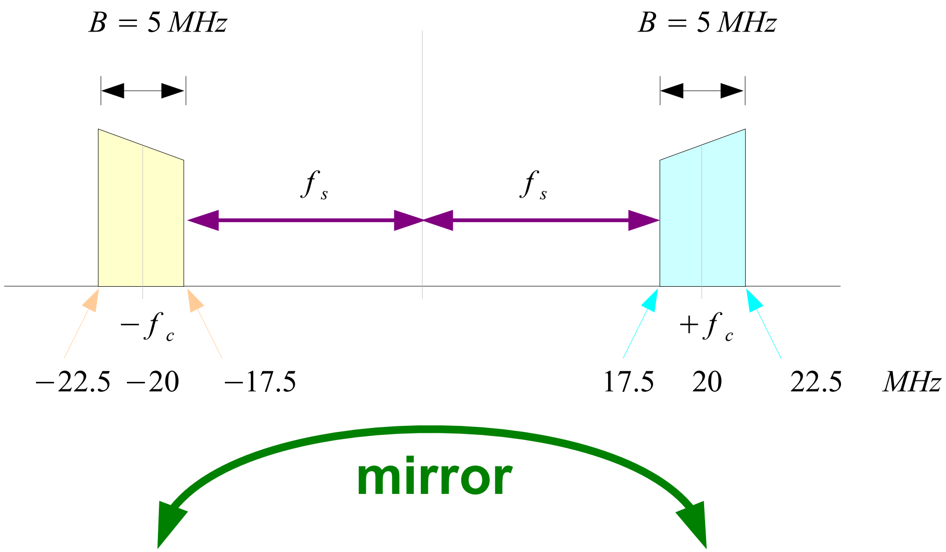
- **Bandpass Sampling**
- **IF filtering**
- **Harmonic Sampling**
- **Sub-Nyquist Sampling**

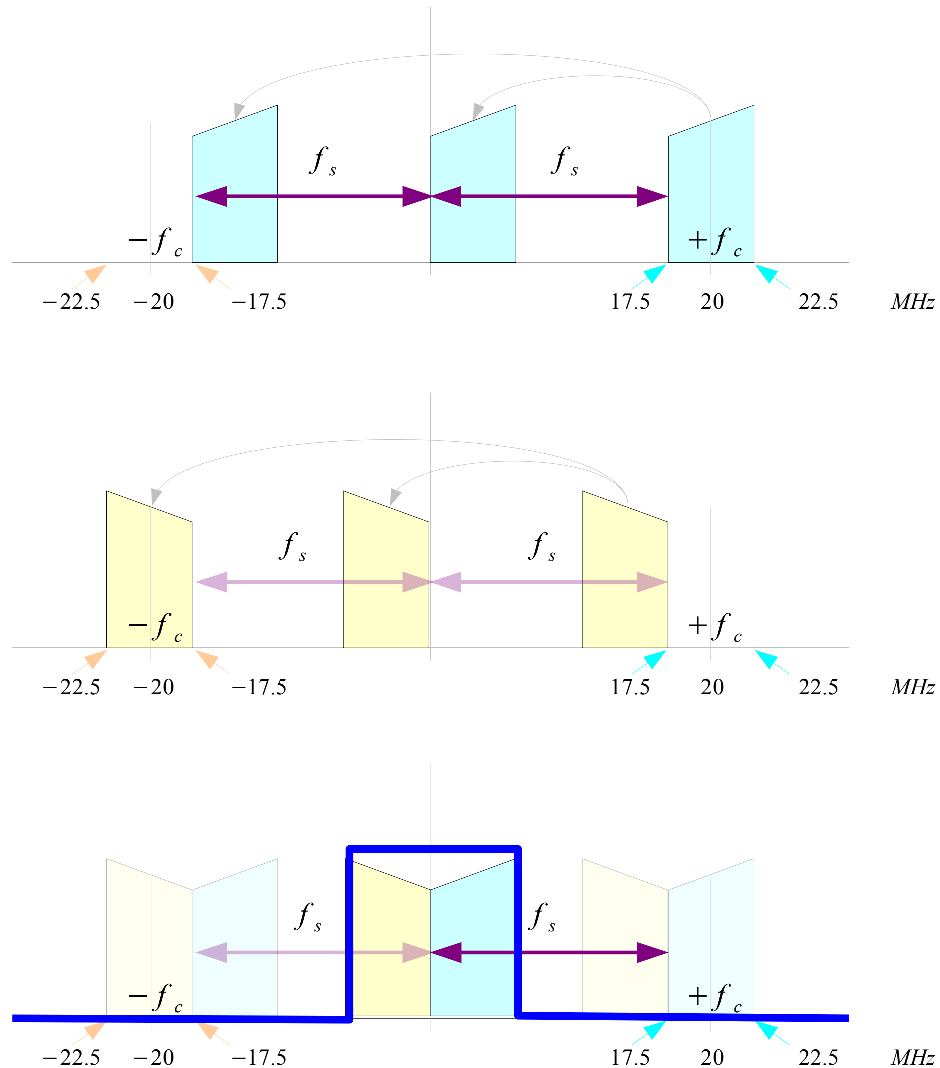


Low-pass Signal Sampling



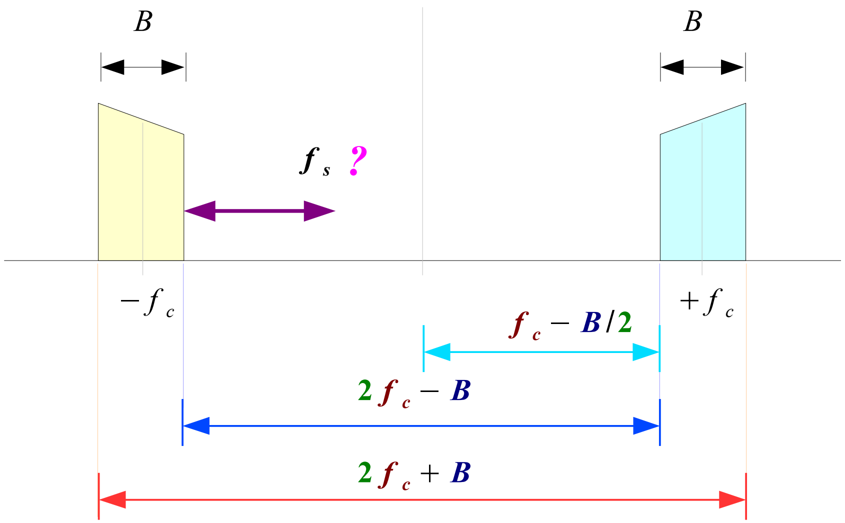
Band-pass Signal Sampling





- **Bandpass Sampling**
- **IF filtering**
- **Harmonic Sampling**
- **Sub-Nyquist Sampling**

Sampling Frequency f_s (1)



- **Bandpass Sampling**
- **IF filtering**
- **Harmonic Sampling**
- **Sub-Nyquist Sampling**

Assume there are m multiples of f_s

Given an integer m

$$2f_c - B = m \cdot f_s$$

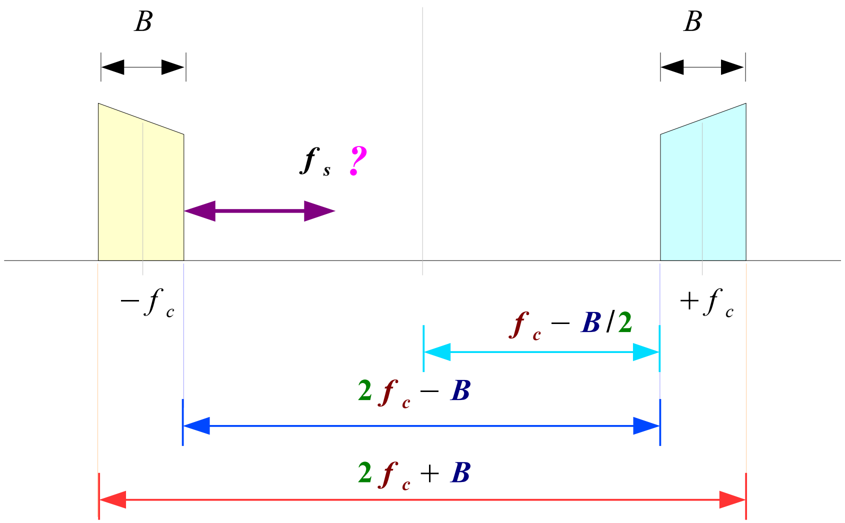
Max f_s ***condition***

f_s *can be decreased according to the following condition without introducing aliasing problems*

$$2f_c + B = (m+1) \cdot f_s$$

Min f_s ***condition***

Sampling Frequency f_s (2)



When $m = 6$ ⇒

$$\text{max } f_s = \frac{2f_c - B}{6}$$

$$\text{min } f_s = \frac{2f_c + B}{7}$$

$$\text{min } f_s \quad \frac{2f_c + B}{7} \le f_s \le \frac{2f_c - B}{6} \quad \text{max } f_s$$

Assume there are m multiples of f_s

Given an integer m

$$2f_c - B = m \cdot f_s$$

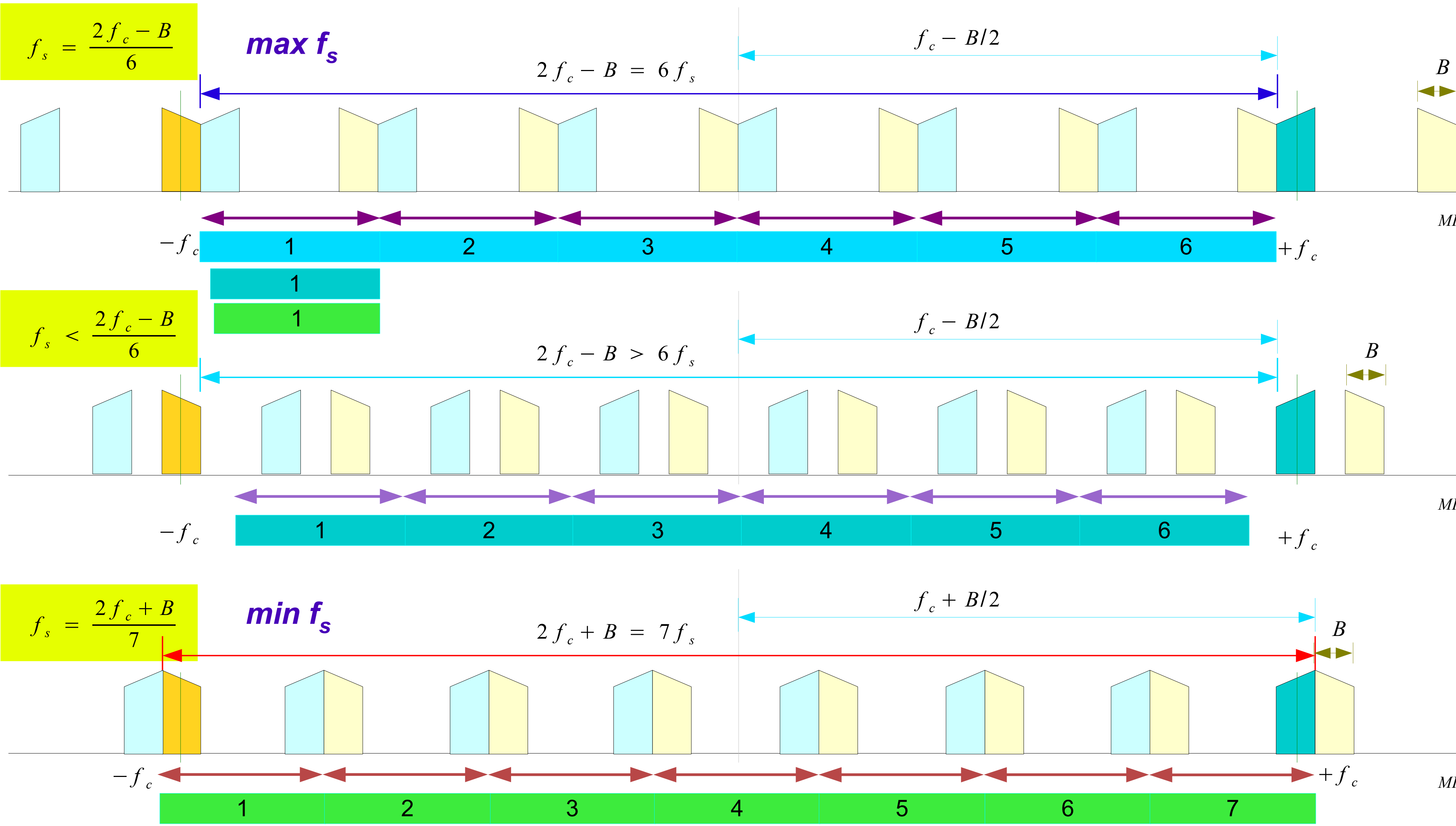
Max f_s condition

f_s *can be decreased according to the following condition without introducing aliasing problems*

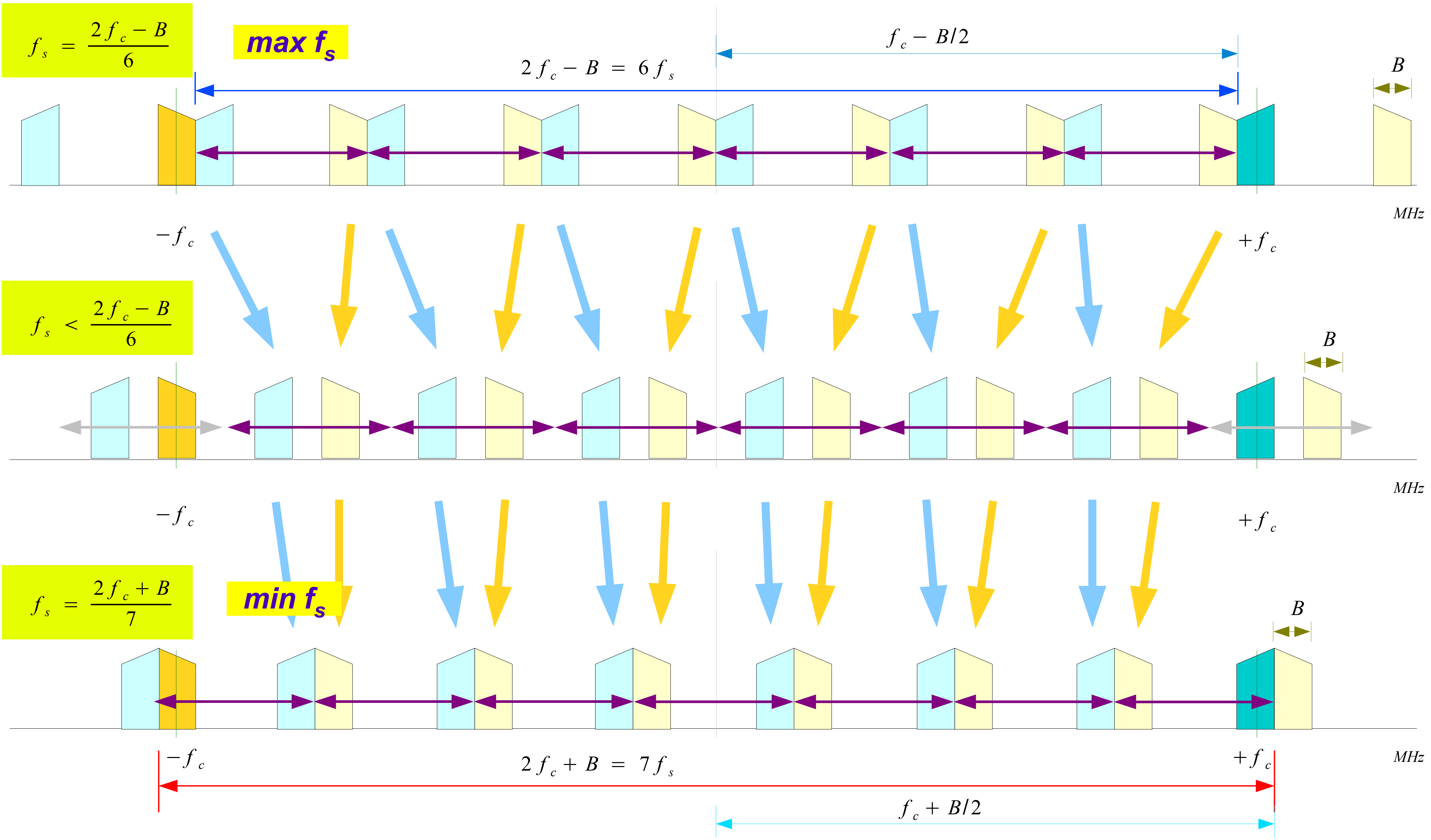
$$2f_c + B = (m+1) \cdot f_s$$

Min f_s condition

Sampling Frequency f_s (3)



Sampling Frequency f_s (4)



Range of f_s (1)

$$\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}$$

$$f_c = 20\,MHz$$
$$B = 5\,MHz$$

$$2B \le f_s$$

Optimum Sampling Frequency

$m = 1$ ➡ $\frac{2\cdot 20 + 5}{1+1} = 22.5 \le f_s \le \frac{2\cdot 20 - 5}{1} = 35$ ➡ $f_s = 22.5\,MHz \quad (10 \le f_s)$

$m = 2$ ➡ $\frac{2\cdot 20 + 5}{2+1} = 15 \le f_s \le \frac{2\cdot 20 - 5}{2} = 17.5$ ➡ $f_s = 17.5\,MHz \quad (10 \le f_s)$

$m = 3$ ➡ $\frac{2\cdot 20 + 5}{3+1} = 11.25 \le f_s \le \frac{2\cdot 20 - 5}{3} = 11.67$ ➡ $f_s = 11.25\,MHz \quad (10 \le f_s)$

$m = 4$ ➡ $\frac{2\cdot 20 + 5}{4+1} = 9 \ge \frac{2\cdot 20 - 5}{4} = 8.75$ ➡ X

$m = 5$ ➡ $\frac{2\cdot 20 + 5}{5+1} = 7.5 \ge \frac{2\cdot 20 - 5}{5} = 7.0$ ➡ X

Range of f_s (2)

$$\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}$$

$f_c = 20\,MHz$

$B = 5\,MHz$

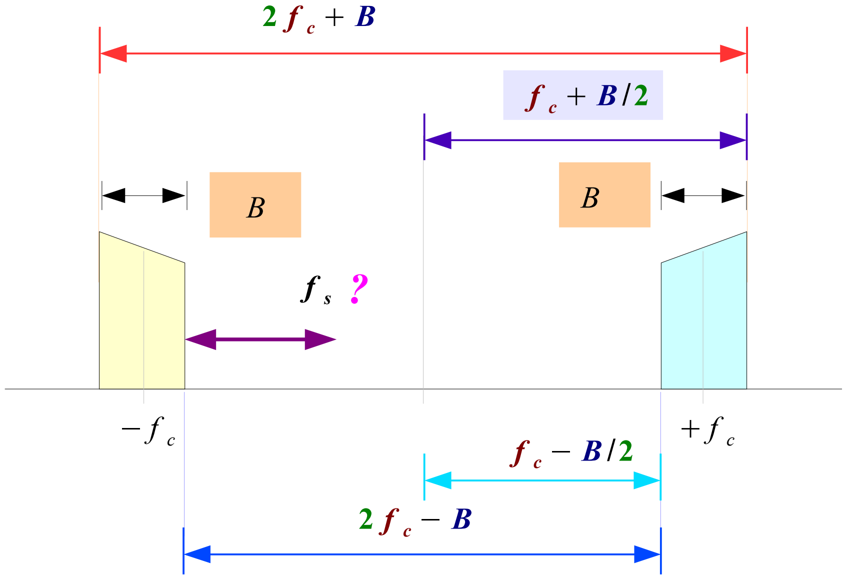
$$2B \le f_s$$

$$\frac{f_c + B/2}{B} = R$$ ➡ **highest signal frequency / bandwidth**

$$\frac{2f_c + B}{(m+1)B} = f(m, R)$$ ➡ **minimum sampling rate / bandwidth**

$$\frac{2(f_c + B/2)}{(m+1)B} = \frac{2R}{m+1} = f(m, R)$$

$m = 1$	$f(1, R) = R$	$m = 5$	$f(5, R) = \frac{1}{3}R$
$m = 2$	$f(2, R) = \frac{2}{3}R$	$m = 6$	$f(6, R) = \frac{2}{7}R$
$m = 3$	$f(3, R) = \frac{1}{2}R$	$m = 7$	$f(7, R) = \frac{1}{4}R$
$m = 4$	$f(4, R) = \frac{2}{5}R$	$m = 8$	$f(8, R) = \frac{2}{9}R$



Young Won Lim
3/9/12

Range of f_s (3)

$$\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}$$

$$f_c = 20\,MHz$$
$$B = 5\,MHz$$

$$2B \le f_s$$

$$\frac{f_c + B/2}{B} = R$$

highest signal frequency / bandwidth

$$f_H = f_c + B/2$$

$$R = f_H / B$$

$$\frac{2f_c + B}{m+1} \cdot \frac{1}{B} = f(m, R)$$

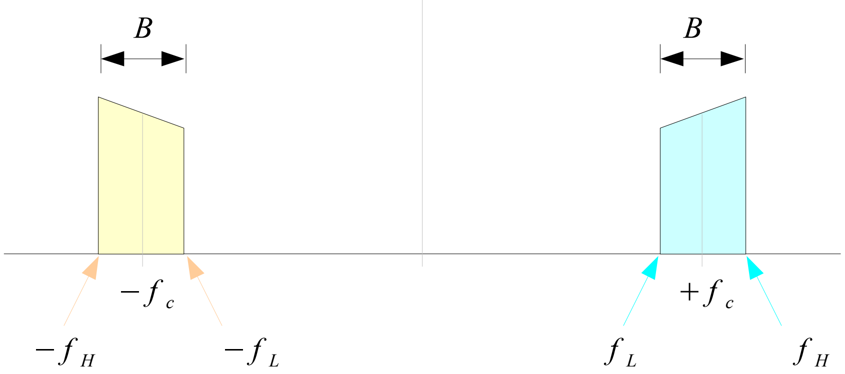
minimum sampling rate / bandwidth

$$f_{s,min} = \frac{2f_c + B}{m+1} = \frac{2f_H}{k}$$

$$\frac{2(f_c + B/2)}{(m+1)B} = \frac{2R}{m+1} = f(m, R)$$

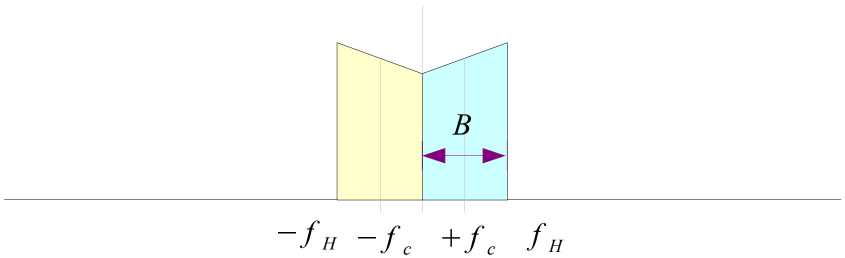
$$f(m, R) = \frac{2f_H}{kB} = \frac{2R}{k}$$

$$m + 1 = k$$



Young Won Lim
3/9/12

$k = 1 \quad (m = 0)$

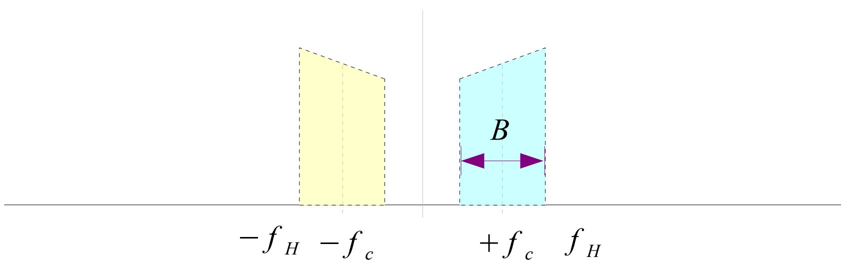


$$f_H = f_c + B/2 = 1\,B$$

$$f_H / B = R = 1$$

$$f_{s,min} = \frac{2 f_H}{k} = \frac{2 f_H}{m+1} = 2\,B$$

$k = 1 \quad (m = 0)$

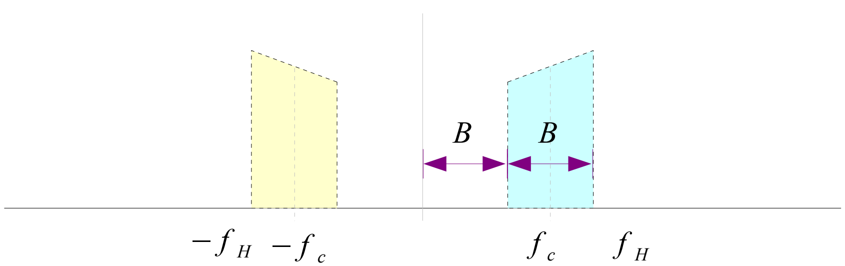


$$f_H = f_c + B/2 = 1.5\,B$$

$$f_H / B = R = 1.5$$

$$f_{s,min} = \frac{2 f_H}{k} = \frac{2 f_H}{m+1} = 3\,B$$

$k = 1 \quad (m = 0)$

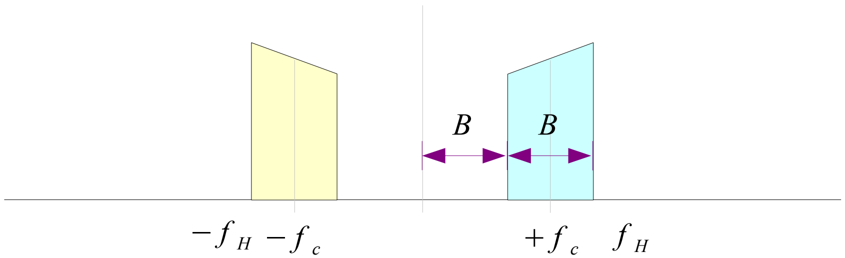


$$f_H = f_c + B/2 = 2\,B$$

$$f_H / B = R = 2$$

$$f_{s,min} = \frac{2 f_H}{k} = \frac{2 f_H}{m+1} = 4\,B$$

$k = 2 \quad (m = 1)$

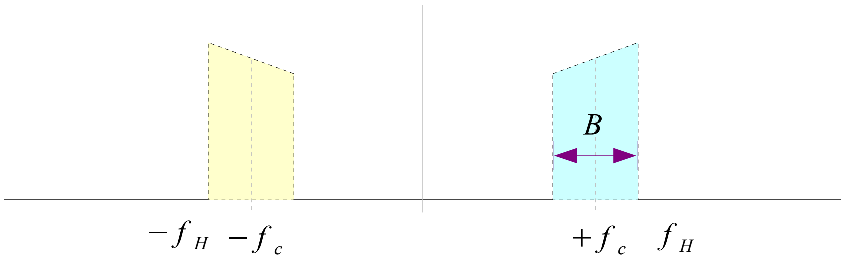


$$f_H = f_c + B/2 = 2B$$

$$f_H / B = R = 2$$

$$f_{s,min} = \frac{2 f_H}{k} = \frac{2 f_H}{m+1} = 2B$$

$k = 2 \quad (m = 1)$

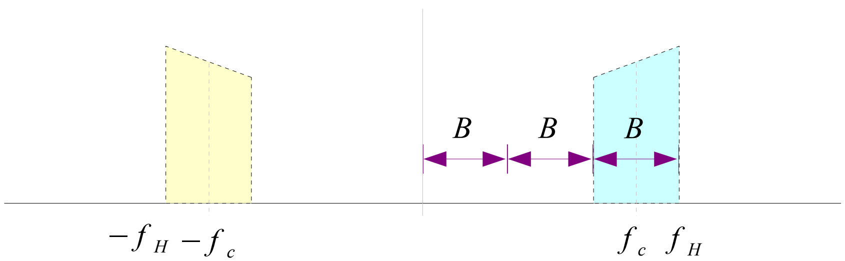


$$f_H = f_c + B/2 = 2.5B$$

$$f_H / B = R = 2.5$$

$$f_{s,min} = \frac{2 f_H}{k} = \frac{2 f_H}{m+1} = 2.5B$$

$k = 2 \quad (m = 3)$

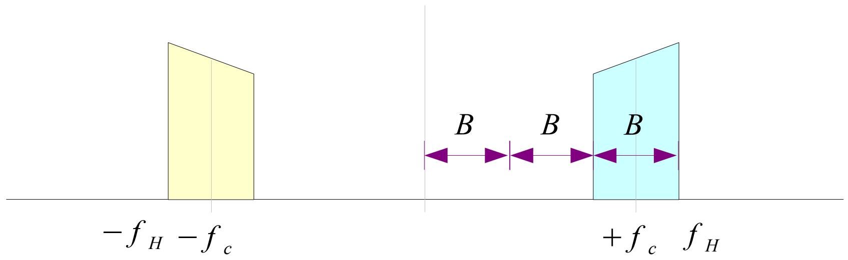


$$f_H = f_c + B/2 = 3B$$

$$f_H / B = R = 3$$

$$f_{s,min} = \frac{2 f_H}{k} = \frac{2 f_H}{m+1} = 3B$$

$k = 3 \ \ (m = 2)$

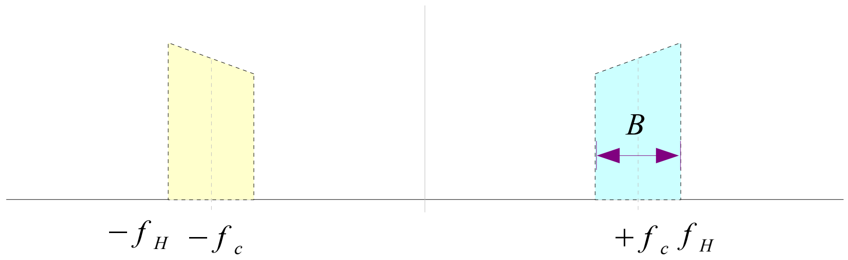


$$f_H = f_c + B/2 = 3B$$

$$f_H / B = R = 3$$

$$f_{s,min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = 2B$$

$k = 3 \ \ (m = 2)$

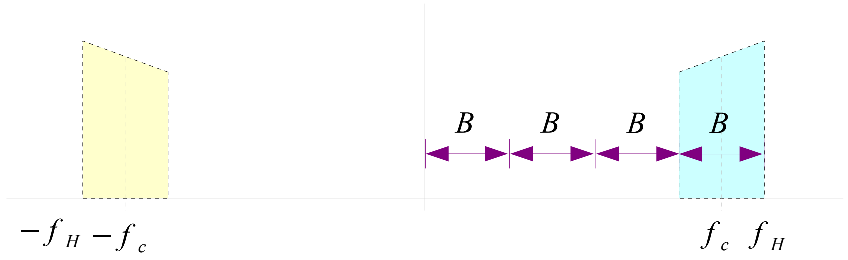


$$f_H = f_c + B/2 = 3.5B$$

$$f_H / B = R = 3.5$$

$$f_{s,min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = \frac{7}{3}B$$

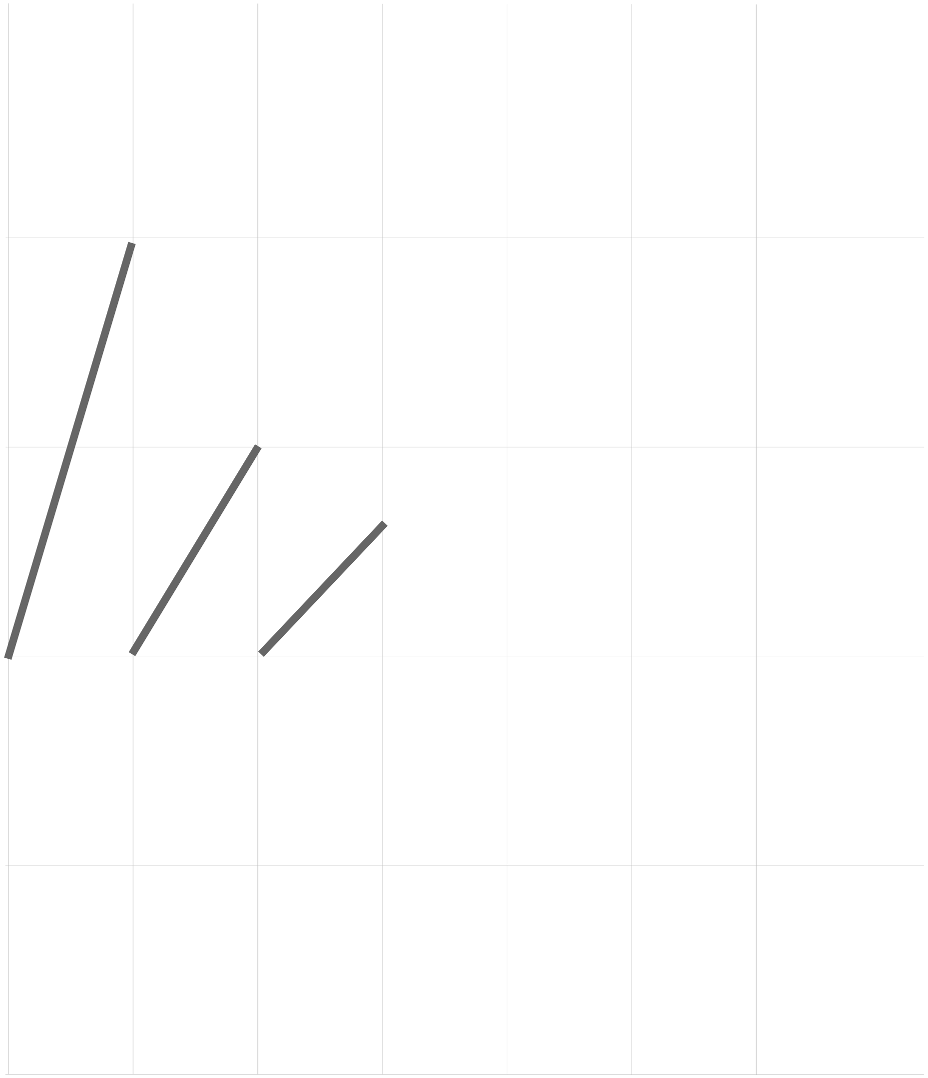
$k = 3 \ \ (m = 2)$



$$f_H = f_c + B/2 = 4B$$

$$f_H / B = R = 4$$

$$f_{s,min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = \frac{8}{3}B$$



References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A “graphical interpretation” of the DFT and FFT, by Steve Mann
[4] R. G. Lyons, Understanding Digital Signal Processing, 1997