General and Interval Type-2 Fuzzy Face-Space Approach to Emotion Recognition

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***Abstract*—Facial expressions of a person representing similar emotion are not always unique. Naturally, the facial features of a subject taken from different instances of the same emotion have wide variations. In the presence of two or more facial features, the variation of the attributes together makes the emotion recognition problem more complicated. This variation is the main source of uncertainty in the emotion recognition problem, which has been addressed here in two steps using type-2 fuzzy sets. First a type-2 fuzzy face space is constructed with the background knowledge of facial features of different subjects for different emotions. Second, the emotion of an unknown facial expression is determined based on the consensus of the measured facial features with the fuzzy face space. Both interval and general type-2 fuzzy sets (GT2FS) have been used separately to model the fuzzy face space. The interval type-2 fuzzy set (IT2FS) involves primary membership functions for m facial features obtained from n-subjects, each having l-instances of facial expressions for a given emotion. The GT2FS in addition to employing the primary membership functions mentioned above also involves the secondary memberships for individual primary membership curve, which has been obtained here by formulating and solving an optimization problem. The optimization problem here attempts to minimize the difference between two decoded signals: the first one being the type-1 defuzzification of the average primary membership functions obtained from the n-subjects, while the second one refers to the type-2 defuzzified signal for a given primary membership function with secondary memberships as unknown. The uncertainty management policy adopted using GT2FS has resulted in a classification accuracy of 98.333% in comparison to 91.667% obtained by its interval type-2 counterpart. A small improvement (approximately 2.5%) in classification accuracy by IT2FS has been attained by pre-processing measurements using the well-known interval approach.**

***Index Terms*—Emotion recognition, facial feature extraction, fuzzy face space, interval and general type-2 fuzzy sets, interval approach (IA).**

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I. Introduction

EMOTION recognition is currently gaining importance for its increasing scope of applications in human–computer interactive systems. Several modalities of emotion recognition, including facial expression, voice, gesture, and posture have been studied in the literature. However, irrespective of the modality, emotion recognition comprises two fundamental steps involving feature extraction and classification [36]. Feature extraction refers to determining a set of features/attributes, preferably independent, which together represents a given emotional expression. Classification aims at mapping emotional features into one of several emotion classes.

Performance of an emotion recognition system greatly depends on feature selection and classifier design. A good classification algorithm sometimes cannot yield high classification accuracy for poorly selected features. On the other hand, even using a large set of features, describing an emotion, we occasionally fail to recognize the emotion correctly because of a poor classifier. Most commonly used techniques for feature selection in the emotion recognition problem include principal component analysis (PCA) [59], independent component analysis [60], rough sets [42], [61], Gabor filter [62], and Fourier descriptors [25]. Among the popularly used techniques for emotion classification, neural net-based mapping [3], [4], [18], fuzzy relational approach [14], linear discriminate analysis [60], support vector machine (SVM) [8], and hidden Markov model [59], gegege[62] need special mention. A brief overview of the existing research on emotion recognition is given next.

Ekman and Friesen took an early attempt to recognize facial expression from the movements of cheek, chin, and wrinkles [24]. Their experiments confirmed the existence of a good correlation between basic movements of the facial action units [13], [19] and facial expressions [1], [2], [5], [7], [10], [19]–[22]. Kobayashi and Hara [15]–[17] designed a scheme for the recognition of human facial expressions using the well-known back-propagation neural networks [38], [43]. Their scheme is capable of recognizing six common facial expressions depicting happiness, sadness, fear, anger, surprise, and disgust. Yamada proposed an alternative method of emotion recognition through classification of visual information [49].

Fernandez-Dols *et al.* proposed a scheme for decoding emotions from facial expressions and content [50]. Kawakami *et al.* [43] designed a method for the construction of emotion space using neural networks. Busso and Narayanan [51] analyzed the scope of facial expressions, speech, and multi-modal information in emotion recognition. Metallinou *et al.* [71] employed content-sensitive learning for audio-visual emotion recognition. In [73], Metallinou *et al.* proposed a novel approach to visual emotion recognition using a compact representation of face and viseme information. In [74], Metallinou *et al.* presented an approach to decision level fusion for handling multi-modal information in emotion recognition. Lee *et al.* [75] employed a hierarchical binary tree for emotion recognition. Mower *et al.* designed an interesting scheme about human perception of audio-visual synthetic emotion character in the presence of conflicting information [76]. Cohen *et al.* [52] developed a scheme for emotion recognition from the temporal variations in facial expressions obtained from the live video sequence of the subjects. They used hidden Markov model to automatically segment and recognize facial expression. Gao *et al.* presented a scheme for facial expression recognition from a single facial image using line based caricatures [53]. Among other significant contributions in emotion recognition, the works presented in [6], [8], [9], [11], [12], [15]–[17], [23]–[28], [30], [31], [32], [35], [40], [46], [56], [57], [60], [70], [72], [77]–[80] need special mention. For a more complete literature survey, which cannot be given here for space restriction, readers may refer to two outstanding papers by Pantic *et al.* [57], [67].

Emotional features greatly depend on the psychological states of the subjects. For example, facial expressions of a subject, while experiencing the same emotion, have wider variations, resulting in significant changes in individual feature. Further, different subjects experiencing the same emotion have differences in their facial features. Repeated experiments with a large number of subjects, each having multiple instances of similar emotional experience, reveal that apparently there exists a small but random variation of facial features around specific fixed points [65]. The variation between different instances of facial expression for similar emotive experience of an individual can be regarded as an *intra-personal level uncertainty*[41]. On the other hand, the variation in facial expression of individuals for similar emotional experience can be treated as *inter-personal level uncertainty*[41].

The variations in features can be modeled with fuzzy sets. Classical (type-1 (T1)) fuzzy sets, pioneered by Zadeh [66], have widely been used over the last five decades for modeling uncertainty of ill-defined systems. T1 fuzzy sets employ a single membership function to represent the degree of uncertainty in measurements of a given feature. Hence, it can capture the variation in measurements of a given feature for different instances of a specific emotion experienced by a subject. In [14], the authors have considered a fixed membership function to model the uncertainty involved in a feature for a given emotion, disregarding the possibility of variation in the membership curves for different subjects.

This paper, however, models the above form of inter-personal level uncertainty by interval type-2 (T2) fuzzy sets (IT2FS). IT2FS employs an upper and a lower membership function (UMF and LMF) to capture the uncertainty involved in a given measurement of a feature within the bounds of its two membership curves at the point of the measurement. However, the degree of correct assignment of membership for each membership curve embedded between the UMF and LMF in IT2FS is treated as unity, which is not always appropriate. General T2 fuzzy set (GT2FS) can overcome the above problem by considering a secondary membership grade that represents the correctness in (primary) membership assignment at each measurement points. Naturally, GT2FS is expected to give us better results in emotion classification for its representational advantage over IT2FS.

One fundamental problem in GT2FS that limits its application in classification problems, perhaps, is due to users' inability to correctly specify the secondary memberships. In this paper, we determine the secondary memberships by extracting certain knowledge from the individual primary assignments for each feature of a given emotion for a subject. The knowledge extracted is encoded as an optimization problem with secondary memberships as unknown. The solution to the optimization problem carried out offline provides the secondary grades. The secondary grades are later aggregated with the primary memberships of individual feature for all subjects at the given measurement point to obtain modified primary memberships.

The paper provides two alternative approaches to emotion recognition from an unknown facial expression, when the emotion class of individual facial expression of a large number of experimental subjects is available. The first approach deals with IT2FS to construct a fuzzy face space based on the measurements of a set of features from a given set of facial expressions carrying different emotions. An unknown facial expression is classified into one of several emotion classes by determining the maximum support of individual emotion classes to a given set of measurements of a facial expression. The class having the maximum support is declared as the emotion of the unknown facial expression. In spirit, this is similar to how a fuzzy rule-based system for classification works.

The second approach employs GT2FS to construct a fuzzy face space, comprising both primary and secondary membership functions, obtained from known facial expressions of several subjects containing multiple instances of the same emotion for each subject. The emotion class of an unknown facial expression is determined by computing the support of each class to the given facial expression. The class with the maximum support is the winner. The maximum support evaluation here employs both primary and secondary memberships, and thus is slightly different than the IT2FS-based classification.

Experiments reveal that the classification accuracy of emotion of an unknown person by the GT2FS-based scheme is as high as 98%. When secondary memberships are ignored, and classification is performed with IT2FS, the classification accuracy falls by a margin of 7%. The additional 7% classification accuracy obtained by GT2FS, however, has to pay a price for additional complexity of $(m \times n \times k)$ multiplications, where m, n, and k denote the number of features, number of subjects, and number of emotion classes, respectively. A 2.5% improvement in classification accuracy by IT2FS has been attained by pre-processing measurements and selecting membership functions using the well-known interval approach (IA) [68].

The paper is divided into eight sections. Section II provides fundamental definitions associated with T2 fuzzy sets, which will be required in the rest of the paper. In Section III, we propose the principle of uncertainty management in fuzzy face space for emotion recognition. Section IV deals with secondary membership evaluation procedure for a given T2 primary membership function. A scheme for selection of membership function and data filtering to eliminate poor measurements to improve the performance of IT2FS-based recognition is given in Section V. Experimental details are given in Section VI, and two methods of performance analysis are undertaken in Section VII. Conclusions are listed in Section VIII.

II. Preliminaries on T2 Fuzzy Sets

In this section, we define some terminologies related to T1 and T2 fuzzy sets. These definitions will be used throughout the paper.

Definition 1: Given a universe of discourse X, a conventional *T1 fuzzy set* A defined on X, is given by a 2-D membership function, also called T1 membership function. The (primary) *membership function*, denoted by $\mu_A(x)$, is a crisp number in [0, 1] for a generic element $x \in X$. Usually, the fuzzy set A is expressed as a two tuple [36], given by

$$A = \{(x, \mu_A(x))\,|\forall x \in X\}. \tag{1}$$

An alternative representation of the fuzzy set A is also found in the literature as given in (2).

$$A = \int_{x\in X} \mu_A(x)|x \tag{2}$$

where $\int$ denotes union of all admissible x.

Definition 2: A *T2 fuzzy set* $\tilde{A}$ is characterized by a 3-D membership function, also called T2 membership function, which itself is fuzzy. The *T2 membership function* is usually denoted by $\mu_{\tilde{A}}(x, u)$, where $x \in X$, and $u \in J_x \subseteq [0, 1]$[39]. Usually, the fuzzy set $\tilde{A}$ is expressed as a two tuple:

$$\tilde{A} = \{((x, u), \mu_{\tilde{A}}(x, u))\,|x \in X, u \in J_x \subseteq [0, 1]\} \tag{3}$$

where $\mu_{\tilde{A}}(x, u) \in [0, 1]$. An alternative form of representation of the T2 fuzzy set is given in (4)

$$\tilde{A} = \int_{x\in X}\int_{u\in J_x} \mu_{\tilde{A}}(x, u)|(x, u), J_x \subseteq [0, 1] \tag{4}$$

$$= \int_{x\in X}\left[\frac{\int_{u\in J_x} f_x(u)}{u}\right]/x, J_x \subseteq [0, 1] \tag{5}$$

where $f_x(u) = \mu_{\tilde{A}}(x, u) \in [0, 1]$. The $\int\int$ denotes union over all admissible x and u [39].

Definition 3: At each point of x, say $x = x^{/}$, the 2-D plane containing axes u and $\mu(x^{/}, u)$ is called the *vertical slice* of $\mu_{\tilde{A}}(x, u)$. A *secondary membership function* is a vertical slice of $\mu_{\tilde{A}}(x, u)$. Symbolically, it is given by $\mu_{\tilde{A}}(x, u)$ at $x = x^{/}$ for $x^{/} \in X$ and $\forall u \in J_{x^{/}} \subseteq [0, 1]$

$$\mu_{\tilde{A}}(x = x', u) = \int_{u\in J_{x^{/}}} f_{x^{/}}(u)|u, J_{x^{/}} \subseteq [0, 1] \tag{6}$$

where $0 \le f_{x^{/}}(u) \le 1$. The amplitude of a secondary membership function is called secondary grade (of membership). In (6)$J_{x^{/}}$ is the primary membership of $x^{/}$.

Definition 4: Uncertainty in the primary membership of a T2 fuzzy set $\tilde{A}$ is represented by a bounded region, called *footprint of uncertainty* (FOU) [39], which is the defined as the union of all primary memberships, i.e.,

$$FOU(\tilde{A}) = \bigcup_{x\in U} J_x. \tag{7}$$

If all the secondary grades of a T2 fuzzy set $\tilde{A}$ are equal to 1, i.e.,

$$\mu_{\tilde{A}}(x, u) = 1 \forall x \in X, \forall u \in J_x \subseteq [0, 1] \tag{8}$$

then $\tilde{A}$ is called *IT2FS*. The FOU is bounded by two curves, called the *Lower* and the *Upper Membership functions*, denoted by $\underline{\mu}_{\tilde{A}}(x)$ and $\overline{\mu}_{\tilde{A}}(x)$, respectively, where $\underline{\mu}_{\tilde{A}}(x)$ and $\overline{\mu}_{\tilde{A}}(x)$ at all x, respectively, take up the minimum and the maximum of the membership functions of the embedded T1 fuzzy sets [38] in the FOU.

III. Uncertainty Management in Fuzzy Face Space for Emotion Recognition

This section provides a general overview of the proposed scheme for emotion recognition using T2 fuzzy sets. Here, the emotion recognition problem is considered as uncertainty management in fuzzy space after encoding the measured facial attributes by T2 fuzzy sets.

Let $F = \{f_1, f_2, \ldots, f_m\}$ be the set of m facial features. Let $\mu_{\tilde{A}}(f_i)$ be the primary membership in [0,1] of the feature f_i to be a member of set $\tilde{A}$, and $\mu(f_i, \mu_{\tilde{A}}(f_i))$ be the secondary membership of the measured variable f_i in [0,1]. A primary and secondary membership function corresponds to a particular emotion class c, are denoted by $\mu_{\tilde{A}c}(f_i)$ and $\mu(f_i, \mu_{\tilde{A}c}(f_i))$, respectively. If the measurement of a facial feature, f_i, is performed p times on the same subject experiencing the same emotion, and the measurements are quantized into q intervals of equal size, we can evaluate the frequency of occurrence of the measured variable f_i in q quantized intervals. The interval containing the highest frequency of occurrence then can be identified, and its center, m_i, approximately represents the mode of the measurement variable f_i. The second moment, σ_i, around m_i is determined and a bell-shaped (Gaussian) membership function centered at m_i and with a spread σ_i is used to represent the membership function of the random variable f_i. This function represents the membership of f_i to be CLOSE-TO the central value, m_i. It may be noted that a bell-shaped (Gaussian-like) membership curve would have a peak at the center with a membership value one, indicating that membership at this point is the largest for an obvious reason of having the highest frequency of f_i at the center.

On repetition of the above experiment for variable f_i on n subjects, each experiencing the same emotion, we obtain n such membership functions, each one for one individual subject. Naturally, the measurement variable f_i now has both intra- and inter-personal level uncertainty. The intra-level uncertainty

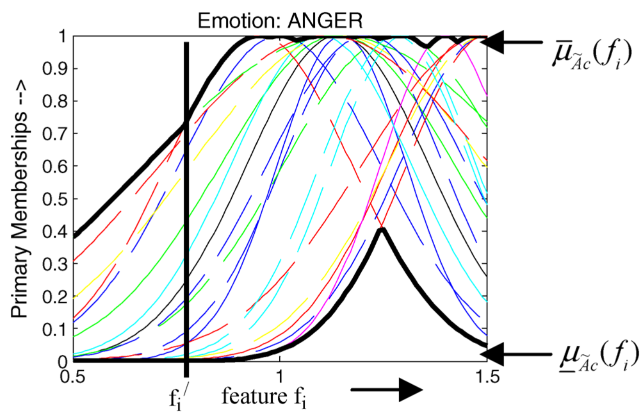


Fig. 1 Experimental FOU for feature f_{i} = Mouth-Opening.

occurs due to the pre-assumption of a specific (Gaussian) primary membership function, and the inter-level uncertainty occurs due to multiplicity of the membership functions for n subjects. Thus, a new measurement for an unknown facial expression can be encoded using all the n-membership curves, giving n possible membership values, thereby giving rise to uncertainty in the fuzzy space.

The uncertainty involved in the present problem has been addressed here by three distinctive approaches: 1) IT2FS, 2) IA-IT2FS, and 3) GT2FS. The first approach is simple, but more error prone as it ignores the intra-level uncertainty. The second and the third approaches are robust as they are capable to take care of both the uncertainties. However, the modality of uncertainty management by the second and the third approaches is significantly different. The second approach models each subject's interval using a uniform probability distribution, and thus the mean and variance of each interval are mapped into an embedded T1 fuzzy set. The third approach handles intra- and inter-personal level uncertainty compositely by fusing the primary and the secondary membership functions into an embedded interval T2 membership function. All three approaches have many common steps. Hence, we first present the steps involved in IT2FS and then explain the two techniques without repeating the common steps further.

A. *Principles Used in the IT2FS Approach*

The primary membership functions for a given feature value f_i corresponding to a particular emotion c taken from n-subjects together forms a IT2FS $\tilde{A}_c$, whose FOU is bounded by a lower and an upper membership curves $\underline{\mu}_{\tilde{A}c}(f_i)$ and $\overline{\mu}_{\tilde{A}c}(f_i)$, respectively, where

$$\underline{\mu}_{\tilde{A}c}(f_i) = Min\left\{\mu^1_{\tilde{A}c}(f_i), \mu^2_{\tilde{A}c}(f_i), \ldots, \mu^n_{\tilde{A}c}(f_i)\right\}, \quad (9)$$
$$\overline{\mu}_{\tilde{A}c}(f_i) = Max\left\{\mu^1_{\tilde{A}c}(f_i), \mu^2_{\tilde{A}c}(f_i), \ldots, \mu^n_{\tilde{A}c}(f_i)\right\} \quad (10)$$

are evaluated for all f_i, and $\mu^j_{\tilde{A}_c}(f_i), 1 \le j \le n$ denotes the primary membership function of feature f_i for subject j in IT2FS $\tilde{A}_c$.

Fig. 1 provides the FOU for a given feature f_i. Now, for a given measurement $f_i^{/}$, we obtain an interval $[\underline{\mu}_{\tilde{A}c}(f_i^{/}), \overline{\mu}_{\tilde{A}c}(f_i^{/})]$, representing the entire span of uncertainty of the measurement variable $f_i^{/}$ in the fuzzy space, induced by n primary membership distributions: $\mu^j_{\tilde{A}c}(f_i), 1 \le j \le n$. The interval $[\underline{\mu}_{\tilde{A}c}(f_i^{/}), \overline{\mu}_{\tilde{A}c}(f_i^{/})]$ is evaluated by replacing f_i by $f_i^{/}$ in (9) and (10), respectively.

If there exist m different facial features, then for each feature, we would have such an interval, and consequently we obtain m such intervals given by

$$\left[\underline{\mu}_{\tilde{A}c}\left(f_1^{/}\right), \overline{\mu}_{\tilde{A}c}\left(f_1^{/}\right)\right], \left[\underline{\mu}_{\tilde{A}c}\left(f_2^{/}\right), \overline{\mu}_{\tilde{A}c}\left(f_2^{/}\right)\right], \ldots\ldots\ldots \times \left[\underline{\mu}_{\tilde{A}c}\left(f_m^{/}\right), \overline{\mu}_{\tilde{A}c}\left(f_m^{/}\right)\right].$$

The proposed IT2FS reasoning system employs a particular format of rules, commonly used in fuzzy classification problems [47]. Consider for instance a fuzzy rule, given by R_c: if f_1 is $\tilde{A}_1$ AND f_2 is $\tilde{A}_2 \ldots$ AND f_m is $\tilde{A}_m$ then emotion class is c.

Here, f_i for $i = 1tom$ are m-measurements (feature values) and $\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_m$ are IT2FS on the respective domains

$$\tilde{A}_i = \left[\underline{\mu}_{\tilde{A}c}(f_i), \overline{\mu}_{\tilde{A}c}(f_i)\right], \forall i. \quad (11)$$

Since an emotion is characterized by all of these m features, to find the overall support of the m features (m measurements made for the unknown subject) to the emotion class c represented by the n primary memberships, we use the fuzzy meet operation

$$S_c^{\min} = Min\left\{\underline{\mu}_{\tilde{A}c}\left(f_1^{/}\right), \underline{\mu}_{\tilde{A}c}\left(f_2^{/}\right) \ldots, \underline{\mu}_{\tilde{A}c}\left(f_m^{/}\right)\right\} \quad (12)$$
$$S_c^{\max} = Min\left\{\overline{\mu}_{\tilde{A}c}\left(f_1^{/}\right), \overline{\mu}_{\tilde{A}c}\left(f_2^{/}\right), \ldots, \overline{\mu}_{\tilde{A}c}\left(f_m^{/}\right)\right\}. \quad (13)$$

Thus, we can say that the unknown subject is experiencing the emotion class c at least to the extent s_c^{min}, and at most to the extent s_c^{max}.

To reduce the nonspecificity associated with the interval $s_{c-i} = [s_c^{min}, s_c^{max}]$, different approaches can be taken. For example, the most conservative approach would be to use lower bound, while the most liberal view would be to use the upper bound of the interval as the support for the class c. In the absence of any additional information, a balanced approach would be to use center of the interval as the support for the class c by the n primary memberships to the unknown subject. This idea is supported by Mendel [42] and Lee [48]. We compute the center S_c of the interval S_{c-i}

$$S_c = \frac{\left(s_c^{min} + s_c^{max}\right)}{2}. \quad (14)$$

Thus, S_c is the degree of support that the unknown facial expression is in emotion class c. Now, to predict the emotion of a person from his facial expression, we determine S_c for each emotion class. Presuming that there exist k emotion classes, let us denote the degree by which the emotion classes $1, 2, \ldots, k$ support the unknown facial expression be $S_1, S_2, \ldots, S_k$, respectively. Since a given facial expression may convey different emotions with different degrees, we resolve the conflict by ranking the S_i for $i = 1 tok$, and thus determine the emotion class r, for which $S_r >= S_\mathrm{i}$ for all i.

The principle of selection of the emotion class r from a set of competitive emotions, satisfying the above inequality holds, since the joint occurrence of the fuzzy memberships, induced by (12)–(14), for all the features of the given facial expression for emotion r is the greatest among the same values for all other emotions.

B. Principles Used in the GT2FS Approach

The previous approach employs a reasoning mechanism to compute the degree of support of k emotion classes induced by m features for each class to an unknown facial expression using a set of $k \times m$ IT2FS. The GT2FS-based reasoning realized with measurements taken from n-subjects, however, requires $k \times m \times n$ GT2FSs to determine the emotion class of an unknown facial expression. The current approach tunes the primary membership values for the given measurements using the secondary memberships of the same measurement, and thus reduces the degree of intra-level uncertainty of the primary distributions. The reduction in the degree of uncertainty helps in improving the classification accuracy of emotion at the cost of additional complexity required to evaluate T2 secondary distributions and also to reason with $k \times m \times n$ fuzzy sets.

Let f_i be the measurement of the ith feature for a subject with an unknown emotion class. Now, by consulting the n primary membership functions that were generated from n-subjects in the training data for a given emotion class, c, we obtain n primary membership values corresponding to f_i for emotion class c as given by $\mu^1_{\tilde{A}c}(f_i), \mu^2_{\tilde{A}c}(f_i), \ldots, \mu^n_{\tilde{A}c}(f_i)$. Let the secondary membership values for each primary membership value, respectively, be $\mu(f_i, \mu^1_{\tilde{A}c}(f_i)), \mu(f_i, \mu^2_{\tilde{A}c}(f_i)), \ldots, \mu(f_i, \mu^n_{\tilde{A}c}(f_i))$. Note that, these secondary membership values correspond to emotion class c. Unless clarity demands, we have avoided (here and elsewhere) use of a subscript to represent the emotion class. We now fuse (aggregate) the evidences provided by the primary and secondary membership values to obtain the modified primary membership supports. A plausible way of fusing would be to use a T-norm. Here, we use the product. The product always lies within the FOU and thus satisfies Mendel-John Representation Theorem [39]. Further higher is the secondary membership, higher is the product representing new embedded fuzzy membership. Since the secondary membership represents the degree of correctness in primary membership, the product helps in reduction of intra-level uncertainty. Thus, for subject j of the training data representing emotion class c, we obtain

$$ {}^{\mathrm{mod}}\mu^j_{\tilde{A}c}(f_i) = \mu^j_{\tilde{A}c}(f_i) \times \mu\left(f_i, \mu^j_{\tilde{A}c}(f_i)\right) \forall j = 1, \ldots, n \qquad (15) $$

where ${}^{\mathrm{mod}}\mu^j_{\tilde{A}c}(f_i)$ denotes the modified primary membership value for jth training subject for cth emotion class. The secondary membership values used in the above product function are evaluated using their primary memberships obtained by a procedure discussed in Section IV.

The next step is to determine the range of ${}^{\mathrm{mod}}\mu^j_{\tilde{A}}(f_i^/)$ for $j = 1\ to\ n$, comprising the minimum and the maximum given by $[{}^{\mathrm{mod}}\underline{\mu}_{\tilde{A}}(f_i^/), {}^{\mathrm{mod}}\overline{\mu}_{\tilde{A}}(f_i^/)]$, where

$$ {}^{\mathrm{mod}}\underline{\mu}_{\tilde{A}}\left(f_i^/\right) = Min\left\{{}^{\mathrm{mod}}\mu^1_{\tilde{A}}\left(f_i^/\right), {}^{\mathrm{mod}}\mu^2_{\tilde{A}}\left(f_i^/\right), \ldots, {}^{\mathrm{mod}}\mu^n_{\tilde{A}}\left(f_i^/\right)\right\} \qquad (16) $$

$$ {}^{\mathrm{mod}}\overline{\mu}_{\tilde{A}}\left(f_i^/\right) = Max\left\{{}^{\mathrm{mod}}\mu^1_{\tilde{A}}\left(f_i^/\right), {}^{\mathrm{mod}}\mu^2_{\tilde{A}}\left(f_i^/\right), \ldots, {}^{\mathrm{mod}}\mu^n_{\tilde{A}}\left(f_i^/\right)\right\}. \qquad (17) $$

Now, for m features, the rule-based T2 classification is performed in a similar manner as in the previous section with the replacement of $\underline{\mu}_{\tilde{A}}(f_i^/)$ and $\overline{\mu}_{\tilde{A}}(f_i^/)$ by ${}^{\mathrm{mod}}\underline{\mu}_{\tilde{A}}(f_i^/)$ and ${}^{\mathrm{mod}}\overline{\mu}_{\tilde{A}}(f_i^/)$, respectively.

C. Methodology

We briefly discuss the main steps involved in fuzzy face-space construction based on the measurements of m facial features for n-subjects, each having l instances of facial expression for a particular emotion. We need to classify a facial expression of an unknown person into one of k emotion classes.

IT2FS-Based Emotion Recognition:

1) We extract m facial features for n subjects, each having l (l could be different for different emotion classes) instances of facial expression for a particular emotion. The above features are extracted for k-emotion classes.
2) We construct a fuzzy face space for each emotion class separately. The fuzzy face space for an emotion class comprises a set of n primary membership functions for each feature. Thus, we have m groups (denoted by m rows of blocks in Fig. 2) of n-primary membership functions (containing n blocks under each row of Fig. 2). Each primary membership curve is constructed from l-facial instances of a subject attempted to exhibit a particular emotion in her facial expression by acting.
3) For a given set of features $f_1^/, f_2^/, \ldots, f_m^/$ obtained from an unknown facial expression, we determine the range of membership for feature $f_i^/$, given by $[\underline{\mu}_{\tilde{A}}(f_i^/), \overline{\mu}_{\tilde{A}}(f_i^/)]$, where $\tilde{A}$ is an IT2FS with a primary membership function defined as CLOSE-TO-center-value-m of the respective membership function.
4) Now, for an emotion class j, we take fuzzy meet operation over the ranges for each feature to evaluate the range of uncertainty for individual emotion class. The meet operation here is computed by taking cumulative t-norm (here we use min) of $\underline{\mu}_{\tilde{A}}(f_i^/)$ and $\overline{\mu}_{\tilde{A}}(f_i^/)$ separately for $i = 1 tom$, and thus obtaining S_j^{min} and S_j^{max}, respectively (see top of Fig. 2).
5) The support of the j-th emotion class to the measurements is evaluated by computing the average S_j of $S_j^{\min}$ and $S_j^{\max}$.

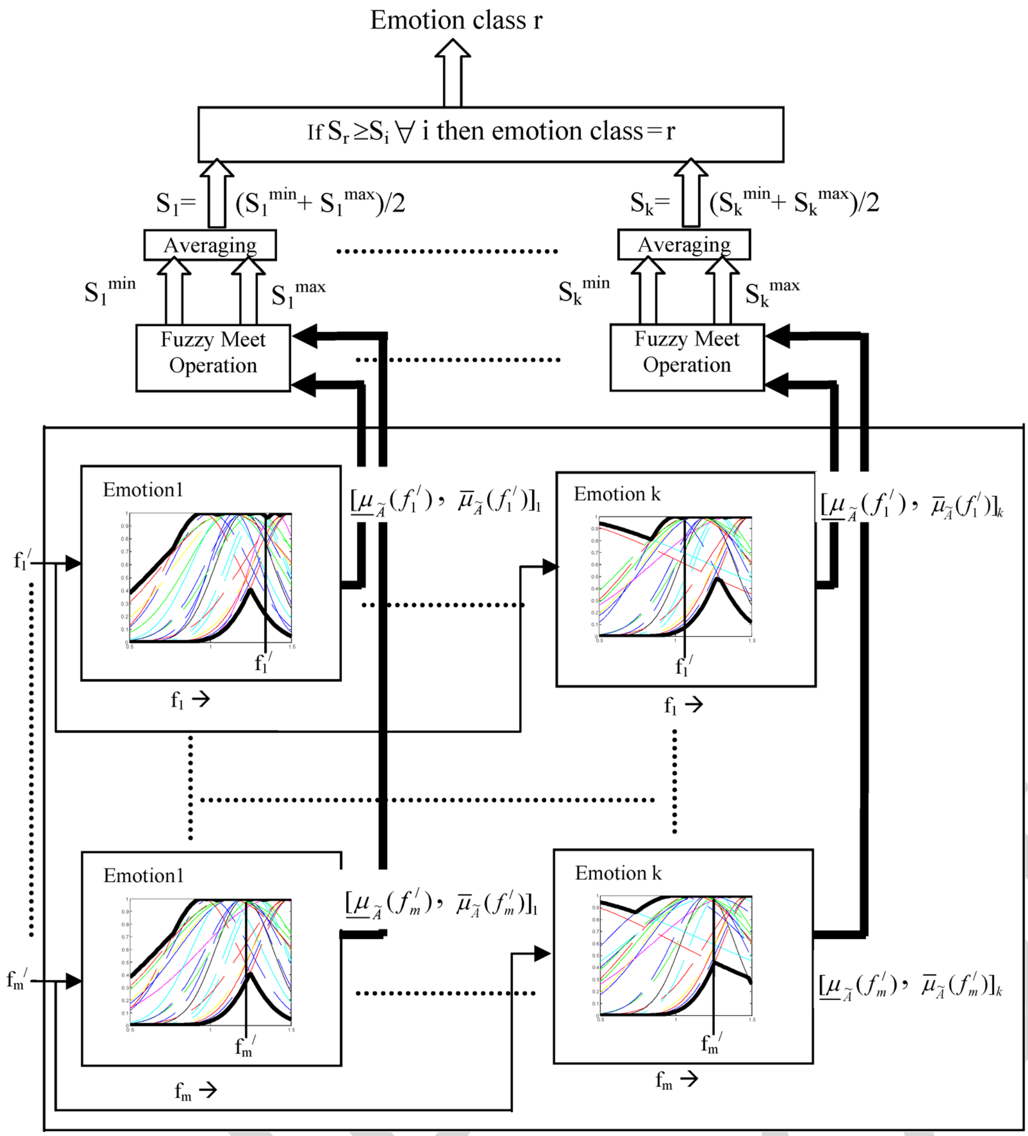


Fig. 2. The IT2FSS-based emotion recognition.

6) Now, we determine the maximum support offered by all the k emotion classes, and declare the unknown facial expression to have emotion r, if $S_r \geq S_i$ for all emotion class $i = 1$ to k. The suffix j in $[\mu_A^{\min}(f_i^{/}), \mu_A^{\max}(f_i^{/})]_j$ refers to the range in that interval for emotion j.

GT2FS-Based Emotion Recognition:

1) This step is same as the step 1 of IT2FS-based emotion recognition.
2) The construction of the primary membership functions here follows the same procedure as given in step 2 of IT2FS-based recognition scheme. In addition, we need to construct secondary membership functions for individual primary membership curves. The procedure for construction of secondary membership functions will be discussed in Section IV. The complete scheme of construction of T2FFS, considering all k emotion classes, is given in Fig. 3.
3) For a given feature $f_i^{/}$, we consult each primary and secondary membership curve under a given emotion class, and take the product of primary and secondary membership at $f_i = f_i^{/}$. The resulting membership value obtained for the membership curves for the subject w in the training data is given by

$$^{\text{mod}}\mu_{\tilde{A}}^{w}\left(f_i^{/}\right) = \mu_{\tilde{A}}^{w}\left(f_i^{/}\right) \times \mu\left(f_i^{/}, \mu_{\tilde{A}}^{w}\left(f_i^{/}\right)\right) \tag{18}$$

where the notations have their usual meaning. Now, for $w = 1 to n$, we evaluate $^{\text{mod}}\mu_{\tilde{A}}^{w}(f_i^{/})$, and thus obtain the minimum and the maximum values of $^{\text{mod}}\mu_{\tilde{A}}^{w}(f_i^{/})$, to obtain a range of uncertainty $[^{\text{mod}}\underline{\mu}_{\tilde{A}}(f_i^{/}), {}^{\text{mod}}\overline{\mu}_{\tilde{A}}(f_i^{/})]$. This is repeated for all features under each emotion class. In Fig. 4 we, unlike conventional approaches, present secondary membership functions against feature $f_i^{/}$, for $i = 1 to m$. Such representation is required to demonstrate the computation of $^{\text{mod}}\underline{\mu}_{\tilde{A}}(f_i^{/})$.
4) Step 4 is the same as that in IT2FS-based recognition scheme with the replacement of $\underline{\mu}_{\tilde{A}}(f_i^{/})$ and $\overline{\mu}_{\tilde{A}}(f_i^{/})$,

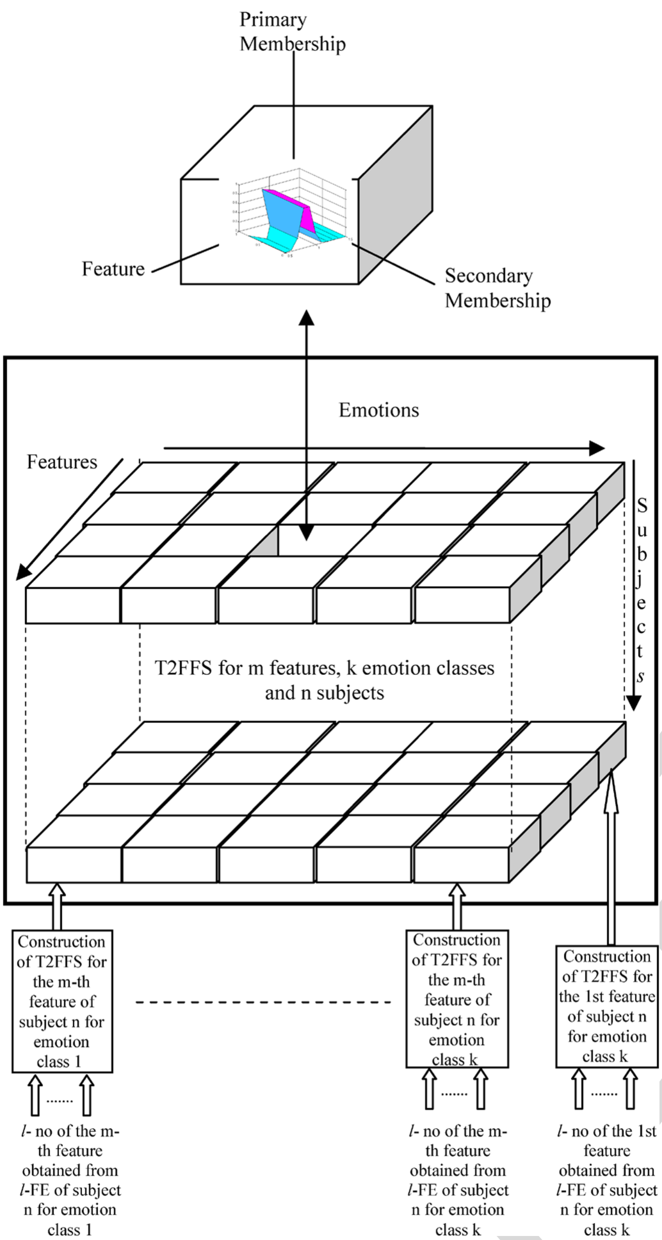


Fig. 3. General type-2 fuzzy face-space construction for m features, k emotion classes, and n subjects.

respectively, by ${}^{\text{mod}}\underline{\mu}_{\tilde{A}}(f_i^{/})$ and ${}^{\text{mod}}\overline{\mu}_{\tilde{A}}(f_i^{/})$. Steps 5 and 6 are exactly similar to those in IT2FS-based recognition scheme. A complete scheme for GT2FS-based emotion recognition, considering support of k-emotion classes is given in Fig. 5.

IV. Fuzzy T2 Membership Evaluation

In this, we discuss T2 membership evaluation [37]–[39]. Although theoretically very sound, T2 fuzzy set has limitedly been used over the last two decades because of the users' inadequate knowledge to correctly assign the secondary memberships. This paper, however, overcomes this problem by extracting T2 membership function from its T1 counterpart by an evolutionary algorithm. A brief outline to the construction of secondary membership function is given in this section.

Intuitively, when an expert assigns a grade of membership, she is relatively more certain to determine the location of the peaks and the minima of the function, but may not have enough background knowledge to correctly assign the membership values at other points. Presuming that the (secondary) membership values at the peak and the minima are close to 1, we attempt to compute secondary memberships at the remaining part of the secondary membership function. The following assumptions are used to construct an objective function, which is minimized to obtain the solution of the problem.

1) Let $x = x_p$ and $x = x_q$ be two successive optima (peak/minimum) on the primary membership function $\mu_A(x)$. Then, at any point x lying between x_p and x_q, the secondary membership $\mu(x, \mu_A(x))$ will be smaller than both $\mu(x_p, \mu_A(x_p))$ and $\mu(x_q, \mu_A(x_q))$.
2) The fall-off in secondary membership at a point x away from its value at a peak/minimum $\mu(x_p, \mu_A(x_p))$ is exponential, given by

$$\mu\left(x, \mu_A(x)\right) = \mu\left(x_p, \mu_A(x_p)\right).\exp\left(-|x - x_p|\right). \tag{19}$$

3) The secondary membership at any point x between two consecutive optima at $x = x_p$ and $x = x_q$ in the primary membership is selected from the range $[\alpha, \beta]$, where

$$\left.\begin{array}{l}\alpha = \mu\left(x_p, \mu_A(x_p)\right).\exp\left(-|x - x_p|\right)\\ \beta = \mu\left(x_q, \mu_A(x_q)\right).\exp\left(-|x - x_q|\right)\end{array}\right\}. \tag{20}$$

T1 defuzzification over the average of n primary membership functions should return the same value as obtained by T2 defuzzification for a given primary membership function for any given source. This assumption holds because the two modalities of defuzzification, representing the same real-world parameter, should return close values, ignoring the average inter-personal level of uncertainty while taking the average of n-primary membership functions.
4) The unknown secondary membership at two values of x separated by a small positive δ should have a small difference. This is required to avoid sharp changes in the secondary grade.

Let the primary membership functions for feature $f_i = x$ from n sources be $\mu^1_{\tilde{A}}(x), \mu^2_{\tilde{A}}(x), \ldots, \mu^n_{\tilde{A}}(x)$. Then, the average membership function which represents a special form of fuzzy aggregation is given by

$$\mu_{\tilde{\mathrm{A}}}(x) = \frac{\sum_{\mathrm{i}=1}^{\mathrm{n}} \mu^{\mathrm{i}}_{\tilde{\mathrm{A}}}(x)}{n}, \forall x \tag{21}$$

i.e., at each position of $x = x_j$, the above membership aggregation is employed to evaluate a new composite membership profile $\mu_{\tilde{A}}(x)$. The defuzzified signal obtained by the centroid method [36] from the averaged primary membership function is given by

$$\bar{\bar{c}} = \frac{\sum_{\forall \mathrm{x}} x.\mu_{\tilde{\mathrm{A}}}(x)}{\sum_{\forall \mathrm{x}} \mu_{\tilde{\mathrm{A}}}(x)}. \tag{22}$$

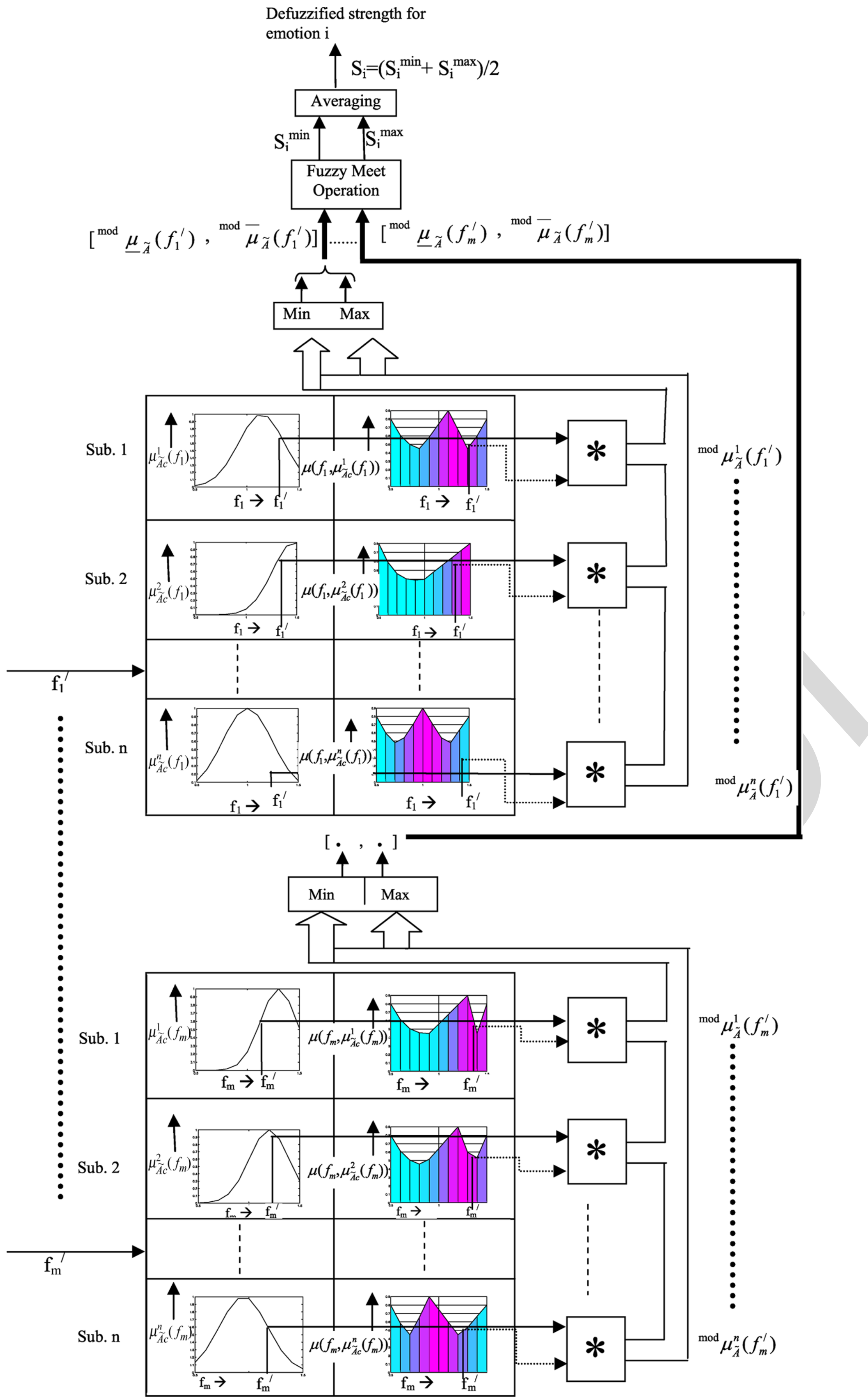


Fig. 4. Computing support of the general type-2 fuzzy FS for emotion class i.

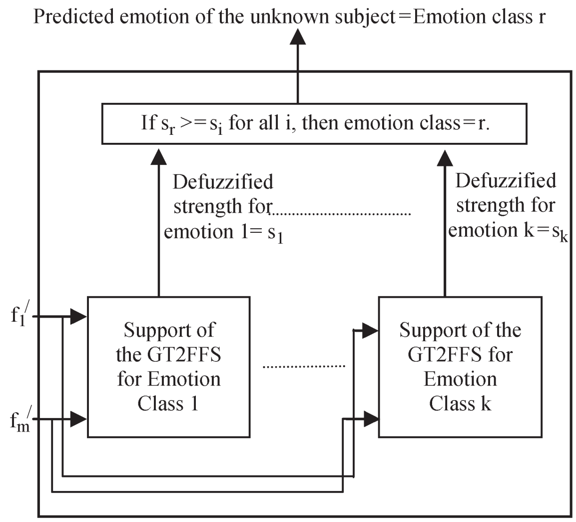


Fig. 5. GT2FFS-based emotion classification.

Further, the T2 centroidal defuzzified signal obtained from the ith primary and secondary membership functions here is defined as

$$\overline{c_i} = \frac{\sum_{\forall x} x.\mu^{\mathrm{i}}_{\tilde{A}}(x).\mu\left(x, \mu^{\mathrm{i}}_{\tilde{A}}(x)\right)}{\sum_{\forall x} \mu^{\mathrm{i}}_{\tilde{A}}(x).\mu\left(x, \mu^{\mathrm{i}}_{\tilde{A}}(x)\right)}. \tag{23}$$

The products of primary and secondary memberships are used in (23) to refine the primary memberships by the degree of certainty of the corresponding secondary values.

Using assumptions 3 and 4, we construct a performance index J_{i} to compute secondary membership for the ith subject for a given emotion

$$J_i = (\overline{c_i} - \overline{\overline{c}})^2 + \sum_{x=x_1}^{x_{R-1}} \left\{\mu\left((x+\delta), \mu^i_{\tilde{A}}(x+\delta)\right) - \mu\left(x, \mu^i_{\tilde{A}}(x)\right)\right\}^2. \tag{24}$$

The second term in (24) acts as a regularizing term to prevent abrupt changes in the membership function. In (24),x_1 and x_R are the smallest and the largest values of a given feature considered over R sampled points of $\mu^i_{\tilde{A}}(x)$. In (24), $\delta = (x_R - x_1)/(R-1)$ and $x_k = x_1 + (k-1).\delta$ for $k = 1, \ldots, R$. The secondary membership evaluation problem now transforms to minimization of J_i by selecting $\mu(x, \mu^i_{\tilde{A}}(x))$ from a given range $[\alpha, \beta]$, where α and β are the secondary memberships at the two optima in secondary membership around the point x. Expressions (20) are used to compute α and β for each x separately. Note that, for each subject carrying individual emotion, we have to define (23) and (24) and find the optimal secondary membership functions.

Any derivative-free optimization algorithm can be used to minimize J_i with respect to secondary memberships, and obtain $\mu(x, \mu^i_{\tilde{A}}(x))$ at each x except the optima on the secondary membership. Differential evolution (DE) [34] is one such derivative-free optimization algorithm, which has fewer control parameters, and has outperformed the well-known binary coded genetic algorithm [54] and particle swarm optimization algorithms [55] with respect to standard benchmark functions [45]. Further, DE is simple and involves only a few lines code, which motivated us to employ it to solve the above optimization problem.

An outline to basic DE [34] is given in the Appendix. An algorithm to compute the secondary membership function of a T2 fuzzy set from its primary counterpart using DE is given below.

1) Obtain the averaged primary membership function $\mu_{\tilde{A}}(x)$ from the primary membership functions $\mu^i_{\tilde{A}}(x)$ obtained from n sources, i.e., $i = 1, \ldots, n$. Evaluate $\overline{\overline{c}}$, and also $\overline{c_i}$ for a selected primary membership distribution $\mu^i_{\tilde{A}}(x)$ using (22) and (23), respectively.
2) Find the optima on $\mu^j_{\tilde{A}}(x)$ for a given j. Let the set of x corresponding to the optima be S. Set the secondary membership $\mu(x, \mu^j_{\tilde{A}}(x))$ to 0.99 (close to one) for all $x \in S$.
3) For each $x \in X$, where $x \notin S$, identify the optima closest around x from S. Let they be located at $x = x_p$ and $x = x_q$, where $x_p < x < x_q$. Determine α and β for each x, given by (20).
4) For each x, where $\mu(x, \mu^j_{\tilde{A}}(x))$ lies in $[\alpha, \beta]$, minimize J_{j} by DE.
5) Obtain $\mu(x, \mu^j_{\tilde{A}}(x))$ for all x after the DE converges.
6) Repeat step 2 onwards for all j.

For a Gaussian primary membership function, the minimum occurs at infinity, but the minimum value is practically zero when x is $m \pm 4\sigma$, where m and σ are mean and standard deviation of x. In Step 2, the minimum is taken as $m \pm 4\sigma$, and we obtain x by dividing the range $[m - 4\sigma, m + 4\sigma]$ into equal intervals of same length (here 20 intervals).

An illustrative plot of secondary membership function for a given primary is given in Fig. 6.

V. Filtering Unwanted Data Points in Feature Space Using Interval Approach

The IT2FS-based scheme for emotion recognition given in Section III is computationally efficient with good classification accuracy. However, its performance depends greatly on the measurements obtained from facial expressions of the experimental subjects. In order to reduce the effect of outliers, we here present a scheme of data pre-processing/filtering and selection of membership functions following the well-known IA [68].

The important steps of IA used in the present context are re-structured for the present application as outlined below. Let $[a^{(i)}, b^{(i)}]$ be the end-point interval of measurements of a given facial feature for the ith subject obtained from l instances of her facial expressions for a specific emotion.

Step 1) (Outlier processing): This step divides the two sets of lower and upper data end-points: $a^{(i)}$ and $b^{(i)}$, respectively, for $i = 1$ to n subjects in quartiles, and tests the acceptability of each data end-point by satisfying the following criteria:

$$\left.\begin{array}{l} a^{(i)} \in [Q_a(0.25) - 1.5IQR_a, Q_a(0.75) + 1.5IQR_a] \\ b^{(i)} \in [Q_b(0.25) - 1.5IQR_b, Q_b(0.75) + 1.5IQR_b] \\ L^{(i)} \in [Q_L(0.25) - 1.5IQR_L, Q_L(0.75) + 1.5IQR_L] \end{array}\right\} \tag{25}$$

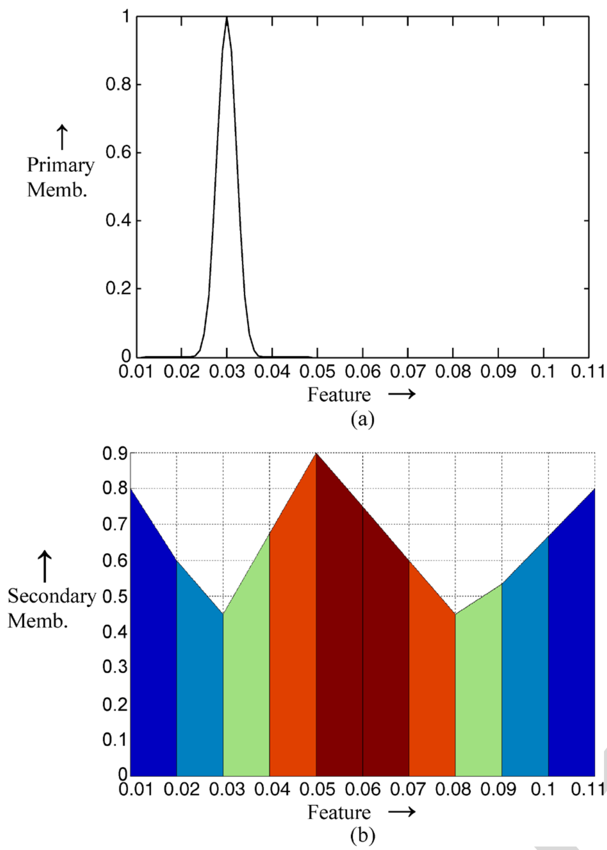


Fig. 6. (a) The primary membership function for a given feature and (b) its corresponding secondary membership function obtained by minimizing J_i.

where $Q_j(x)$ denotes the quartile ranges containing the first $x\%$ of the data points in the *i-th* data set. Here, $j \in \{a, b, L\}$ and a, b denote lower, upper end points of intervals, and L is the length of an interval. IQR denotes intra-quartile range and is defined by $Q(0.75)$ minus $Q(0.25)$. The suffixes a, b and L in IQR denote the IQR for left, right end points and interval length, respectively. $L^{(i)}$ is defined as the length of data interval $= b^{(i)} - a^{(i)}$, for $i = 1ton$. The reduced set of data end-points after outlier processing is $n^{/}$.

Step 2) (Tolerance limit processing): This step deals with tolerance limit processing by presuming the data distributions to be Gaussian, and testing whether lower/upper data end-points: $a^{(i)}, b^{(i)}$ and interval length $L^{(i)}$ lie within mean plus/minus $k(= 2.752)$ times the standard deviation of the data points. The number 2.752 appears in the scenario for statistical validation with 20 data end-point intervals for 20 subjects [68].

If a data interval $[a^{(i)}, b^{(i)}]$ and its length $L^{(i)}$ satisfy (26), the interval is accepted, otherwise rejected:

$$\left.\begin{array}{l} a^{(i)} \in [m_l - ks_l, m_l + ks_l] \\ b^{(i)} \in [m_r - ks_r, m_r + ks_r] \\ L^{(i)} \in [m_L - ks_L, m_L + ks_L] \end{array}\right\} \tag{26}$$

where, m_j and s_j denotes sample mean and standard deviation for $j \in \{l, r, L\}$, for the $n^{/}$ set of data points/intervals. After tolerance processing, the reduced set of data end-points is $n^{//}$.

Step 3) (*Reasonable-interval test*): This step checks whether intervals are reasonable, i.e., they are overlapped. This has been performed by computing ξ^*, given in (27) and then by testing whether lower bounds of each interval $a^{(i)} < \xi^*$ and upper bound $b^{(i)} > \xi^*$, where $\xi^*$is one of the possible values of

$$\xi^* = \frac{(m_r\sigma_l^2 - m_l\sigma_r^2) \pm \sigma_l\sigma_r\left[(m_l - m_r)^2 + 2\left(\sigma_l^2 - \sigma_r^2\right)\ln\left(\frac{\sigma_l}{\sigma_r}\right)\right]^{\frac{1}{2}}}{\sigma_l^2 - \sigma_r^2} \tag{27}$$

where m_l and σ_l are sample mean and variance of the $n^{//}$ left endpoints and m_r and σ_r are sample mean and variance of the $n^{//}$ right endpoints. If $m_l <= \xi^* <= m_r$ is satisfied, then the data intervals are retained and dropped otherwise. The remaining number of data points after the drop of some intervals is called $n^{///}$.

Step 4) (FOU selection): This step is used for the selection of the right FOU among triangle, left shoulder, and right shoulder. For each FOU, the criteria can be found in [68]. We here reproduce the results for triangular FOU only, as our results to be given in Section VI yields triangular FOU. For triangular FOU, the conditions are

$$\left.\begin{array}{l} m_r \leq 5.831m_l - 1.328\frac{s_c}{\sqrt{n^{///}}} \\ m_r \leq 0.171m_l + 8.29 - 1.328\frac{s_d}{\sqrt{n^{///}}} \\ m_r \geq m_l \end{array}\right\} \tag{28}$$

where s_c = standard deviation of $[b^{(i)} - 5.831a^{(i)}]$ for $i = 1ton^{///}$, s_d = standard deviation of $[b^{(i)} - 0.17a^{(i)} - 8.29]$ for $i = 1ton^{///}$.

Step 5) (FOU parameter evaluation): This step deals with parameter evaluation of the triangular membership functions for the existing data intervals $[a^{(i)}, b^{(i)}]$. For each interval $[a^{(i)}, b^{(i)}]$, we obtain the parameters $a_{MF}^{(i)}$ and $b_{MF}^{(i)}$ representing the end-points of the x-coordinates of the base for a symmetric triangular membership function as reproduced below [68]:

$$\left.\begin{array}{l} a_{MF}^{(i)} = \frac{1}{2}\left[\left(a^{(i)} + b^{(i)}\right) - \sqrt{2}\left(b^{(i)} - a^{(i)}\right)\right]. \\ b_{MF}^{(i)} = \frac{1}{2}\left[\left(a^{(i)} + b^{(i)}\right) + \sqrt{2}\left(b^{(i)} - a^{(i)}\right)\right] \end{array}\right\}. \tag{29}$$

We use these membership functions in place of Gaussian membership functions in our IT2FS approach and call this approach as IT-IT2FS.

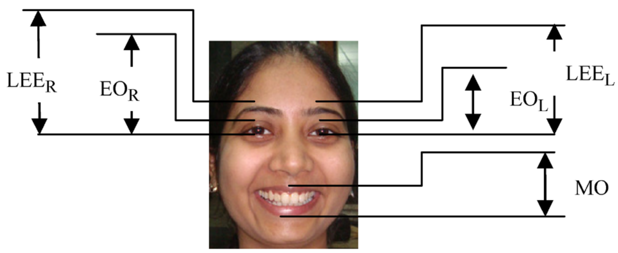
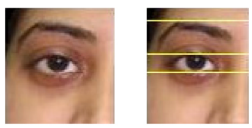


Fig. 7. Facial features.



(a) (b)

Fig. 8. (a) Localized eye search region, and (b) detection of eye features.

(a) (b) (c)

Fig. 9. (a) Mouth search area, (b) lip cluster, and (c) graph of average intensity over each row against the row position.

VI. Experiments Details

In this section, we present the experimental details of emotion recognition using the principles introduced in Sections III–V. Here, we consider the following $k(=5)$ emotion classes: anger, fear, disgust, happiness, and relaxation. The experiment is conducted with two sets of subjects: 1) the first set of $n(=20)$ subjects is considered for designing the fuzzy face space and 2) the other set of 40 facial expressions taken from six unknown subjects is considered to validate the result of the proposed emotion classification scheme. Five facial features (i.e., $m = 5$) have been used here to design the T2 fuzzy face space.

We now briefly overview the main steps of feature extraction followed by fuzzy face-space construction and emotion recognition of an unknown subject using the pre-constructed face space.

A. Feature Extraction

Feature extraction is a fundamental step in emotion recognition. This paper considers extraction of features from emotionally rich facial expressions synthesized by the subjects by acting. Existing research results [14], [28] reveal that the most important facial regions responsible for the manifestation of emotion are the eyes and the lips. This motivated us to select the following features: Left eye opening ($\mathrm{EO_L}$), right eye opening ($\mathrm{EO_R}$), Distance between the Lower Eyelid to the Eyebrow for the Left Eye ($\mathrm{LEE_L}$), distance between the lower eyelid to eyebrow for the right eye ($\mathrm{LEE_R}$), and the maximum mouth opening (MO) including the lower and the upper lips. Fig. 7 explains the above facial features on a selected facial image.

For extraction of any of the features mentioned above, the first step that needs to be carried out is to separate out the skin and the non-skin regions of the image.

Estimation of Eye Features ($\mathrm{EO_L}$, $\mathrm{LEE_L}$, $\mathrm{EO_R}$, and $\mathrm{LEE_R}$): To compute the eye features, we first localize the eye region as shown in Fig. 8(a). The image in Fig. 8(a) is now transformed to gray scale, and average intensity over each row of pixels is evaluated. Now, we identify the row with the maximum dip in average intensity, while scanning the image from top. This row indicates the first dark region from top, i.e., the eyebrow region (Fig. 8(b)). Similarly, we detect the lower eyelid by identifying the row with the maximum dip in intensity in the gray scale image, while scanning the face up from the bottommost row. The location of the top eyelid region is identified by scanning the face up from the marked lower eyelid until the maximum dip occurs in the gray scale image.

Estimation of MO: In order to estimate the MO, we first localize the mouth region as shown in Fig. 9(a). Then, a conversion from R-G-B to perceptually uniform $L^* - a^* - b^*$ color space is undertaken in order to represent the perceptual difference in color by Euclidean distance [69]. The k-means clustering algorithm is applied next on this image to segment the image into three clusters, namely skin, lip, and teeth regions. Each cluster is now transformed to gray scale, and the one with the highest average gradient of the boundary points (in intensity) is declared as the lip region. Now, to obtain the MO, we plot the average intensity over each row of Fig. 9(b) against the row number. The width of the zero-crossing zone in the plot (Fig. 9(c)) provides a measure of MO.

Experiments are undertaken both on colored image database such as the Indian Women (Jadavpur University) database, and gray scale images including Japanese Female Facial Expression (JAFFE) and Cohn-Kanade databases. The principles of feature extraction introduced above are equally applicable in both color and gray scale images. However, for color images, we need a conversion to gray scale to determine the features of eye and mouth of the subject. In addition, for the gray scale facial images, segmentation of lip-, skin-, and teeth-regions is performed with intensity data only, unlike the case in color images, where we use the 3-D data points (L^*, a^*, b^*) as the input to the k-means algorithm for segmentation.

Selective images from three facial expression databases are given in Fig. 10. Training and test image data partition for three experimental databases is given in Table I. The training data in Table I include l instances for n subjects for k distinct emotions.

The following explanation in this section is given with respect to Indian Woman Database (Jadavpur University).

B. Creating the T2 Fuzzy Face Space

The interval T2 fuzzy face space contains only the primary membership distributions for each facial feature. Since we have five facial features, and the experiment includes five distinct emotions of 20 subjects, we obtain $20 \times 5 \times 5 = 500$ primary membership curves. To compute primary memberships, ten instances of a given emotion are used. These 500 membership curves are grouped into 25 heads, each containing

Fig. 10. Experiment done on different databases: a) JAFFE, b) Indian women database (Jadavpur University), c) Cohn-Kanade database.

TABLE I
TRAINING AND TEST DATA FOR THREE DATABASES

Databases used	Training Images ($n \times l \times k$)	Test Images selected at random
JAFFE	5×3×5	40
Indian Woman(J.U)	20×10×5	40
Cohn-Kanade	10×5×5	40

20 membership curves of 20 subjects for a specific feature for a given emotion. Fig. 11 gives an illustration of one such group of 20 membership functions for the feature EO_L for the emotion: Anger.

For each primary membership function, we have a corresponding secondary membership function. Thus, we obtain 500 secondary membership functions. Two illustrative T2 secondary memberships for subjects 1 and 2 for the feature EO_L for the emotion anger are given in Fig. 12. The axes in the figure represent feature (EO_L), primary and secondary membership values as indicated.

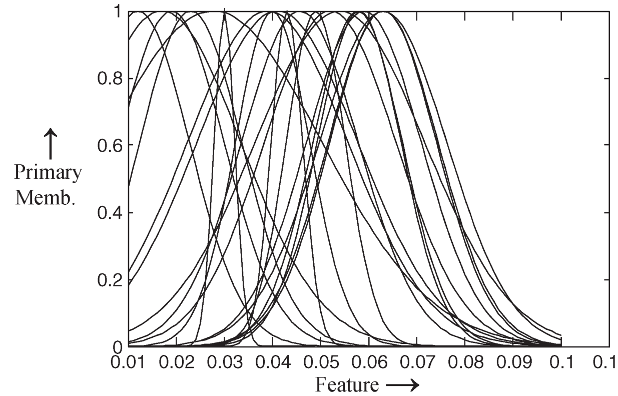


Fig. 11. Membership distributions for emotion anger and feature EO_L.

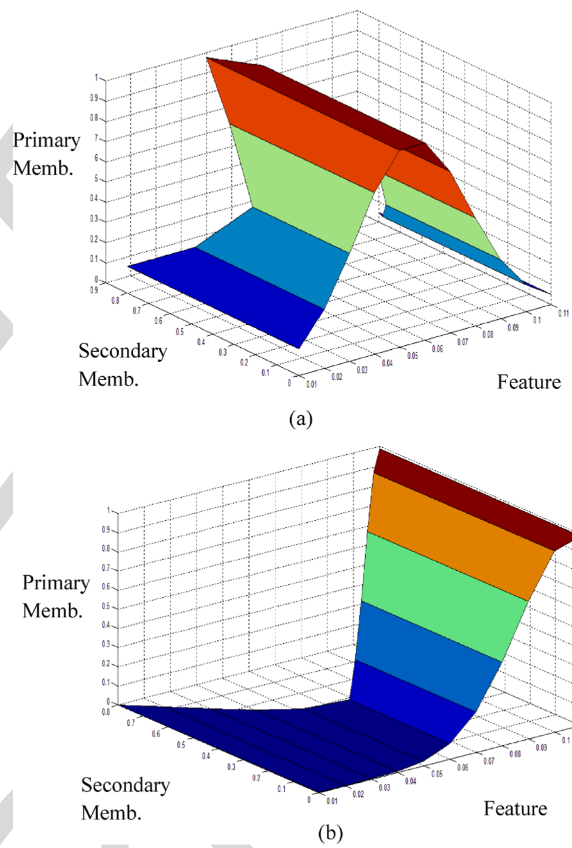


Fig. 12. (a) Secondary membership curve of subject 1. (b) Secondary membership curve of subject 2 for emotion anger.

C. Emotion Recognition of an Unknown Facial Expression

The emotion recognition problem addressed here attempts to determine the emotion of an unknown person from her facial expression. To keep the measurements in an emotional expression normalized and free from distance variation from the camera focal plane, we construct a bounding box, covering only the face region, and the reciprocal of the diagonal of the bounding box is used as a scale factor for normalization of the measurements. The normalized features obtained from Fig. 13 are listed in Table II. We now briefly explain the experimental results obtained by two alternative reasoning methodologies incorporating IT2FS and GT2FS.

Fig. 13. Facial image of an unknown subject.

TABLE II
EXTRACTED FEATURES OF FIG. 13

EO_L	EO_R	MO	LEE_L	LEE_R
0.026	0.026	0.135	0.115	0.115

TABLE III
CALCULATED TYPE-2 PRIMARY MEMBERSHIP VALUES FOR THE FEATURE: EO_L UNDER EMOTION: DISGUST

Feature: EO_L(pri)									
Primary Memberships (μ_{pri})									
0.65	0.10	0.15	0.45	0.18	0.55	0.06	0.41	0.16	0.12
0.38	0.45	0.09	0.19	0.67	0.68	0.52	0.44	0.37	0.55
Range (min{ μ_{pri}}, max{ μ_{pri}})= [0.06, 0.68]									

TABLE IV
CALCULATED RANGES OF PRIMARY MEMBERSHIPS AND CENTRE VALUE FOR EACH EMOTION

Emotion	Range of Primary Membership for Features					Range S_c^j after fuzzy Meet operation **(centre)**
	EO_L	EO_R	MO	LEE_L	LEE_R	
Anger	0-0.18	0-0.24	0.076 -0.764	0-0.215	0.001-0.234	0-0.18 (0.09)
Disgust	0.06-0.68	0.064-0.65	0-0.52	0-0.58	0-0.78	0-0.52 (0.26)
Fear	0 - 0.067	0-0.071	0.194-0.914	0.042-0.74	0.038-0.729	0-0.067 (0.0335)
Happiness	0 - 0.687	0-0.694	0.12-0.897	0.57-0.85	0.64-0.89	0-0.687 **(0.3435)**
Relaxed	0 - 0.384	0-0.393	0-0.052	0.076-0.89	0.081-0.92	0-0.052 (0.026)

IT2FS-Based Recognition: The IT2FS-based recognition scheme considers a fuzzy face space of five sets of 20 primary membership functions as in Fig. 11, where each set refers to one particular feature obtained from 20 sources for an individual emotion. Consequently, for five distinct emotions, we have 25 such sets of primary membership functions. Table III provides the evaluation of T2 primary membership values for the feature, EO_L, consulting 20 primary functions obtained from 20 subjects, representing the facial expression for disgust. The range of these memberships is given in the last row of Table III. For each feature, we obtain five tables like Table III, each one for a given emotion. Thus, for five features, we would have altogether 25 such tables.

Table IV provides the results of individual range in primary membership for each feature experimented under different emotional conditions. For example, the entry (0–0.18) corresponding to the row anger and column EO_L gives an idea about the extent of the EO_L for the unknown subject matches with known subjects from the emotion class anger. The results of computing fuzzy meet operation over the range of individual features taken from facial expressions of the subjects under the same emotional condition are given in Table IV. The average of the ranges along with its center value is also given in Table IV. It is observed that the center has the largest value $(= 0.3435)$ for the emotion: happiness.

TABLE V
RESULTS OF EXECUTION OF IA ON FEATURE EO_L DATA SET FOR EMOTION: ANGER

Data Preprocessing
Data points taken: 20 pairs of $a^{(i)}$, $b^{(i)}$ for i=1 to 20 subjects
Step-1: Outlier Processing Result: deleted point is [0.021, 0.113]
Step- 2: Tolerance Limit Processing Result: no deletion
Step- 3: Reasonable- interval Rest Result: no deletion
FOU Selection:
Step- 4: Computed values for: S_c= 0.0934; S_d= 0.0179; Test Condition (given in Fig. 14) Result: FOU = Triangle as (m_l, m_r) = (0.0755, 0.12257) lies in the interior FOU (triangle) obtained by satisfying (28)
Triangle Parameter Evaluation
Step- 5: a_{MF}, b_{MF} evaluated from (29) Result: Given in Fig. 15.

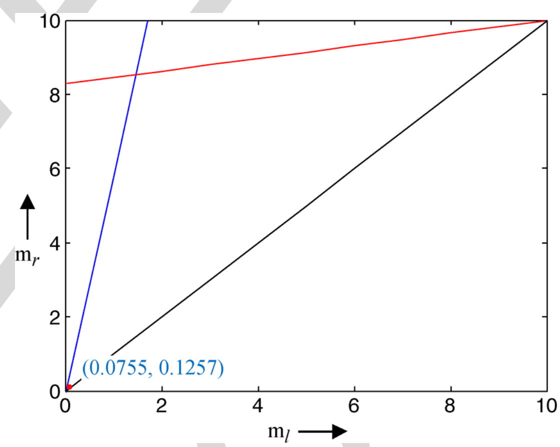


Fig. 14. Graphical selection of FOU by testing that the point $(m_l, m_r) = (0.0755, 0.1257)$ plotted in the figure lies under the triangular zone obtained by satisfying inequalities in (28).

IT2FS-Based Recognition With Pre-Processing of Features Using the Interval Approach (Hereafter IA-IT2FS): The IA introduced in Section V has two fundamental merits. It eliminates noisy data points obtained from facial data of the subjects. It also helps in identifying the primary membership functions for each feature of a facial expression representing a specific emotion by a statistically meaningful approach. The results of execution of adapted IA algorithm given in the last section for the feature EO_L for the emotion anger are given in Table V for convenience. After similar tables for all features of all possible emotions are determined, we use the statistically significant FOU for each feature of each emotion. In Fig. 14, we provide an illustrative experimental FOU for the feature EO_L for emotion

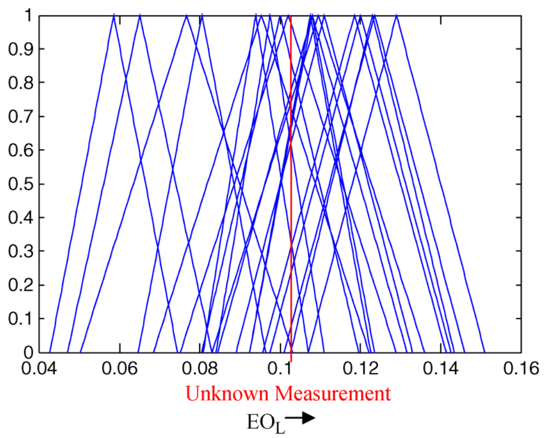


Fig. 15. Constructed symmetric triangular membership functions using (29).

TABLE VI
CALCULATED TYPE-2 MEMBERSHIP VALUES FOR THE FEATURE: EO$_L$ UNDER EMOTION: DISGUST

Feature	Primary Memberships (μ_{pri})	Secondary memberships (μ_{sec})	$\mu^{mod}=\mu_{pri}\times\mu_{sec}$	Range ($\min\{\mu^{mod}\}$, $\max\{\mu^{mod}\}$)
EO$_L$	0.65	0.06	0.039	0.039-0.4355
	0.1	0.87	0.087	
	0.15	0.85	0.1275	
	0.45	0.53	0.2385	
	0.18	0.74	0.1332	
	0.55	0.52	0.286	
	0.08	0.88	0.0704	
	0.41	0.53	0.2173	
	0.16	0.78	0.1248	
	0.12	0.81	0.0972	
	0.38	0.67	0.2546	
	0.45	0.58	0.261	
	0.09	0.89	0.0801	
	0.19	0.72	0.1368	
	0.67	0.65	0.4355	
	0.68	0.58	0.3944	
	0.52	0.55	0.286	
	0.44	0.67	0.2948	
	0.37	0.78	0.2886	
	0.55	0.53	0.2915	

anger by performing step 4 of Section V. The parameters of the FOU, here triangles, are evaluated by step 5 of Section V. Now, for an unknown facial expression, we follow the steps of IT2FS-based approach to recognize the emotion exhibited in the facial expression. Our experiments reveal that the pre-processing steps by IA help in improving the recognition accuracy of the IT2FS scheme by 2.5% (Fig. 15).

GT2FS-Based Recognition: We now briefly illustrate the GT2FS-based reasoning for emotion classification. Here, the secondary membership function corresponding to the individual primary membership function of five features obtained from facial expressions carrying five distinct emotions for 20 different subjects are determined using membership functions like Fig. 12.

Table VI provides the summary of the primary and secondary memberships obtained for EO$_L$ for the emotion: disgust. The range computation for the feature EO$_L$ is also shown in the last column of Table VI. The same computations are repeated for all emotions, and the range evaluated in the last column of Table VII indicates that the center of this range here too has the largest value $(= 0.301)$ for the emotion: happiness.

TABLE VII
CALCULATED RANGES OF PRIMARY MEMBERSHIP CENTRE VALUE FOR EACH EMOTION

Emotion	Range of Secondary Membership for Features					Range S_c^j after fuzzy Meet operation **(centre)**
	EO$_L$	EO$_R$	MO	LEE$_L$	LEE$_R$	
Anger	0-0.21	0 - 0.27	0.26 - 0.983	0.0006 - 0.763	0.0006- 0.790	0-0.21 (0.105)
Disgust	0.039- 0.4355	0.031- 0.433	0-0	0-0.15	0.-0.13	0-0 (0)
Fear	0 - 0.312	0-0.295	0.04- 0.713	0.044- 0.564	0.038- 0.571	0-0.295 (0.1475)
Happiness	0 - 0.602	0-0.606	0.273- 0.98	0.06- 0.93	0.064- 0.97	0-0.602 (**0.301**)
Relaxed	0 - 0.425	0-0.421	0-0	0.001- 0.758	0.001- 0.742	0-0 (0)

TABLE VIII
PERCENTAGE ACCURACY OF OUR PROPOSED METHODS OVER THREE DATABASES

	JAFFE	Indian Women (Jadavpur University)	Cohn-Kanade	Average Accuracy (of last 3 columns)
IT2FS	90%	92.5%	92.5%	91.667%
IA-IT2FS	92.5%	95%	95%	94.167%
GT2FS	97.5%	100%	97.5%	98.333%

VII. PERFORMANCE ANALYSIS

Performance analysis for emotion recognition itself is an open-ended research problem, as there is a dearth of literature on this topic. This paper, compares the relative performance of the proposed GT2FS algorithms with five traditional emotion recognition algorithms/techniques and the IA-IT2FS and IT2FS-based schemes introduced here, considering a common framework in terms of their features and databases. The algorithms used for comparison include linear SVM classifier [28], (T1) fuzzy relational approach [14], PCA [33], multi-layer perceptron (MLP) [1], [29], radial basis function network (RBFN) [1], [29], IT2FS, and IA-IT2FS [68].

Table VIII shows the classification accuracy of our proposed three algorithms using three facial image databases, i.e., JAFFE, Indian Women Face Database (Jadavpur University), and Cohn-Kanade database. Experimental classification accuracy obtained for different other algorithms mentioned above using the three databases is given in Table X.

Two statistical tests called McNemar's test [58] and Friedman test [59], and one new test, called root mean square error test are undertaken to analyze the relative performance of the proposed algorithms over existing ones.

A. McNemar's Test

Let f_A and f_B be two classifiers obtained by algorithms A and B, when both the algorithms have a common training set R.

Let n_{01} be the number of examples misclassified by f_A but not by f_B, and n_{10} be the number of examples misclassified by f_B but not by f_A. Then, under the null hypothesis that

TABLE IX
STATISTICAL COMPARISON OF PERFORMANCE USING MC NEMAR'S TEST WITH THREE DATABASES

Reference Algorithm A=GT2FS						
Classifier Algorithm B used for comparison	JAFFE Database		Indian Database (Jadavpur University)		Cohn-Kanade Database	
	Z_j	Comments on acceptance/ rejection of hypothesis	Z_j	Comments on acceptance/ rejection of hypothesis	Z_j	Comments on acceptance/ rejection of hypothesis
IT2FS	1.333	Accept	1.333	Accept	0.5	Accept
IA-IT2FS	0.5	Accept	0.5	Accept	0	Accept
SVM	1.333	Accept	0	Accept	1.333	Accept
Fuzzy Relational Approach	2.25	Accept	1.333	Accept	1.333	Accept
PCA	0	Accept	2.25	Accept	2.25	Accept
MLP	8.1	Reject	8.1	Reject	8.1	Reject
RBFN	11.077	Reject	10.083	Reject	10.083	Reject

both algorithms have the same error rate, the statistic Z in (30) follows a χ^2 with degree of freedom equals to 1 [59]:

$$Z = \frac{(|n_{01} - n_{10}| - 1)^2}{n_{01} + n_{10}}. \tag{30}$$

Let A be the proposed GT2FS algorithm and B is one of the other seven algorithms. We thus evaluate $Z = Z_1$ through Z_7, where Z_j denotes the comparator statistic of misclassification between the GT2FS (Algorithm: A) and the jth of the seven algorithms (Algorithm: B), where the suffix j refers to the algorithm in row number j of Table IX.

Table IX is evaluated to obtain Z_1 through Z_7 and the hypothesis has been rejected, if $Z_j > \chi_1^2, 0.95 = 3.84$, where $\chi_1^2, 0.95 = 3.84$ is the value of the chi square distribution for 1 degree of freedom at probability of 0.05 [81].

The last inequality indicates that if the null hypothesis is true, then the probability of χ^2 to be more than 3.84 is less than 0.05. If the hypothesis is not rejected, we consider its acceptance. The decision about acceptance or rejection is also included in Table IX.

It is evident from Table IX that McNemar's test cannot distinguish the performance of the five classification algorithms: IT2FS, IA-IT2FS, SVM, fuzzy relational approach, and PCA that support the hypothesis. Hence, next we use the Friedman test for ranking the algorithms.

B. Friedman Test

The Friedman test [58] ranks the algorithms for each data sets separately. The best performing algorithm gets rank 1. In case of ties, average ranks are used.

TABLE X
AVERAGE RANKING OF CLASSIFICATION ALGORITHMS BY FRIEDMAN TEST, WHERE, CA = Classifier Algorithm, A = GT2FS, B_1 = SVM, B_2 = IT2FS, B_3 = IA-IT2FS, B_4 = Fuzzy Relational Approach, B_5 = PCA, B_6 = MLP, B_7 = RBFN

CA	Classification Accuracy tested by databases			Ranks obtained through experiments with databases			Average Rank (R_j)
	JAFFE	Indian	Cohn-Kanade	JAFFE	Indian	Cohn-Kanade	
A	97.5	100	97.5	1	1	1	1
B_1	90.33	97.57	88.11	4	2	5	3.667
B_2	90	92.5	92.5	5	4	3	4
B_3	92.5	95	95	3	3	2	2.667
B_4	87.5	92	90	6	5	4	5
B_5	95	87.5	87.5	2	6	6	4.667
B_6	72.5	75	72.5	7	7	7	7
B_7	65	67.5	67.5	8	8	8	8

Let r_i^j be the rank of jth algorithm on the ith data set. The average rank of algorithm j then is evaluated by

$$R_j = \frac{1}{N} \sum_{\forall i} r_i^j. \tag{31}$$

The null hypothesis here states that all the algorithms are equivalent, so their individual ranks R_j should be equal. Under the null hypothesis, for large enough N and k, the Friedman statistic χ_F^2 in (32) is distributed as a χ^2 with k-1 degrees of freedom. Here, $k = 8$ and $N = 3$. A larger N of course is desirable; however, emotion databases being fewer, finding large N is not feasible. Here, we consider percentage accuracy of classification as the basis of rank. Table X provides the percentage accuracy of classification with respect to three databases, JAFFE, Indian Woman (Jadavpur University), and Cohn-Kanade and the respective ranks of the algorithm

$$\chi_F^2 = \frac{12N}{k(k+1)} \left[\sum_j R_j^2 - \frac{k(k+1)^2}{4} \right]. \tag{32}$$

Now, using $N = 3$, $k = 8$, and the ranks in Table X, we obtain $\chi_F^2 = 17.67 > \chi_{7,0.95}^2 = 14.067$ [81], where $\chi_7^2, 0.95 = 14.067$ is the value of the chi square distribution for 7° of freedom at probability of 0.05 [81]

$$\chi_F^2 = \frac{12N}{k(k+1)} \left[\sum_j R_j^2 - \frac{k(k+1)^2}{4} \right]$$
$$= 17.67 > \chi_{7,0.95}^2 (14.067).$$

Thus, the hypothesis that the algorithms are equivalent is rejected. Therefore, the performances of the algorithms are determined by their ranks only. The order of ranking of the algorithm is apparent from their average ranks. The smaller the average rank, the better is the algorithm. Let ">" be a comparator of relative ranks where $x > y$ means the algorithm x is better in rank than algorithm y. Table X indicates that the relative order of ranking of the algorithm by Friedman test as, GT2FS > IA − IT2FS > SVM > IT2FS > PCA >Fuzzy Relational Approach > MLP > RBFN. It is clear from Table X that the average rank of GT2FS is 1 and average rank of IT2FS and IA-IT2FS are 4 and 2, respectively, claiming GT2FS outperforms all the algorithms by Friedman test.

VIII. Conclusion

The paper presents three automatic emotion recognition systems based on IT2FS, IA-IT2FS, and GT2FS. In order to classify an unknown facial expression, these systems make use of the background knowledge about a large face database with known emotion classes. The GT2FS-based recognition scheme requires T2 secondary membership functions, which are obtained using an innovative evolutionary approach that is also proposed in this paper. All the schemes first construct a fuzzy face space, and then infer the emotion class of the unknown facial expression by determining the maximum support of the individual emotion classes using the pre-constructed fuzzy face space. The class with the highest support is assigned as the emotion of the unknown facial expression.

The IT2FS-based recognition scheme takes care of the inter-subject level uncertainty in computing the maximum support of individual emotion class. The GT2FS-based recognition scheme, however, takes care of both the inter- and intra-subject level uncertainty, and thus offers higher classification accuracy for the same set of features. Using three data sets, the classification accuracy obtained by employing GT2FS is 98.333%, by IT2FS is 91.667%, and by IA-IT2FS is 94.167%.

The more the number of subjects used for constructing the fuzzy face space, the better would be the fuzzy face space, and thus better would be the classification accuracy. Since the fuzzy face space is created offline, the online computational load to recognize emotion is insignificantly small in IT2FS. The computational load in GT2FS, however, is large as it includes an optimization procedure to determine the secondary membership for each emotion and for each subject. However, this additional complexity in GT2FS, offers approximately 7% improvement in classification accuracy in comparison to that by IT2FS. The IA-IT2FS has around 2.5% gain in classification accuracy with no more additional computational complexity than IT2FS. It may be noted that the necessary computations in IA-IT2FS for data filtering and membership function selection is performed offline. The statistical tests employed clearly indicate that GT2FS outperforms the seven selected algorithms.

The problems that may be taken up as future research are briefly outlined below. First, new alternative strategies are to be designed to determine secondary memberships without using optimization techniques. Second, a more formal and systematic approach to fuse secondary and primary memberships to reduce uncertainty in the fuzzy face space is to be developed. Last, we would try to explore the power of fuzzy logic to determine emotion classes in absence of sufficient (or even no) measurements. Facial features, for example MO, may be directly encoded into fuzzy features with fuzzy sets, such as "a little," "more," and "not so large," and then an IT2FS-based model may be adopted to recognize emotion of unknown subjects. Classification accuracy under this circumstance could be poor, but a more human-like interpretation of emotion can be given in the absence of precise measurements.

Appendix
The Classical Differential Evolution Algorithm [34]

An iteration of the classical DE algorithm consists of the four basic steps—initialization of a population of vectors, mutation, crossover or recombination, and finally selection. The main steps of classical DE are given below:

I. Set the generation number $t = 0$ and randomly initialize a population of NP individuals $\vec{P}_t = \{\vec{X}_1(t), \vec{X}_2(t), \ldots, \vec{X}_{NP}(t)\}$ with $\vec{X}_1(t) = \{x_{i,1}(t), x_{i,2}(t), \ldots, x_{i,D}(t)\}$ and each individual uniformly distributed in the range $[\vec{X}_{\min}, \vec{X}_{\max}]$, where $X_{\min} = \{x_{\min,1}, x_{\min,2}, \ldots, x_{\min,D}\}$ and $\vec{X}_{\max} = \{x_{\max,1}, x_{\max,2}, \ldots, x_{\max,D}\}$ with $i = [1, 2, \ldots, NP]$.

II. **while** stopping criterion is not reached, **do**
for$i = 1$**to**NP
a. **Mutation**:
Generate a donor vector $\vec{V}(t) = \{v_{i,1}(t), v_{i,2}(t), \ldots\ldots, v_{i,D}(t)\}$ corresponding to the ith target vector $\vec{X}_1(t)$ by the following scheme $\vec{V}_1(t) = \vec{X}_{r1}(t) + F * (\vec{X}_{r2}(t) - \vec{X}_{r3}(t))$ where r_1, r_2 and r_3 are distinct random integers in the range [1, NP]
b. **Crossover**:
Generate trial vector $\vec{U}_i(t) = \{u_{i,1}(t), u_{i,2}(t), \ldots, u_{i,D}(t)\}$ for the ith target vector $\vec{X}_1(t)$ by binomial crossover as

$$\begin{aligned}\vec{u}_{i,j}(t) &= \vec{v}_{i,j}(t)\ if\ rand\,(0,1) < Cr\\ &= \vec{x}_{i,j}(t)\ otherwise.\end{aligned}$$

c. **Selection**:
Evaluate the trial vector $\vec{U}_i(t)$
if$f(\vec{U}_i(t)) \le f(\vec{X}_i(t))$,
then$vecX_i(t+1) = vecU_i(t)$
$f(\vec{X}_i(t+1)) = f(vecU_i(t))$
end if
end for
d. Increase the counter value t = t + 1.
end while

The parameters used in the algorithm namely scaling factor "F" and crossover rate "Cr" should be initialized before calling the "while" loop. The terminate condition can be defined in many ways, a few of which include: 1) fixing the number of iterations N, 2) when best fitness of population does not change appreciably over successive iterations, and 3) either of 1) and 2), whichever occurs earlier.

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General and Interval Type-2 Fuzzy Face-Space Approach to Emotion Recognition

Anisha Halder, Amit Konar, Rajshree Mandal, Aruna Chakraborty, Pavel Bhowmik, Nikhil R. Pal, and Atulya Nagar

***Abstract*—Facial expressions of a person representing similar emotion are not always unique. Naturally, the facial features of a subject taken from different instances of the same emotion have wide variations. In the presence of two or more facial features, the variation of the attributes together makes the emotion recognition problem more complicated. This variation is the main source of uncertainty in the emotion recognition problem, which has been addressed here in two steps using type-2 fuzzy sets. First a type-2 fuzzy face space is constructed with the background knowledge of facial features of different subjects for different emotions. Second, the emotion of an unknown facial expression is determined based on the consensus of the measured facial features with the fuzzy face space. Both interval and general type-2 fuzzy sets (GT2FS) have been used separately to model the fuzzy face space. The interval type-2 fuzzy set (IT2FS) involves primary membership functions for m facial features obtained from n-subjects, each having l-instances of facial expressions for a given emotion. The GT2FS in addition to employing the primary membership functions mentioned above also involves the secondary memberships for individual primary membership curve, which has been obtained here by formulating and solving an optimization problem. The optimization problem here attempts to minimize the difference between two decoded signals: the first one being the type-1 defuzzification of the average primary membership functions obtained from the n-subjects, while the second one refers to the type-2 defuzzified signal for a given primary membership function with secondary memberships as unknown. The uncertainty management policy adopted using GT2FS has resulted in a classification accuracy of 98.333% in comparison to 91.667% obtained by its interval type-2 counterpart. A small improvement (approximately 2.5%) in classification accuracy by IT2FS has been attained by pre-processing measurements using the well-known interval approach.**

***Index Terms*—Emotion recognition, facial feature extraction, fuzzy face space, interval and general type-2 fuzzy sets, interval approach (IA).**

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I. Introduction

EMOTION recognition is currently gaining importance for its increasing scope of applications in human–computer interactive systems. Several modalities of emotion recognition, including facial expression, voice, gesture, and posture have been studied in the literature. However, irrespective of the modality, emotion recognition comprises two fundamental steps involving feature extraction and classification [36]. Feature extraction refers to determining a set of features/attributes, preferably independent, which together represents a given emotional expression. Classification aims at mapping emotional features into one of several emotion classes.

Performance of an emotion recognition system greatly depends on feature selection and classifier design. A good classification algorithm sometimes cannot yield high classification accuracy for poorly selected features. On the other hand, even using a large set of features, describing an emotion, we occasionally fail to recognize the emotion correctly because of a poor classifier. Most commonly used techniques for feature selection in the emotion recognition problem include principal component analysis (PCA) [59], independent component analysis [60], rough sets [42], [61], Gabor filter [62], and Fourier descriptors [25]. Among the popularly used techniques for emotion classification, neural net-based mapping [3], [4], [18], fuzzy relational approach [14], linear discriminate analysis [60], support vector machine (SVM) [8], and hidden Markov model [59], gegege[62] need special mention. A brief overview of the existing research on emotion recognition is given next.

Ekman and Friesen took an early attempt to recognize facial expression from the movements of cheek, chin, and wrinkles [24]. Their experiments confirmed the existence of a good correlation between basic movements of the facial action units [13], [19] and facial expressions [1], [2], [5], [7], [10], [19]–[22]. Kobayashi and Hara [15]–[17] designed a scheme for the recognition of human facial expressions using the well-known back-propagation neural networks [38], [43]. Their scheme is capable of recognizing six common facial expressions depicting happiness, sadness, fear, anger, surprise, and disgust. Yamada proposed an alternative method of emotion recognition through classification of visual information [49].

Fernandez-Dols *et al.* proposed a scheme for decoding emotions from facial expressions and content [50]. Kawakami *et al.* [43] designed a method for the construction of emotion space using neural networks. Busso and Narayanan [51] analyzed the scope of facial expressions, speech, and multi-modal information in emotion recognition. Metallinou *et al.* [71] employed content-sensitive learning for audio-visual emotion recognition. In [73], Metallinou *et al.* proposed a novel approach to visual emotion recognition using a compact representation of face and viseme information. In [74], Metallinou *et al.* presented an approach to decision level fusion for handling multi-modal information in emotion recognition. Lee *et al.* [75] employed a hierarchical binary tree for emotion recognition. Mower *et al.* designed an interesting scheme about human perception of audio-visual synthetic emotion character in the presence of conflicting information [76]. Cohen *et al.* [52] developed a scheme for emotion recognition from the temporal variations in facial expressions obtained from the live video sequence of the subjects. They used hidden Markov model to automatically segment and recognize facial expression. Gao *et al.* presented a scheme for facial expression recognition from a single facial image using line based caricatures [53]. Among other significant contributions in emotion recognition, the works presented in [6], [8], [9], [11], [12], [15]–[17], [23]–[28], [30], [31], [32], [35], [40], [46], [56], [57], [60], [70], [72], [77]–[80] need special mention. For a more complete literature survey, which cannot be given here for space restriction, readers may refer to two outstanding papers by Pantic *et al.* [57], [67].

Emotional features greatly depend on the psychological states of the subjects. For example, facial expressions of a subject, while experiencing the same emotion, have wider variations, resulting in significant changes in individual feature. Further, different subjects experiencing the same emotion have differences in their facial features. Repeated experiments with a large number of subjects, each having multiple instances of similar emotional experience, reveal that apparently there exists a small but random variation of facial features around specific fixed points [65]. The variation between different instances of facial expression for similar emotive experience of an individual can be regarded as an *intra-personal level uncertainty*[41]. On the other hand, the variation in facial expression of individuals for similar emotional experience can be treated as *inter-personal level uncertainty*[41].

The variations in features can be modeled with fuzzy sets. Classical (type-1 (T1)) fuzzy sets, pioneered by Zadeh [66], have widely been used over the last five decades for modeling uncertainty of ill-defined systems. T1 fuzzy sets employ a single membership function to represent the degree of uncertainty in measurements of a given feature. Hence, it can capture the variation in measurements of a given feature for different instances of a specific emotion experienced by a subject. In [14], the authors have considered a fixed membership function to model the uncertainty involved in a feature for a given emotion, disregarding the possibility of variation in the membership curves for different subjects.

This paper, however, models the above form of inter-personal level uncertainty by interval type-2 (T2) fuzzy sets (IT2FS). IT2FS employs an upper and a lower membership function (UMF and LMF) to capture the uncertainty involved in a given measurement of a feature within the bounds of its two membership curves at the point of the measurement. However, the degree of correct assignment of membership for each membership curve embedded between the UMF and LMF in IT2FS is treated as unity, which is not always appropriate. General T2 fuzzy set (GT2FS) can overcome the above problem by considering a secondary membership grade that represents the correctness in (primary) membership assignment at each measurement points. Naturally, GT2FS is expected to give us better results in emotion classification for its representational advantage over IT2FS.

One fundamental problem in GT2FS that limits its application in classification problems, perhaps, is due to users' inability to correctly specify the secondary memberships. In this paper, we determine the secondary memberships by extracting certain knowledge from the individual primary assignments for each feature of a given emotion for a subject. The knowledge extracted is encoded as an optimization problem with secondary memberships as unknown. The solution to the optimization problem carried out offline provides the secondary grades. The secondary grades are later aggregated with the primary memberships of individual feature for all subjects at the given measurement point to obtain modified primary memberships.

The paper provides two alternative approaches to emotion recognition from an unknown facial expression, when the emotion class of individual facial expression of a large number of experimental subjects is available. The first approach deals with IT2FS to construct a fuzzy face space based on the measurements of a set of features from a given set of facial expressions carrying different emotions. An unknown facial expression is classified into one of several emotion classes by determining the maximum support of individual emotion classes to a given set of measurements of a facial expression. The class having the maximum support is declared as the emotion of the unknown facial expression. In spirit, this is similar to how a fuzzy rule-based system for classification works.

The second approach employs GT2FS to construct a fuzzy face space, comprising both primary and secondary membership functions, obtained from known facial expressions of several subjects containing multiple instances of the same emotion for each subject. The emotion class of an unknown facial expression is determined by computing the support of each class to the given facial expression. The class with the maximum support is the winner. The maximum support evaluation here employs both primary and secondary memberships, and thus is slightly different than the IT2FS-based classification.

Experiments reveal that the classification accuracy of emotion of an unknown person by the GT2FS-based scheme is as high as 98%. When secondary memberships are ignored, and classification is performed with IT2FS, the classification accuracy falls by a margin of 7%. The additional 7% classification accuracy obtained by GT2FS, however, has to pay a price for additional complexity of $(m \times n \times k)$ multiplications, where m, n, and k denote the number of features, number of subjects, and number of emotion classes, respectively. A 2.5% improvement in classification accuracy by IT2FS has been attained by pre-processing measurements and selecting membership functions using the well-known interval approach (IA) [68].

The paper is divided into eight sections. Section II provides fundamental definitions associated with T2 fuzzy sets, which will be required in the rest of the paper. In Section III, we propose the principle of uncertainty management in fuzzy face space for emotion recognition. Section IV deals with secondary membership evaluation procedure for a given T2 primary membership function. A scheme for selection of membership function and data filtering to eliminate poor measurements to improve the performance of IT2FS-based recognition is given in Section V. Experimental details are given in Section VI, and two methods of performance analysis are undertaken in Section VII. Conclusions are listed in Section VIII.

II. Preliminaries on T2 Fuzzy Sets

In this section, we define some terminologies related to T1 and T2 fuzzy sets. These definitions will be used throughout the paper.

Definition 1: Given a universe of discourse X, a conventional *T1 fuzzy set* A defined on X, is given by a 2-D membership function, also called T1 membership function. The (primary) *membership function*, denoted by $\mu_A(x)$, is a crisp number in [0, 1] for a generic element $x \in X$. Usually, the fuzzy set A is expressed as a two tuple [36], given by

$$A = \{(x, \mu_A(x)) \,|\forall x \in X\}. \tag{1}$$

An alternative representation of the fuzzy set A is also found in the literature as given in (2).

$$A = \int_{x \in X} \mu_A(x)|x \tag{2}$$

where $\int$ denotes union of all admissible x.

Definition 2: A *T2 fuzzy set* $\tilde{A}$ is characterized by a 3-D membership function, also called T2 membership function, which itself is fuzzy. The *T2 membership function* is usually denoted by $\mu_{\tilde{A}}(x, u)$, where $x \in X$, and $u \in J_x \subseteq [0, 1]$[39]. Usually, the fuzzy set $\tilde{A}$ is expressed as a two tuple:

$$\tilde{A} = \{((x, u), \mu_{\tilde{A}}(x, u)) \,| x \in X, u \in J_x \subseteq [0, 1]\} \tag{3}$$

where $\mu_{\tilde{A}}(x, u) \in [0, 1]$. An alternative form of representation of the T2 fuzzy set is given in (4)

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x, u)|(x, u), J_x \subseteq [0, 1] \tag{4}$$

$$= \int_{x \in X} \left[\frac{\int_{u \in J_x} f_x(u)}{u} \right] /x, J_x \subseteq [0, 1] \tag{5}$$

where $f_x(u) = \mu_{\tilde{A}}(x, u) \in [0, 1]$. The $\int\int$ denotes union over all admissible x and u [39].

Definition 3: At each point of x, say $x = x^{/}$, the 2-D plane containing axes u and $\mu(x^{/}, u)$ is called the *vertical slice* of $\mu_{\tilde{A}}(x, u)$. A *secondary membership function* is a vertical slice of $\mu_{\tilde{A}}(x, u)$. Symbolically, it is given by $\mu_{\tilde{A}}(x, u)$ at $x = x^{/}$ for $x^{/} \in X$ and $\forall u \in J_{x^{/}} \subseteq [0, 1]$

$$\mu_{\tilde{A}}(x = x', u) = \int_{u \in J_{x^{/}}} f_{x^{/}}(u)|u, J_{x^{/}} \subseteq [0, 1] \tag{6}$$

where $0 \leq f_{x^{/}}(u) \leq 1$. The amplitude of a secondary membership function is called secondary grade (of membership). In (6)$J_{x^{/}}$ is the primary membership of $x^{/}$.

Definition 4: Uncertainty in the primary membership of a T2 fuzzy set $\tilde{A}$ is represented by a bounded region, called *footprint of uncertainty* (FOU) [39], which is the defined as the union of all primary memberships, i.e.,

$$FOU(\tilde{A}) = \underset{x \in U}{\cup} J_x. \tag{7}$$

If all the secondary grades of a T2 fuzzy set $\tilde{A}$ are equal to 1, i.e.,

$$\mu_{\tilde{A}}(x, u) = 1 \forall x \in X, \forall u \in J_x \subseteq [0, 1] \tag{8}$$

then $\tilde{A}$ is called *IT2FS*. The FOU is bounded by two curves, called the *Lower* and the *Upper Membership functions*, denoted by $\underline{\mu}_{\tilde{A}}(x)$ and $\overline{\mu}_{\tilde{A}}(x)$, respectively, where $\underline{\mu}_{\tilde{A}}(x)$ and $\overline{\mu}_{\tilde{A}}(x)$ at all x, respectively, take up the minimum and the maximum of the membership functions of the embedded T1 fuzzy sets [38] in the FOU.

III. Uncertainty Management in Fuzzy Face Space for Emotion Recognition

This section provides a general overview of the proposed scheme for emotion recognition using T2 fuzzy sets. Here, the emotion recognition problem is considered as uncertainty management in fuzzy space after encoding the measured facial attributes by T2 fuzzy sets.

Let $F = \{f_1, f_2, \ldots, f_m\}$ be the set of m facial features. Let $\mu_{\tilde{A}}(f_i)$ be the primary membership in [0,1] of the feature f_i to be a member of set $\tilde{A}$, and $\mu(f_i, \mu_{\tilde{A}}(f_i))$ be the secondary membership of the measured variable f_i in [0,1]. A primary and secondary membership function corresponds to a particular emotion class c, are denoted by $\mu_{\tilde{A}c}(f_i)$ and $\mu(f_i, \mu_{\tilde{A}c}(f_i))$, respectively. If the measurement of a facial feature, f_i, is performed p times on the same subject experiencing the same emotion, and the measurements are quantized into q intervals of equal size, we can evaluate the frequency of occurrence of the measured variable f_i in q quantized intervals. The interval containing the highest frequency of occurrence then can be identified, and its center, m_i, approximately represents the mode of the measurement variable f_i. The second moment, σ_i, around m_i is determined and a bell-shaped (Gaussian) membership function centered at m_i and with a spread σ_i is used to represent the membership function of the random variable f_i. This function represents the membership of f_i to be CLOSE-TO the central value, m_i. It may be noted that a bell-shaped (Gaussian-like) membership curve would have a peak at the center with a membership value one, indicating that membership at this point is the largest for an obvious reason of having the highest frequency of f_i at the center.

On repetition of the above experiment for variable f_i on n subjects, each experiencing the same emotion, we obtain n such membership functions, each one for one individual subject. Naturally, the measurement variable f_i now has both intra- and inter-personal level uncertainty. The intra-level uncertainty

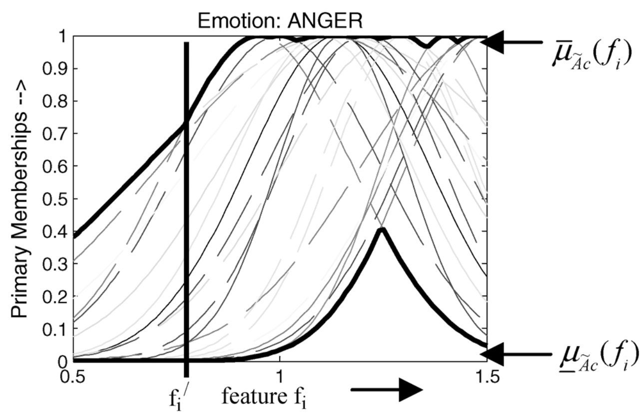


Fig. 1 Experimental FOU for feature $f_{\mathrm{i}} =$ Mouth-Opening.

occurs due to the pre-assumption of a specific (Gaussian) primary membership function, and the inter-level uncertainty occurs due to multiplicity of the membership functions for n subjects. Thus, a new measurement for an unknown facial expression can be encoded using all the n-membership curves, giving n possible membership values, thereby giving rise to uncertainty in the fuzzy space.

The uncertainty involved in the present problem has been addressed here by three distinctive approaches: 1) IT2FS, 2) IA-IT2FS, and 3) GT2FS. The first approach is simple, but more error prone as it ignores the intra-level uncertainty. The second and the third approaches are robust as they are capable to take care of both the uncertainties. However, the modality of uncertainty management by the second and the third approaches is significantly different. The second approach models each subject's interval using a uniform probability distribution, and thus the mean and variance of each interval are mapped into an embedded T1 fuzzy set. The third approach handles intra- and inter-personal level uncertainty compositely by fusing the primary and the secondary membership functions into an embedded interval T2 membership function. All three approaches have many common steps. Hence, we first present the steps involved in IT2FS and then explain the two techniques without repeating the common steps further.

A. *Principles Used in the IT2FS Approach*

The primary membership functions for a given feature value f_i corresponding to a particular emotion c taken from n-subjects together forms a IT2FS $\tilde{A}_c$, whose FOU is bounded by a lower and an upper membership curves $\underline{\mu}_{\tilde{A}c}(f_i)$ and $\overline{\mu}_{\tilde{A}c}(f_i)$, respectively, where

$$\underline{\mu}_{\tilde{A}c}(f_i) = Min\left\{\mu^1_{\tilde{A}c}(f_i), \mu^2_{\tilde{A}c}(f_i), \ldots, \mu^n_{\tilde{A}c}(f_i)\right\}, \tag{9}$$

$$\overline{\mu}_{\tilde{A}c}(f_i) = Max\left\{\mu^1_{\tilde{A}c}(f_i), \mu^2_{\tilde{A}c}(f_i), \ldots, \mu^n_{\tilde{A}c}(f_i)\right\} \tag{10}$$

are evaluated for all f_i, and $\mu^j_{\tilde{A}_c}(f_i), 1 \leq j \leq n$ denotes the primary membership function of feature f_i for subject j in IT2FS $\tilde{A}_c$.

Fig. 1 provides the FOU for a given feature f_i. Now, for a given measurement $f_i^{/}$, we obtain an interval $[\underline{\mu}_{\tilde{A}c}(f_i^{/}), \overline{\mu}_{\tilde{A}c}(f_i^{/})]$, representing the entire span of uncertainty of the measurement variable $f_i^{/}$ in the fuzzy space, induced by n primary membership distributions: $\mu^j_{\tilde{A}c}(f_i), 1 \leq j \leq n$. The interval $[\underline{\mu}_{\tilde{A}c}(f_i^{/}), \overline{\mu}_{\tilde{A}c}(f_i^{/})]$ is evaluated by replacing f_i by $f_i^{/}$ in (9) and (10), respectively.

If there exist m different facial features, then for each feature, we would have such an interval, and consequently we obtain m such intervals given by

$$\left[\underline{\mu}_{\tilde{A}c}\left(f_1^{/}\right), \overline{\mu}_{\tilde{A}c}\left(f_1^{/}\right)\right], \left[\underline{\mu}_{\tilde{A}c}\left(f_2^{/}\right), \overline{\mu}_{\tilde{A}c}\left(f_2^{/}\right)\right], \ldots\ldots\ldots \times \left[\underline{\mu}_{\tilde{A}c}\left(f_m^{/}\right), \overline{\mu}_{\tilde{A}c}\left(f_m^{/}\right)\right].$$

The proposed IT2FS reasoning system employs a particular format of rules, commonly used in fuzzy classification problems [47]. Consider for instance a fuzzy rule, given by R_c: if f_1 is $\tilde{A}_1$ AND f_2 is $\tilde{A}_2 \ldots$ AND f_m is $\tilde{A}_m$ then emotion class is c.

Here, f_i for $i = 1tom$ are m-measurements (feature values) and $\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_m$ are IT2FS on the respective domains

$$\tilde{A}_i = \left[\underline{\mu}_{\tilde{A}c}(f_i), \overline{\mu}_{\tilde{A}c}(f_i)\right], \forall i. \tag{11}$$

Since an emotion is characterized by all of these m features, to find the overall support of the m features (m measurements made for the unknown subject) to the emotion class c represented by the n primary memberships, we use the fuzzy meet operation

$$S_c^{\min} = Min\left\{\underline{\mu}_{\tilde{A}c}\left(f_1^{/}\right), \underline{\mu}_{\tilde{A}c}\left(f_2^{/}\right) \ldots, \underline{\mu}_{\tilde{A}c}\left(f_m^{/}\right)\right\} \tag{12}$$

$$S_c^{\max} = Min\left\{\overline{\mu}_{\tilde{A}c}\left(f_1^{/}\right), \overline{\mu}_{\tilde{A}c}\left(f_2^{/}\right), \ldots, \overline{\mu}_{\tilde{A}c}\left(f_m^{/}\right)\right\}. \tag{13}$$

Thus, we can say that the unknown subject is experiencing the emotion class c at least to the extent s_c^{min}, and at most to the extent s_c^{max}.

To reduce the nonspecificity associated with the interval $s_{c-i} = [s_c^{min}, s_c^{max}]$, different approaches can be taken. For example, the most conservative approach would be to use lower bound, while the most liberal view would be to use the upper bound of the interval as the support for the class c. In the absence of any additional information, a balanced approach would be to use center of the interval as the support for the class c by the n primary memberships to the unknown subject. This idea is supported by Mendel [42] and Lee [48]. We compute the center S_c of the interval S_{c-i}

$$S_c = \frac{\left(s_c^{min} + s_c^{max}\right)}{2}. \tag{14}$$

Thus, S_c is the degree of support that the unknown facial expression is in emotion class c. Now, to predict the emotion of a person from his facial expression, we determine S_c for each emotion class. Presuming that there exist k emotion classes, let us denote the degree by which the emotion classes $1, 2, \ldots, k$ support the unknown facial expression be $S_1, S_2, \ldots, S_k$, respectively. Since a given facial expression may convey different emotions with different degrees, we resolve the conflict by ranking the S_i for $i = 1tok$, and thus determine the emotion class r, for which $S_r >= S_{\mathrm{i}}$ for all i.

The principle of selection of the emotion class r from a set of competitive emotions, satisfying the above inequality holds, since the joint occurrence of the fuzzy memberships, induced by (12)–(14), for all the features of the given facial expression for emotion r is the greatest among the same values for all other emotions.

B. *Principles Used in the GT2FS Approach*

The previous approach employs a reasoning mechanism to compute the degree of support of k emotion classes induced by m features for each class to an unknown facial expression using a set of $k \times m$ IT2FS. The GT2FS-based reasoning realized with measurements taken from n-subjects, however, requires $k \times m \times n$ GT2FSs to determine the emotion class of an unknown facial expression. The current approach tunes the primary membership values for the given measurements using the secondary memberships of the same measurement, and thus reduces the degree of intra-level uncertainty of the primary distributions. The reduction in the degree of uncertainty helps in improving the classification accuracy of emotion at the cost of additional complexity required to evaluate T2 secondary distributions and also to reason with $k \times m \times n$ fuzzy sets.

Let f_i be the measurement of the ith feature for a subject with an unknown emotion class. Now, by consulting the n primary membership functions that were generated from n-subjects in the training data for a given emotion class, c, we obtain n primary membership values corresponding to f_i for emotion class c as given by $\mu^1_{\tilde{A}c}(f_i), \mu^2_{\tilde{A}c}(f_i), \dots, \mu^n_{\tilde{A}c}(f_i)$. Let the secondary membership values for each primary membership value, respectively, be $\mu(f_i, \mu^1_{\tilde{A}c}(f_i)), \mu(f_i, \mu^2_{\tilde{A}c}(f_i)), \dots, \mu(f_i, \mu^n_{\tilde{A}c}(f_i))$. Note that, these secondary membership values correspond to emotion class c. Unless clarity demands, we have avoided (here and elsewhere) use of a subscript to represent the emotion class. We now fuse (aggregate) the evidences provided by the primary and secondary membership values to obtain the modified primary membership supports. A plausible way of fusing would be to use a T-norm. Here, we use the product. The product always lies within the FOU and thus satisfies Mendel-John Representation Theorem [39]. Further higher is the secondary membership, higher is the product representing new embedded fuzzy membership. Since the secondary membership represents the degree of correctness in primary membership, the product helps in reduction of intra-level uncertainty. Thus, for subject j of the training data representing emotion class c, we obtain

$$ {}^{\mathrm{mod}}\mu^j_{\tilde{A}c}(f_i) = \mu^j_{\tilde{A}c}(f_i) \times \mu\left(f_i, \mu^j_{\tilde{A}c}(f_i)\right) \forall j = 1, \dots, n \quad (15) $$

where ${}^{\mathrm{mod}}\mu^j_{\tilde{A}c}(f_i)$ denotes the modified primary membership value for jth training subject for cth emotion class. The secondary membership values used in the above product function are evaluated using their primary memberships obtained by a procedure discussed in Section IV.

The next step is to determine the range of ${}^{\mathrm{mod}}\mu^j_{\tilde{A}}(f_i^{/})$ for $j = 1$ *to* n, comprising the minimum and the maximum given by $[{}^{\mathrm{mod}}\underline{\mu}_{\tilde{A}}(f_i^{/}), {}^{\mathrm{mod}}\overline{\mu}_{\tilde{A}}(f_i^{/})]$, where

$$ {}^{\mathrm{mod}}\underline{\mu}_{\tilde{A}}\left(f_i^{/}\right) = Min\Big\{ {}^{\mathrm{mod}}\mu^1_{\tilde{A}}\left(f_i^{/}\right), {}^{\mathrm{mod}}\mu^2_{\tilde{A}}\left(f_i^{/}\right), \dots, {}^{\mathrm{mod}}\mu^n_{\tilde{A}}\left(f_i^{/}\right)\Big\} \quad (16) $$

$$ {}^{\mathrm{mod}}\overline{\mu}_{\tilde{A}}\left(f_i^{/}\right) = Max\Big\{ {}^{\mathrm{mod}}\mu^1_{\tilde{A}}\left(f_i^{/}\right), {}^{\mathrm{mod}}\mu^2_{\tilde{A}}\left(f_i^{/}\right), \dots, {}^{\mathrm{mod}}\mu^n_{\tilde{A}}\left(f_i^{/}\right)\Big\}. \quad (17) $$

Now, for m features, the rule-based T2 classification is performed in a similar manner as in the previous section with the replacement of $\underline{\mu}_{\tilde{A}}(f_i^{/})$ and $\overline{\mu}_{\tilde{A}}(f_i^{/})$ by ${}^{\mathrm{mod}}\underline{\mu}_{\tilde{A}}(f_i^{/})$ and ${}^{\mathrm{mod}}\overline{\mu}_{\tilde{A}}(f_i^{/})$, respectively.

C. *Methodology*

We briefly discuss the main steps involved in fuzzy face-space construction based on the measurements of m facial features for n-subjects, each having l instances of facial expression for a particular emotion. We need to classify a facial expression of an unknown person into one of k emotion classes.

IT2FS-Based Emotion Recognition:

1) We extract m facial features for n subjects, each having l (l could be different for different emotion classes) instances of facial expression for a particular emotion. The above features are extracted for k-emotion classes.
2) We construct a fuzzy face space for each emotion class separately. The fuzzy face space for an emotion class comprises a set of n primary membership functions for each feature. Thus, we have m groups (denoted by m rows of blocks in Fig. 2) of n-primary membership functions (containing n blocks under each row of Fig. 2). Each primary membership curve is constructed from l-facial instances of a subject attempted to exhibit a particular emotion in her facial expression by acting.
3) For a given set of features $f_1^{/}, f_2^{/}, \dots, f_m^{/}$ obtained from an unknown facial expression, we determine the range of membership for feature $f_i^{/}$, given by $[\underline{\mu}_{\tilde{A}}(f_i^{/}), \overline{\mu}_{\tilde{A}}(f_i^{/})]$, where $\tilde{A}$ is an IT2FS with a primary membership function defined as CLOSE-TO-center-value-m of the respective membership function.
4) Now, for an emotion class j, we take fuzzy meet operation over the ranges for each feature to evaluate the range of uncertainty for individual emotion class. The meet operation here is computed by taking cumulative t-norm (here we use min) of $\underline{\mu}_{\tilde{A}}(f_i^{/})$ and $\overline{\mu}_{\tilde{A}}(f_i^{/})$ separately for $i = 1tom$, and thus obtaining S_j^{min} and S_j^{max}, respectively (see top of Fig. 2).
5) The support of the j-th emotion class to the measurements is evaluated by computing the average S_j of $S_j^{\min}$ and $S_j^{\max}$.

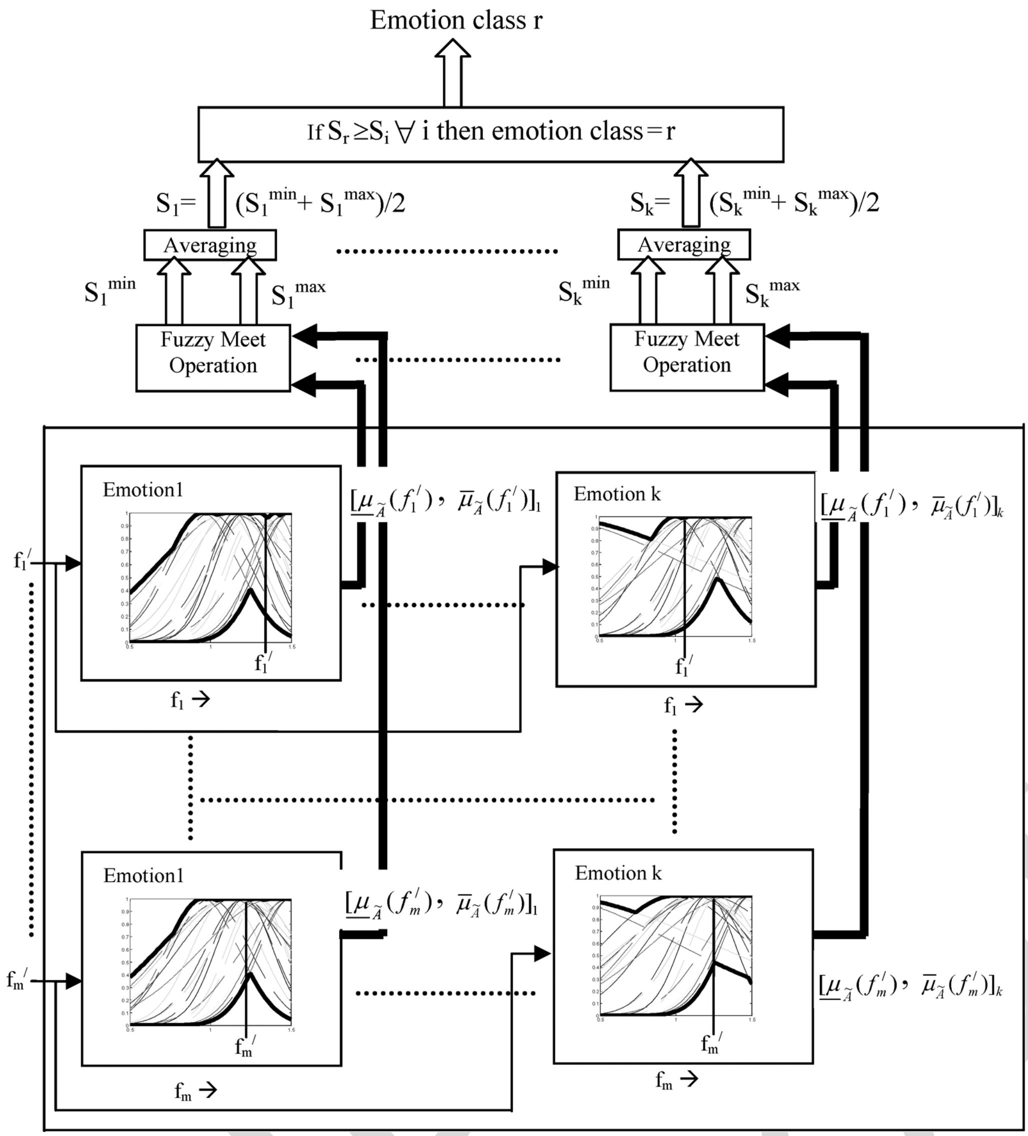


Fig. 2. The IT2FSS-based emotion recognition.

6) Now, we determine the maximum support offered by all the k emotion classes, and declare the unknown facial expression to have emotion r, if $S_r \geq S_i$ for all emotion class $i = 1$ to k. The suffix j in $[\mu_A^{\min}(f_i^/), \mu_A^{\max}(f_i^/)]_j$ refers to the range in that interval for emotion j.

GT2FS-Based Emotion Recognition:

1) This step is same as the step 1 of IT2FS-based emotion recognition.
2) The construction of the primary membership functions here follows the same procedure as given in step 2 of IT2FS-based recognition scheme. In addition, we need to construct secondary membership functions for individual primary membership curves. The procedure for construction of secondary membership functions will be discussed in Section IV. The complete scheme of construction of T2FFS, considering all k emotion classes, is given in Fig. 3.
3) For a given feature $f_i^/$, we consult each primary and secondary membership curve under a given emotion class, and take the product of primary and secondary membership at $f_i = f_i^/$. The resulting membership value obtained for the membership curves for the subject w in the training data is given by

$$ {}^{\text{mod}}\mu_{\tilde{A}}^{w}\left(f_i^/\right) = \mu_{\tilde{A}}^{w}\left(f_i^/\right) \times \mu\left(f_i^/, \mu_{\tilde{A}}^{w}\left(f_i^/\right)\right) \tag{18} $$

where the notations have their usual meaning. Now, for $w = 1 ton$, we evaluate ${}^{\text{mod}}\mu_{\tilde{A}}^{w}(f_i^/)$, and thus obtain the minimum and the maximum values of ${}^{\text{mod}}\mu_{\tilde{A}}^{w}(f_i^/)$, to obtain a range of uncertainty $[{}^{\text{mod}}\underline{\mu}_{\tilde{A}}(f_i^/), {}^{\text{mod}}\overline{\mu}_{\tilde{A}}(f_i^/)]$. This is repeated for all features under each emotion class. In Fig. 4 we, unlike conventional approaches, present secondary membership functions against feature $f_i^/$, for $i = 1 tom$. Such representation is required to demonstrate the computation of ${}^{\text{mod}}\underline{\mu}_{\tilde{A}}(f_i^/)$.
4) Step 4 is the same as that in IT2FS-based recognition scheme with the replacement of $\underline{\mu}_{\tilde{A}}(f_i^/)$ and $\overline{\mu}_{\tilde{A}}(f_i^/)$,

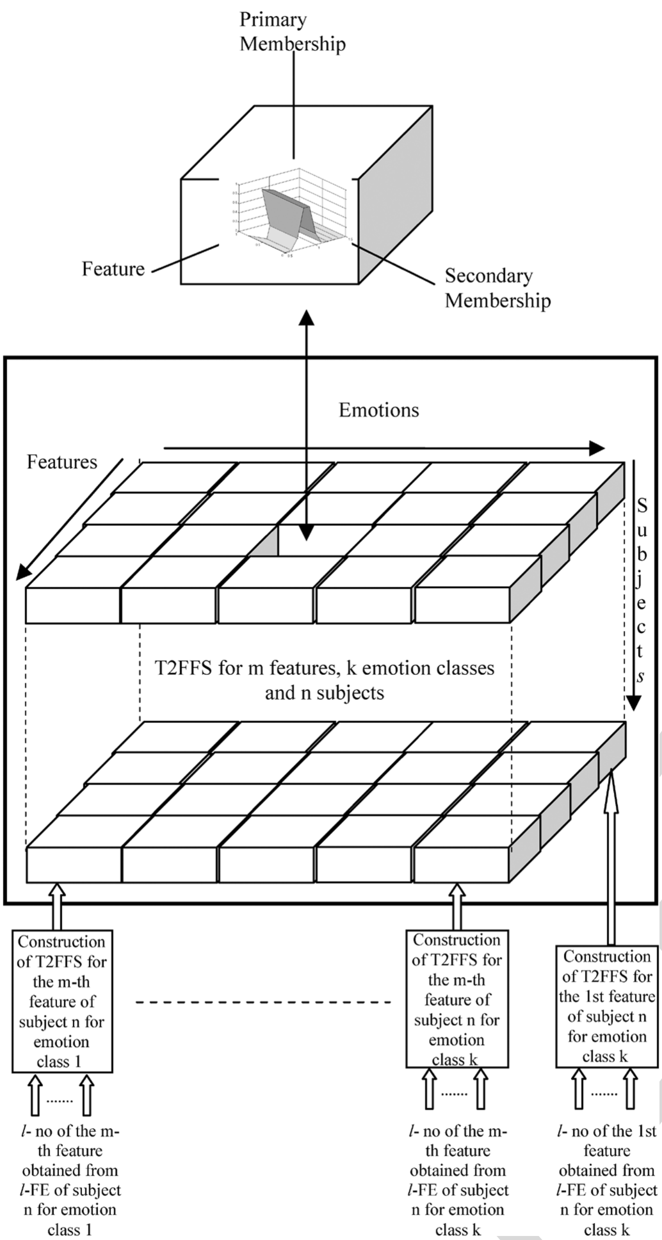


Fig. 3. General type-2 fuzzy face-space construction for m features, k emotion classes, and n subjects.

respectively, by ${}^{\text{mod}}\underline{\mu}_{\tilde{A}}(f_i^{/})$ and ${}^{\text{mod}}\overline{\mu}_{\tilde{A}}(f_i^{/})$. Steps 5 and 6 are exactly similar to those in IT2FS-based recognition scheme. A complete scheme for GT2FS-based emotion recognition, considering support of k-emotion classes is given in Fig. 5.

IV. Fuzzy T2 Membership Evaluation

In this, we discuss T2 membership evaluation [37]–[39]. Although theoretically very sound, T2 fuzzy set has limitedly been used over the last two decades because of the users' inadequate knowledge to correctly assign the secondary memberships. This paper, however, overcomes this problem by extracting T2 membership function from its T1 counterpart by an evolutionary algorithm. A brief outline to the construction of secondary membership function is given in this section.

Intuitively, when an expert assigns a grade of membership, she is relatively more certain to determine the location of the peaks and the minima of the function, but may not have enough background knowledge to correctly assign the membership values at other points. Presuming that the (secondary) membership values at the peak and the minima are close to 1, we attempt to compute secondary memberships at the remaining part of the secondary membership function. The following assumptions are used to construct an objective function, which is minimized to obtain the solution of the problem.

1) Let $x = x_p$ and $x = x_q$ be two successive optima (peak/minimum) on the primary membership function $\mu_A(x)$. Then, at any point x lying between x_p and x_q, the secondary membership $\mu(x, \mu_A(x))$ will be smaller than both $\mu(x_p, \mu_A(x_p))$ and $\mu(x_q, \mu_A(x_q))$.
2) The fall-off in secondary membership at a point x away from its value at a peak/minimum $\mu(x_p, \mu_A(x_p))$ is exponential, given by

$$\mu\left(x, \mu_A(x)\right) = \mu\left(x_p, \mu_A(x_p)\right).\exp\left(-|x - x_p|\right). \tag{19}$$

3) The secondary membership at any point x between two consecutive optima at $x = x_p$ and $x = x_q$ in the primary membership is selected from the range $[\alpha, \beta]$, where

$$\left.\begin{array}{l}\alpha = \mu\left(x_p, \mu_A(x_p)\right).\exp\left(-|x - x_p|\right)\\ \beta = \mu\left(x_q, \mu_A(x_q)\right).\exp\left(-|x - x_q|\right)\end{array}\right\}. \tag{20}$$

T1 defuzzification over the average of n primary membership functions should return the same value as obtained by T2 defuzzification for a given primary membership function for any given source. This assumption holds because the two modalities of defuzzification, representing the same real-world parameter, should return close values, ignoring the average inter-personal level of uncertainty while taking the average of n-primary membership functions.
4) The unknown secondary membership at two values of x separated by a small positive δ should have a small difference. This is required to avoid sharp changes in the secondary grade.

Let the primary membership functions for feature $f_i = x$ from n sources be $\mu^1_{\tilde{A}}(x), \mu^2_{\tilde{A}}(x), \ldots, \mu^n_{\tilde{A}}(x)$. Then, the average membership function which represents a special form of fuzzy aggregation is given by

$$\mu_{\tilde{\mathrm{A}}}(x) = \frac{\sum_{\mathrm{i=1}}^{\mathrm{n}} \mu^{\mathrm{i}}_{\tilde{\mathrm{A}}}(x)}{n}, \forall x \tag{21}$$

i.e., at each position of $x = x_j$, the above membership aggregation is employed to evaluate a new composite membership profile $\mu_{\tilde{A}}(x)$. The defuzzified signal obtained by the centroid method [36] from the averaged primary membership function is given by

$$\bar{\bar{c}} = \frac{\sum_{\forall \mathrm{x}} x.\mu_{\tilde{\mathrm{A}}}(x)}{\sum_{\forall \mathrm{x}} \mu_{\tilde{\mathrm{A}}}(x)}. \tag{22}$$

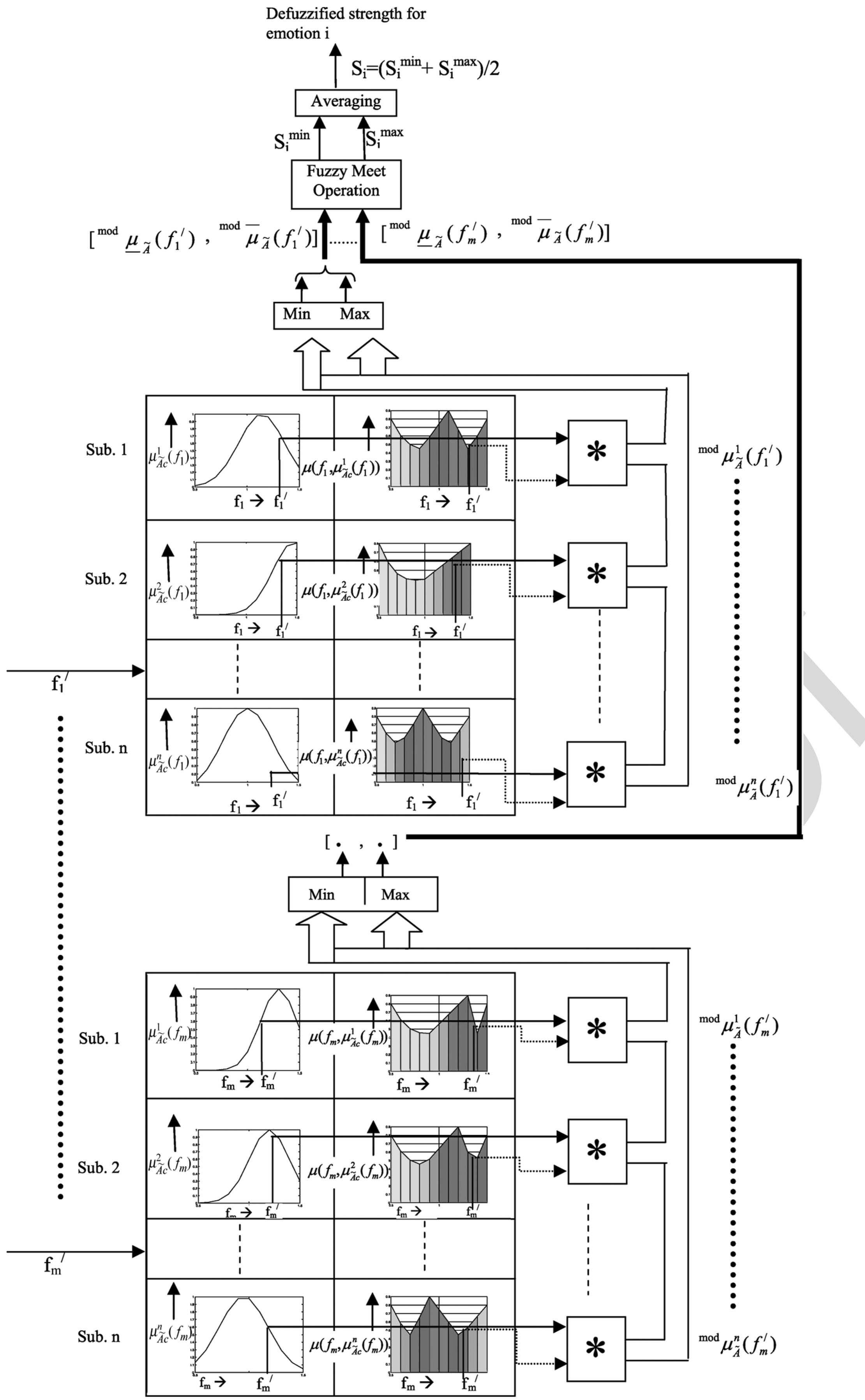


Fig. 4. Computing support of the general type-2 fuzzy FS for emotion class i.

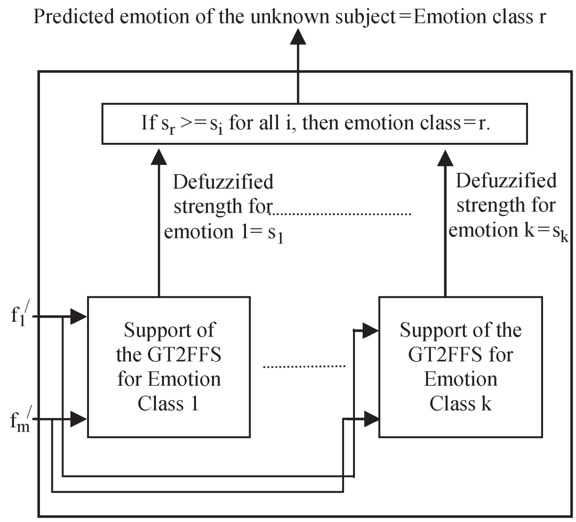


Fig. 5. GT2FFS-based emotion classification.

Further, the T2 centroidal defuzzified signal obtained from the ith primary and secondary membership functions here is defined as

$$\overline{c_i} = \frac{\sum_{\forall x} x.\mu^{\mathrm{i}}_{\tilde{A}}(x).\mu\left(x,\mu^{\mathrm{i}}_{\tilde{A}}(x)\right)}{\sum_{\forall x}\mu^{\mathrm{i}}_{\tilde{A}}(x).\mu\left(x,\mu^{\mathrm{i}}_{\tilde{A}}(x)\right)}. \tag{23}$$

The products of primary and secondary memberships are used in (23) to refine the primary memberships by the degree of certainty of the corresponding secondary values.

Using assumptions 3 and 4, we construct a performance index J_{i} to compute secondary membership for the ith subject for a given emotion

$$J_i=(\overline{c_i}-\overline{\overline{c}})^2+\sum_{x=x_1}^{x_{R-1}}\left\{\mu\left((x+\delta),\mu^{i}_{\tilde{A}}(x+\delta)\right)-\mu\left(x,\mu^{i}_{\tilde{A}}(x)\right)\right\}^2. \tag{24}$$

The second term in (24) acts as a regularizing term to prevent abrupt changes in the membership function. In (24),x_1 and x_R are the smallest and the largest values of a given feature considered over R sampled points of $\mu^i_{\tilde{A}}(x)$. In (24), $\delta = (x_R - x_1)/(R-1)$ and $x_k = x_1 + (k-1).\delta$ for $k = 1,\ldots,R$. The secondary membership evaluation problem now transforms to minimization of J_i by selecting $\mu(x,\mu^i_{\tilde{A}}(x))$ from a given range $[\alpha,\beta]$, where α and β are the secondary memberships at the two optima in secondary membership around the point x. Expressions (20) are used to compute α and β for each x separately. Note that, for each subject carrying individual emotion, we have to define (23) and (24) and find the optimal secondary membership functions.

Any derivative-free optimization algorithm can be used to minimize J_i with respect to secondary memberships, and obtain $\mu(x,\mu^i_{\tilde{A}}(x))$ at each x except the optima on the secondary membership. Differential evolution (DE) [34] is one such derivative-free optimization algorithm, which has fewer control parameters, and has outperformed the well-known binary coded genetic algorithm [54] and particle swarm optimization algorithms [55] with respect to standard benchmark functions [45]. Further, DE is simple and involves only a few lines code, which motivated us to employ it to solve the above optimization problem.

An outline to basic DE [34] is given in the Appendix. An algorithm to compute the secondary membership function of a T2 fuzzy set from its primary counterpart using DE is given below.

1) Obtain the averaged primary membership function $\mu_{\tilde{A}}(x)$ from the primary membership functions $\mu^i_{\tilde{A}}(x)$ obtained from n sources, i.e., $i = 1,\ldots,n$. Evaluate $\overline{\overline{c}}$, and also $\overline{c_i}$ for a selected primary membership distribution $\mu^i_{\tilde{A}}(x)$ using (22) and (23), respectively.
2) Find the optima on $\mu^j_{\tilde{A}}(x)$ for a given j. Let the set of x corresponding to the optima be S. Set the secondary membership $\mu(x,\mu^j_{\tilde{A}}(x))$ to 0.99 (close to one) for all $x \in S$.
3) For each $x \in X$, where $x \notin S$, identify the optima closest around x from S. Let they be located at $x = x_p$ and $x = x_q$, where $x_p < x < x_q$. Determine α and β for each x, given by (20).
4) For each x, where $\mu(x,\mu^j_{\tilde{A}}(x))$ lies in $[\alpha,\beta]$, minimize J_{j} by DE.
5) Obtain $\mu(x,\mu^j_{\tilde{A}}(x))$ for all x after the DE converges.
6) Repeat step 2 onwards for all j.

For a Gaussian primary membership function, the minimum occurs at infinity, but the minimum value is practically zero when x is $m \pm 4\sigma$, where m and σ are mean and standard deviation of x. In Step 2, the minimum is taken as $m \pm 4\sigma$, and we obtain x by dividing the range $[m-4\sigma, m+4\sigma]$ into equal intervals of same length (here 20 intervals).

An illustrative plot of secondary membership function for a given primary is given in Fig. 6.

V. Filtering Unwanted Data Points in Feature Space Using Interval Approach

The IT2FS-based scheme for emotion recognition given in Section III is computationally efficient with good classification accuracy. However, its performance depends greatly on the measurements obtained from facial expressions of the experimental subjects. In order to reduce the effect of outliers, we here present a scheme of data pre-processing/filtering and selection of membership functions following the well-known IA [68].

The important steps of IA used in the present context are re-structured for the present application as outlined below. Let $[a^{(i)}, b^{(i)}]$ be the end-point interval of measurements of a given facial feature for the ith subject obtained from l instances of her facial expressions for a specific emotion.

Step 1) (Outlier processing): This step divides the two sets of lower and upper data end-points: $a^{(i)}$ and $b^{(i)}$, respectively, for $i = 1$ to n subjects in quartiles, and tests the acceptability of each data end-point by satisfying the following criteria:

$$\left.\begin{array}{l} a^{(i)} \in [Q_a(0.25)-1.5IQR_a, Q_a(0.75)+1.5IQR_a] \\ b^{(i)} \in [Q_b(0.25)-1.5IQR_b, Q_b(0.75)+1.5IQR_b] \\ L^{(i)} \in [Q_L(0.25)-1.5IQR_L, Q_L(0.75)+1.5IQR_L] \end{array}\right\} \tag{25}$$

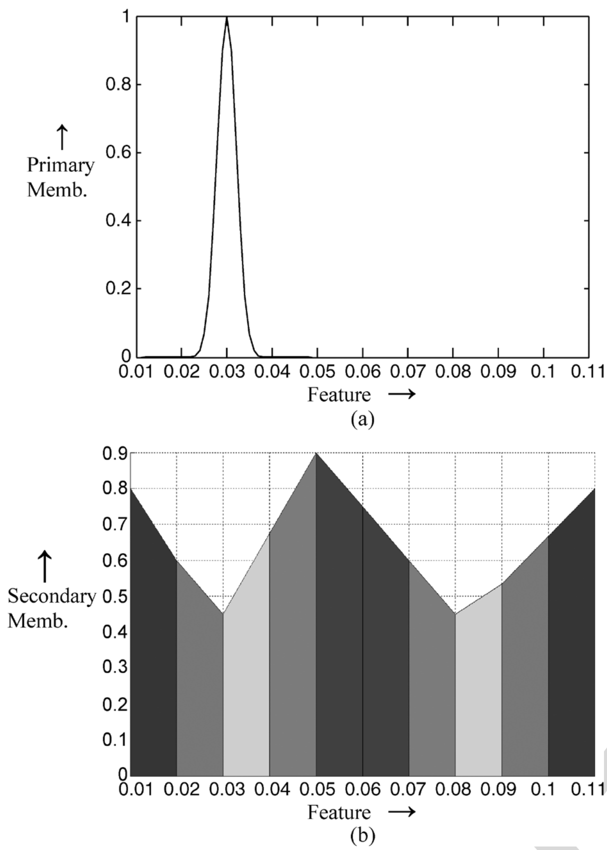


Fig. 6. (a) The primary membership function for a given feature and (b) its corresponding secondary membership function obtained by minimizing J_i.

where $Q_j(x)$ denotes the quartile ranges containing the first $x\%$ of the data points in the i-*th* data set. Here, $j \in \{a, b, L\}$ and a, b denote lower, upper end points of intervals, and L is the length of an interval. IQR denotes intra-quartile range and is defined by $Q(0.75)$ minus $Q(0.25)$. The suffixes a, b and L in IQR denote the IQR for left, right end points and interval length, respectively. $L^{(i)}$ is defined as the length of data interval $= b^{(i)} - a^{(i)}$, for $i = 1 ton$. The reduced set of data end-points after outlier processing is $n^{/}$.

Step 2) (Tolerance limit processing): This step deals with tolerance limit processing by presuming the data distributions to be Gaussian, and testing whether lower/upper data end-points: $a^{(i)}, b^{(i)}$ and interval length $L^{(i)}$ lie within mean plus/minus $k(= 2.752)$ times the standard deviation of the data points. The number 2.752 appears in the scenario for statistical validation with 20 data end-point intervals for 20 subjects [68].

If a data interval $[a^{(i)}, b^{(i)}]$ and its length $L^{(i)}$ satisfy (26), the interval is accepted, otherwise rejected:

$$\left.\begin{array}{l} a^{(i)} \in [m_l - ks_l, m_l + ks_l] \\ b^{(i)} \in [m_r - ks_r, m_r + ks_r] \\ L^{(i)} \in [m_L - ks_L, m_L + ks_L] \end{array}\right\} \qquad (26)$$

where, m_j and s_j denotes sample mean and standard deviation for $j \in \{l, r, L\}$, for the $n^{/}$ set of data points/intervals. After tolerance processing, the reduced set of data end-points is $n^{//}$.

Step 3) (*Reasonable-interval test*): This step checks whether intervals are reasonable, i.e., they are overlapped. This has been performed by computing ξ^*, given in (27) and then by testing whether lower bounds of each interval $a^{(i)} < \xi^*$ and upper bound $b^{(i)} > \xi^*$, where $\xi^*$is one of the possible values of

$$\xi^* = \frac{\left(m_r\sigma_l^2 - m_l\sigma_r^2\right) \pm \sigma_l\sigma_r\left[(m_l - m_r)^2 + 2\left(\sigma_l^2 - \sigma_r^2\right)\ln\left(\frac{\sigma_l}{\sigma_r}\right)\right]^{\frac{1}{2}}}{\sigma_l^2 - \sigma_r^2} \qquad (27)$$

where m_l and σ_l are sample mean and variance of the $n^{//}$ left endpoints and m_r and σ_r are sample mean and variance of the $n^{//}$ right endpoints. If $m_l <= \xi^* <= m_r$ is satisfied, then the data intervals are retained and dropped otherwise. The remaining number of data points after the drop of some intervals is called $n^{///}$.

Step 4) (FOU selection): This step is used for the selection of the right FOU among triangle, left shoulder, and right shoulder. For each FOU, the criteria can be found in [68]. We here reproduce the results for triangular FOU only, as our results to be given in Section VI yields triangular FOU. For triangular FOU, the conditions are

$$\left.\begin{array}{l} m_r \leq 5.831 m_l - 1.328\frac{s_c}{\sqrt{n^{///}}} \\ m_r \leq 0.171 m_l + 8.29 - 1.328\frac{s_d}{\sqrt{n^{///}}} \\ m_r \geq m_l \end{array}\right\} \qquad (28)$$

where $s_c =$ standard deviation of $[b^{(i)} - 5.831a^{(i)}]$ for $i = 1ton^{///}$, $s_d =$ standard deviation of $[b^{(i)} - 0.17a^{(i)} - 8.29]$ for $i = 1ton^{///}$.

Step 5) (FOU parameter evaluation): This step deals with parameter evaluation of the triangular membership functions for the existing data intervals $[a^{(i)}, b^{(i)}]$. For each interval $[a^{(i)}, b^{(i)}]$, we obtain the parameters $a_{MF}^{(i)}$ and $b_{MF}^{(i)}$ representing the end-points of the x-coordinates of the base for a symmetric triangular membership function as reproduced below [68]:

$$\left.\begin{array}{l} a_{MF}^{(i)} = \frac{1}{2}\left[\left(a^{(i)} + b^{(i)}\right) - \sqrt{2}\left(b^{(i)} - a^{(i)}\right)\right]. \\ b_{MF}^{(i)} = \frac{1}{2}\left[\left(a^{(i)} + b^{(i)}\right) + \sqrt{2}\left(b^{(i)} - a^{(i)}\right)\right] \end{array}\right\}. \qquad (29)$$

We use these membership functions in place of Gaussian membership functions in our IT2FS approach and call this approach as IT-IT2FS.

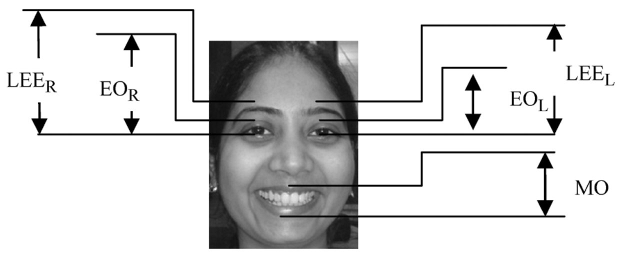
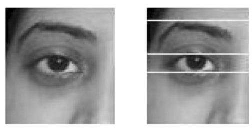


Fig. 7. Facial features.



(a) (b)

Fig. 8. (a) Localized eye search region, and (b) detection of eye features.

(a) (b) (c)

Fig. 9. (a) Mouth search area, (b) lip cluster, and (c) graph of average intensity over each row against the row position.

VI. Experiments Details

In this section, we present the experimental details of emotion recognition using the principles introduced in Sections III–V. Here, we consider the following $k(=5)$ emotion classes: anger, fear, disgust, happiness, and relaxation. The experiment is conducted with two sets of subjects: 1) the first set of $n(=20)$ subjects is considered for designing the fuzzy face space and 2) the other set of 40 facial expressions taken from six unknown subjects is considered to validate the result of the proposed emotion classification scheme. Five facial features (i.e., $m = 5$) have been used here to design the T2 fuzzy face space.

We now briefly overview the main steps of feature extraction followed by fuzzy face-space construction and emotion recognition of an unknown subject using the pre-constructed face space.

A. Feature Extraction

Feature extraction is a fundamental step in emotion recognition. This paper considers extraction of features from emotionally rich facial expressions synthesized by the subjects by acting. Existing research results [14], [28] reveal that the most important facial regions responsible for the manifestation of emotion are the eyes and the lips. This motivated us to select the following features: Left eye opening $(\mathrm{EO_L})$, right eye opening $(\mathrm{EO_R})$, Distance between the Lower Eyelid to the Eyebrow for the Left Eye $(\mathrm{LEE_L})$, distance between the lower eyelid to eyebrow for the right eye $(\mathrm{LEE_R})$, and the maximum mouth opening (MO) including the lower and the upper lips. Fig. 7 explains the above facial features on a selected facial image.

For extraction of any of the features mentioned above, the first step that needs to be carried out is to separate out the skin and the non-skin regions of the image.

Estimation of Eye Features ($\mathrm{EO_L}$, $\mathrm{LEE_L}$, $\mathrm{EO_R}$, and $\mathrm{LEE_R}$): To compute the eye features, we first localize the eye region as shown in Fig. 8(a). The image in Fig. 8(a) is now transformed to gray scale, and average intensity over each row of pixels is evaluated. Now, we identify the row with the maximum dip in average intensity, while scanning the image from top. This row indicates the first dark region from top, i.e., the eyebrow region (Fig. 8(b)). Similarly, we detect the lower eyelid by identifying the row with the maximum dip in intensity in the gray scale image, while scanning the face up from the bottommost row. The location of the top eyelid region is identified by scanning the face up from the marked lower eyelid until the maximum dip occurs in the gray scale image.

Estimation of MO: In order to estimate the MO, we first localize the mouth region as shown in Fig. 9(a). Then, a conversion from R-G-B to perceptually uniform $L^* - a^* - b^*$ color space is undertaken in order to represent the perceptual difference in color by Euclidean distance [69]. The k-means clustering algorithm is applied next on this image to segment the image into three clusters, namely skin, lip, and teeth regions. Each cluster is now transformed to gray scale, and the one with the highest average gradient of the boundary points (in intensity) is declared as the lip region. Now, to obtain the MO, we plot the average intensity over each row of Fig. 9(b) against the row number. The width of the zero-crossing zone in the plot (Fig. 9(c)) provides a measure of MO.

Experiments are undertaken both on colored image database such as the Indian Women (Jadavpur University) database, and gray scale images including Japanese Female Facial Expression (JAFFE) and Cohn-Kanade databases. The principles of feature extraction introduced above are equally applicable in both color and gray scale images. However, for color images, we need a conversion to gray scale to determine the features of eye and mouth of the subject. In addition, for the gray scale facial images, segmentation of lip-, skin-, and teeth-regions is performed with intensity data only, unlike the case in color images, where we use the 3-D data points (L^*, a^*, b^*) as the input to the k-means algorithm for segmentation.

Selective images from three facial expression databases are given in Fig. 10. Training and test image data partition for three experimental databases is given in Table I. The training data in Table I include l instances for n subjects for k distinct emotions.

The following explanation in this section is given with respect to Indian Woman Database (Jadavpur University).

B. Creating the T2 Fuzzy Face Space

The interval T2 fuzzy face space contains only the primary membership distributions for each facial feature. Since we have five facial features, and the experiment includes five distinct emotions of 20 subjects, we obtain $20 \times 5 \times 5 = 500$ primary membership curves. To compute primary memberships, ten instances of a given emotion are used. These 500 membership curves are grouped into 25 heads, each containing

Fig. 10. Experiment done on different databases: a) JAFFE, b) Indian women database (Jadavpur University), c) Cohn-Kanade database.

TABLE I
TRAINING AND TEST DATA FOR THREE DATABASES

Databases used	Training Images ($n \times l \times k$)	Test Images selected at random
JAFFE	5×3×5	40
Indian Woman(J.U)	20×10×5	40
Cohn-Kanade	10×5×5	40

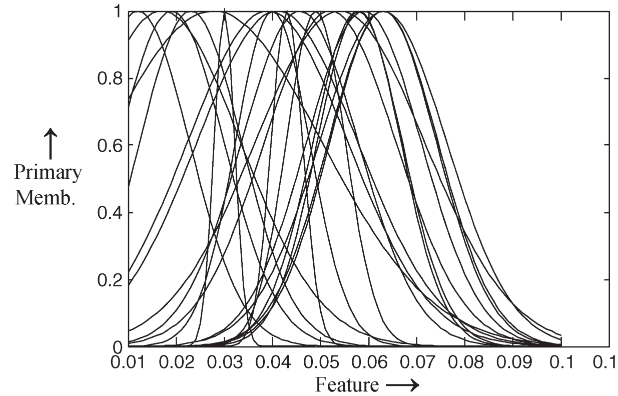


Fig. 11. Membership distributions for emotion anger and feature EO_L.

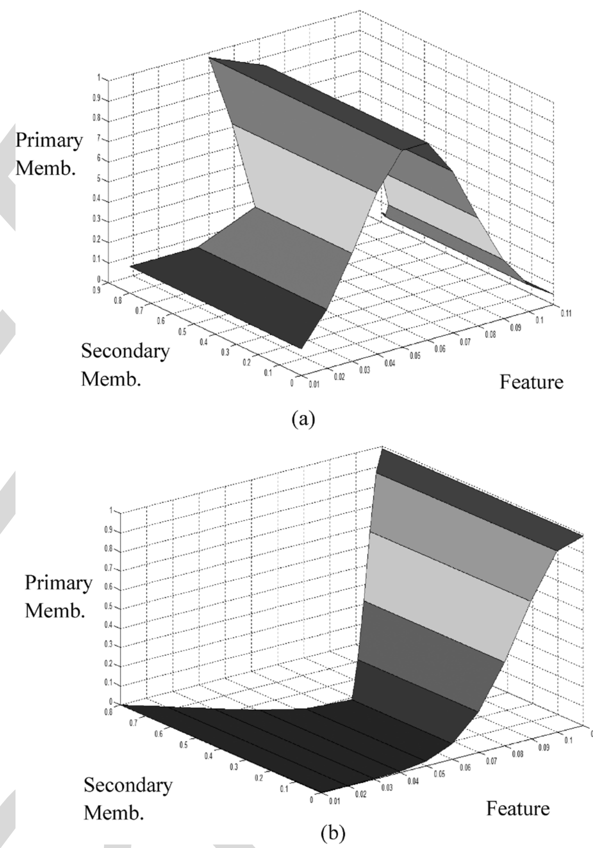


Fig. 12. (a) Secondary membership curve of subject 1. (b) Secondary membership curve of subject 2 for emotion anger.

20 membership curves of 20 subjects for a specific feature for a given emotion. Fig. 11 gives an illustration of one such group of 20 membership functions for the feature EO_L for the emotion: Anger.

For each primary membership function, we have a corresponding secondary membership function. Thus, we obtain 500 secondary membership functions. Two illustrative T2 secondary memberships for subjects 1 and 2 for the feature EO_L for the emotion anger are given in Fig. 12. The axes in the figure represent feature (EO_L), primary and secondary membership values as indicated.

C. Emotion Recognition of an Unknown Facial Expression

The emotion recognition problem addressed here attempts to determine the emotion of an unknown person from her facial expression. To keep the measurements in an emotional expression normalized and free from distance variation from the camera focal plane, we construct a bounding box, covering only the face region, and the reciprocal of the diagonal of the bounding box is used as a scale factor for normalization of the measurements. The normalized features obtained from Fig. 13 are listed in Table II. We now briefly explain the experimental results obtained by two alternative reasoning methodologies incorporating IT2FS and GT2FS.

Fig. 13. Facial image of an unknown subject.

TABLE II
EXTRACTED FEATURES OF FIG. 13

EO_L	EO_R	MO	LEE_L	LEE_R
0.026	0.026	0.135	0.115	0.115

TABLE III
CALCULATED TYPE-2 PRIMARY MEMBERSHIP VALUES FOR THE FEATURE: EO_L UNDER EMOTION: DISGUST

Feature: EO_L(pri)									
Primary Memberships (μ_{pri})									
0.65	0.10	0.15	0.45	0.18	0.55	0.06	0.41	0.16	0.12
0.38	0.45	0.09	0.19	0.67	0.68	0.52	0.44	0.37	0.55
Range (min{ μ_{pri}}, max{ μ_{pri}})= [0.06, 0.68]									

TABLE IV
CALCULATED RANGES OF PRIMARY MEMBERSHIPS AND CENTRE VALUE FOR EACH EMOTION

Emotion	Range of Primary Membership for Features					Range S_c^j after fuzzy Meet operation **(centre)**
	EO_L	EO_R	MO	LEE_L	LEE_R	
Anger	0-0.18	0-0.24	0.076 -0.764	0-0.215	0.001-0.234	0-0.18 (0.09)
Disgust	0.06-0.68	0.064-0.65	0-0.52	0-0.58	0-0.78	0-0.52 (0.26)
Fear	0 - 0.067	0-0.071	0.194-0.914	0.042-0.74	0.038-0.729	0-0.067 (0.0335)
Happiness	0 - 0.687	0-0.694	0.12-0.897	0.57-0.85	0.64-0.89	0-0.687 (**0.3435**)
Relaxed	0 - 0.384	0-0.393	0-0.052	0.076-0.89	0.081-0.92	0-0.052 (0.026)

IT2FS-Based Recognition: The IT2FS-based recognition scheme considers a fuzzy face space of five sets of 20 primary membership functions as in Fig. 11, where each set refers to one particular feature obtained from 20 sources for an individual emotion. Consequently, for five distinct emotions, we have 25 such sets of primary membership functions. Table III provides the evaluation of T2 primary membership values for the feature, EO_L, consulting 20 primary functions obtained from 20 subjects, representing the facial expression for disgust. The range of these memberships is given in the last row of Table III. For each feature, we obtain five tables like Table III, each one for a given emotion. Thus, for five features, we would have altogether 25 such tables.

Table IV provides the results of individual range in primary membership for each feature experimented under different emotional conditions. For example, the entry (0–0.18) corresponding to the row anger and column EO_L gives an idea about the extent of the EO_L for the unknown subject matches with known subjects from the emotion class anger. The results of computing fuzzy meet operation over the range of individual

TABLE V
RESULTS OF EXECUTION OF IA ON FEATURE EO_L DATA SET FOR EMOTION: ANGER

Data Preprocessing
Data points taken: 20 pairs of $a^{(i)}$, $b^{(i)}$ for i=1 to 20 subjects
Step-1: Outlier Processing Result: deleted point is [0.021, 0.113]
Step- 2: Tolerance Limit Processing Result: no deletion
Step- 3: Reasonable- interval Rest Result: no deletion
FOU Selection:
Step- 4: Computed values for: S_c= 0.0934; S_d= 0.0179; Test Condition (given in Fig. 14) Result: FOU = Triangle as (m_l, m_r) = (0.0755, 0.12257) lies in the interior FOU (triangle) obtained by satisfying (28)
Triangle Parameter Evaluation
Step- 5: a_{MF}, b_{MF} evaluated from (29) Result: Given in Fig. 15.

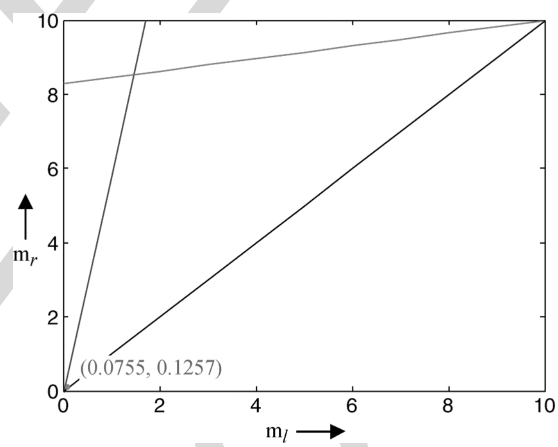


Fig. 14. Graphical selection of FOU by testing that the point $(m_l, m_r) = (0.0755, 0.1257)$ plotted in the figure lies under the triangular zone obtained by satisfying inequalities in (28).

features taken from facial expressions of the subjects under the same emotional condition are given in Table IV. The average of the ranges along with its center value is also given in Table IV. It is observed that the center has the largest value $(= 0.3435)$ for the emotion: happiness.

IT2FS-Based Recognition With Pre-Processing of Features Using the Interval Approach (Hereafter IA-IT2FS): The IA introduced in Section V has two fundamental merits. It eliminates noisy data points obtained from facial data of the subjects. It also helps in identifying the primary membership functions for each feature of a facial expression representing a specific emotion by a statistically meaningful approach. The results of execution of adapted IA algorithm given in the last section for the feature EO_L for the emotion anger are given in Table V for convenience. After similar tables for all features of all possible emotions are determined, we use the statistically significant FOU for each feature of each emotion. In Fig. 14, we provide an illustrative experimental FOU for the feature EO_L for emotion

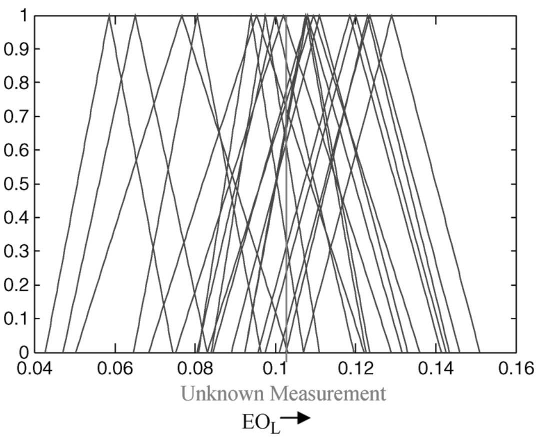


Fig. 15. Constructed symmetric triangular membership functions using (29).

TABLE VI
CALCULATED TYPE-2 MEMBERSHIP VALUES FOR THE FEATURE: EO_L UNDER EMOTION: DISGUST

Feature	Primary Memberships (μ_{pri})	Secondary memberships (μ_{sec})	$\mu^{mod}=\mu_{pri}\times\mu_{sec}$	Range ($\min\{\mu^{mod}\}$, $\max\{\mu^{mod}\}$)
EO_L	0.65	0.06	0.039	0.039-0.4355
	0.1	0.87	0.087	
	0.15	0.85	0.1275	
	0.45	0.53	0.2385	
	0.18	0.74	0.1332	
	0.55	0.52	0.286	
	0.08	0.88	0.0704	
	0.41	0.53	0.2173	
	0.16	0.78	0.1248	
	0.12	0.81	0.0972	
	0.38	0.67	0.2546	
	0.45	0.58	0.261	
	0.09	0.89	0.0801	
	0.19	0.72	0.1368	
	0.67	0.65	0.4355	
	0.68	0.58	0.3944	
	0.52	0.55	0.286	
	0.44	0.67	0.2948	
	0.37	0.78	0.2886	
	0.55	0.53	0.2915	

anger by performing step 4 of Section V. The parameters of the FOU, here triangles, are evaluated by step 5 of Section V. Now, for an unknown facial expression, we follow the steps of IT2FS-based approach to recognize the emotion exhibited in the facial expression. Our experiments reveal that the pre-processing steps by IA help in improving the recognition accuracy of the IT2FS scheme by 2.5% (Fig. 15).

GT2FS-Based Recognition: We now briefly illustrate the GT2FS-based reasoning for emotion classification. Here, the secondary membership function corresponding to the individual primary membership function of five features obtained from facial expressions carrying five distinct emotions for 20 different subjects are determined using membership functions like Fig. 12.

Table VI provides the summary of the primary and secondary memberships obtained for EO_L for the emotion: disgust. The range computation for the feature EO_L is also shown in the last column of Table VI. The same computations are repeated for all emotions, and the range evaluated in the last column of Table VII indicates that the center of this range here too has the largest value $(= 0.301)$ for the emotion: happiness.

TABLE VII
CALCULATED RANGES OF PRIMARY MEMBERSHIP CENTRE VALUE FOR EACH EMOTION

Emotion	Range of Secondary Membership for Features					Range S_c^j after fuzzy Meet operation (**centre**)
	EO_L	EO_R	MO	LEE_L	LEE_R	
Anger	0-0.21	0 - 0.27	0.26 - 0.983	0.0006 - 0.763	0.0006- 0.790	0-0.21 (0.105)
Disgust	0.039- 0.4355	0.031- 0.433	0-0	0-0.15	0.-0.13	0-0 (0)
Fear	0 - 0.312	0-0.295	0.04- 0.713	0.044- 0.564	0.038- 0.571	0-0.295 (0.1475)
Happiness	0 - 0.602	0-0.606	0.273- 0.98	0.06- 0.93	0.064- 0.97	0-0.602 (**0.301**)
Relaxed	0 - 0.425	0-0.421	0-0	0.001- 0.758	0.001- 0.742	0-0 (0)

TABLE VIII
PERCENTAGE ACCURACY OF OUR PROPOSED METHODS OVER THREE DATABASES

	JAFFE	Indian Women (Jadavpur University)	Cohn-Kanade	Average Accuracy (of last 3 columns)
IT2FS	90%	92.5%	92.5%	91.667%
IA-IT2FS	92.5%	95%	95%	94.167%
GT2FS	97.5%	100%	97.5%	98.333%

VII. PERFORMANCE ANALYSIS

Performance analysis for emotion recognition itself is an open-ended research problem, as there is a dearth of literature on this topic. This paper, compares the relative performance of the proposed GT2FS algorithms with five traditional emotion recognition algorithms/techniques and the IA-IT2FS and IT2FS-based schemes introduced here, considering a common framework in terms of their features and databases. The algorithms used for comparison include linear SVM classifier [28], (T1) fuzzy relational approach [14], PCA [33], multilayer perceptron (MLP) [1], [29], radial basis function network (RBFN) [1], [29], IT2FS, and IA-IT2FS [68].

Table VIII shows the classification accuracy of our proposed three algorithms using three facial image databases, i.e., JAFFE, Indian Women Face Database (Jadavpur University), and Cohn-Kanade database. Experimental classification accuracy obtained for different other algorithms mentioned above using the three databases is given in Table X.

Two statistical tests called McNemar's test [58] and Friedman test [59], and one new test, called root mean square error test are undertaken to analyze the relative performance of the proposed algorithms over existing ones.

A. McNemar's Test

Let f_A and f_B be two classifiers obtained by algorithms A and B, when both the algorithms have a common training set R. Let n_{01} be the number of examples misclassified by f_A but not by f_B, and n_{10} be the number of examples misclassified by f_B but not by f_A. Then, under the null hypothesis that

TABLE IX
STATISTICAL COMPARISON OF PERFORMANCE USING MC NEMAR'S TEST WITH THREE DATABASES

Reference Algorithm A=GT2FS						
Classifier Algorithm B used for comparison	JAFFE Database		Indian Database (Jadavpur University)		Cohn-Kanade Database	
	Z_j	Comments on acceptance/ rejection of hypothesis	Z_j	Comments on acceptance/ rejection of hypothesis	Z_j	Comments on acceptance/ rejection of hypothesis
IT2FS	1.333	Accept	1.333	Accept	0.5	Accept
IA-IT2FS	0.5	Accept	0.5	Accept	0	Accept
SVM	1.333	Accept	0	Accept	1.333	Accept
Fuzzy Relational Approach	2.25	Accept	1.333	Accept	1.333	Accept
PCA	0	Accept	2.25	Accept	2.25	Accept
MLP	8.1	Reject	8.1	Reject	8.1	Reject
RBFN	11.077	Reject	10.083	Reject	10.083	Reject

both algorithms have the same error rate, the statistic Z in (30) follows a χ^2 with degree of freedom equals to 1 [59]:

$$Z = \frac{(|n_{01} - n_{10}| - 1)^2}{n_{01} + n_{10}}. \tag{30}$$

Let A be the proposed GT2FS algorithm and B is one of the other seven algorithms. We thus evaluate $Z = Z_1$ through Z_7, where Z_{j} denotes the comparator statistic of misclassification between the GT2FS (Algorithm: A) and the jth of the seven algorithms (Algorithm: B), where the suffix j refers to the algorithm in row number j of Table IX.

Table IX is evaluated to obtain Z_1 through Z_7 and the hypothesis has been rejected, if $Z_{\mathrm{j}} > \chi_1^2$, $0.95 = 3.84$, where χ_1^2, $0.95 = 3.84$ is the value of the chi square distribution for 1 degree of freedom at probability of 0.05 [81].

The last inequality indicates that if the null hypothesis is true, then the probability of χ^2 to be more than 3.84 is less than 0.05. If the hypothesis is not rejected, we consider its acceptance. The decision about acceptance or rejection is also included in Table IX.

It is evident from Table IX that McNemar's test cannot distinguish the performance of the five classification algorithms: IT2FS, IA-IT2FS, SVM, fuzzy relational approach, and PCA that support the hypothesis. Hence, next we use the Friedman test for ranking the algorithms.

B. Friedman Test

The Friedman test [58] ranks the algorithms for each data sets separately. The best performing algorithm gets rank 1. In case of ties, average ranks are used.

TABLE X
AVERAGE RANKING OF CLASSIFICATION ALGORITHMS BY FRIEDMAN TEST, WHERE, CA = Classifier Algorithm, A = GT2FS, B_1 = SVM, B_2 = IT2FS, B_3 = IA-IT2FS, B_4 = Fuzzy Relational Approach, B_5 = PCA, B_6 = MLP, B_7 = RBFN

CA	Classification Accuracy tested by databases			Ranks obtained through experiments with databases			Average Rank (R_j)
	JAFFE	Indian	Cohn-Kanade	JAFFE	Indian	Cohn-Kanade	
A	97.5	100	97.5	1	1	1	1
B_1	90.33	97.57	88.11	4	2	5	3.667
B_2	90	92.5	92.5	5	4	3	4
B_3	92.5	95	95	3	3	2	2.667
B_4	87.5	92	90	6	5	4	5
B_5	95	87.5	87.5	2	6	6	4.667
B_6	72.5	75	72.5	7	7	7	7
B_7	65	67.5	67.5	8	8	8	8

Let r_i^j be the rank of jth algorithm on the ith data set. The average rank of algorithm j then is evaluated by

$$R_j = \frac{1}{N}\sum_{\forall i} r_i^j. \tag{31}$$

The null hypothesis here states that all the algorithms are equivalent, so their individual ranks R_j should be equal. Under the null hypothesis, for large enough N and k, the Friedman statistic χ_F^2 in (32) is distributed as a χ^2 with k-1 degrees of freedom. Here, $k = 8$ and $N = 3$. A larger N of course is desirable; however, emotion databases being fewer, finding large N is not feasible. Here, we consider percentage accuracy of classification as the basis of rank. Table X provides the percentage accuracy of classification with respect to three databases, JAFFE, Indian Woman (Jadavpur University), and Cohn-Kanade and the respective ranks of the algorithm

$$\chi_F^2 = \frac{12N}{k(k+1)}\left[\sum_j R_j^2 - \frac{k(k+1)^2}{4}\right]. \tag{32}$$

Now, using $N = 3$, $k = 8$, and the ranks in Table X, we obtain $\chi_F^2 = 17.67 > \chi_{7,0.95}^2 = 14.067$ [81], where χ_7^2, $0.95 = 14.067$ is the value of the chi square distribution for 7° of freedom at probability of 0.05 [81]

$$\chi_F^2 = \frac{12N}{k(k+1)}\left[\sum_j R_j^2 - \frac{k(k+1)^2}{4}\right]$$
$$= 17.67 > \chi_{7,0.95}^2(14.067).$$

Thus, the hypothesis that the algorithms are equivalent is rejected. Therefore, the performances of the algorithms are determined by their ranks only. The order of ranking of the algorithm is apparent from their average ranks. The smaller the average rank, the better is the algorithm. Let ">" be a comparator of relative ranks where $x > y$ means the algorithm x is better in rank than algorithm y. Table X indicates that the relative order of ranking of the algorithm by Friedman test as, GT2FS > IA − IT2FS > SVM > IT2FS > PCA >Fuzzy Relational Approach > MLP > RBFN. It is clear from Table X that the average rank of GT2FS is 1 and average rank of IT2FS and IA-IT2FS are 4 and 2, respectively, claiming GT2FS outperforms all the algorithms by Friedman test.

VIII. Conclusion

The paper presents three automatic emotion recognition systems based on IT2FS, IA-IT2FS, and GT2FS. In order to classify an unknown facial expression, these systems make use of the background knowledge about a large face database with known emotion classes. The GT2FS-based recognition scheme requires T2 secondary membership functions, which are obtained using an innovative evolutionary approach that is also proposed in this paper. All the schemes first construct a fuzzy face space, and then infer the emotion class of the unknown facial expression by determining the maximum support of the individual emotion classes using the pre-constructed fuzzy face space. The class with the highest support is assigned as the emotion of the unknown facial expression.

The IT2FS-based recognition scheme takes care of the inter-subject level uncertainty in computing the maximum support of individual emotion class. The GT2FS-based recognition scheme, however, takes care of both the inter- and intra-subject level uncertainty, and thus offers higher classification accuracy for the same set of features. Using three data sets, the classification accuracy obtained by employing GT2FS is 98.333%, by IT2FS is 91.667%, and by IA-IT2FS is 94.167%.

The more the number of subjects used for constructing the fuzzy face space, the better would be the fuzzy face space, and thus better would be the classification accuracy. Since the fuzzy face space is created offline, the online computational load to recognize emotion is insignificantly small in IT2FS. The computational load in GT2FS, however, is large as it includes an optimization procedure to determine the secondary membership for each emotion and for each subject. However, this additional complexity in GT2FS, offers approximately 7% improvement in classification accuracy in comparison to that by IT2FS. The IA-IT2FS has around 2.5% gain in classification accuracy with no more additional computational complexity than IT2FS. It may be noted that the necessary computations in IA-IT2FS for data filtering and membership function selection is performed offline. The statistical tests employed clearly indicate that GT2FS outperforms the seven selected algorithms.

The problems that may be taken up as future research are briefly outlined below. First, new alternative strategies are to be designed to determine secondary memberships without using optimization techniques. Second, a more formal and systematic approach to fuse secondary and primary memberships to reduce uncertainty in the fuzzy face space is to be developed. Last, we would try to explore the power of fuzzy logic to determine emotion classes in absence of sufficient (or even no) measurements. Facial features, for example MO, may be directly encoded into fuzzy features with fuzzy sets, such as "a little," "more," and "not so large," and then an IT2FS-based model may be adopted to recognize emotion of unknown subjects. Classification accuracy under this circumstance could be poor, but a more human-like interpretation of emotion can be given in the absence of precise measurements.

Appendix
The Classical Differential Evolution Algorithm [34]

An iteration of the classical DE algorithm consists of the four basic steps—initialization of a population of vectors, mutation, crossover or recombination, and finally selection. The main steps of classical DE are given below:

I. Set the generation number $t = 0$ and randomly initialize a population of NP individuals $\vec{P}_t = \{\vec{X}_1(t), \vec{X}_2(t), \ldots, \vec{X}_{NP}(t)\}$ with $\vec{X}_1(t) = \{x_{i,1}(t), x_{i,2}(t), \ldots, x_{i,D}(t)\}$ and each individual uniformly distributed in the range $[\vec{X}_{\min}, \vec{X}_{\max}]$, where $X_{\min} = \{x_{\min,1}, x_{\min,2}, \ldots, x_{\min,D}\}$ and $\vec{X}_{\max} = \{x_{\max,1}, x_{\max,2}, \ldots, x_{\max,D}\}$ with $i = [1, 2, \ldots, NP]$.

II. **while** stopping criterion is not reached, **do**

for$i = 1$**to**NP

a. **Mutation**:

Generate a donor vector $\vec{V}(t) = \{v_{i,1}(t), v_{i,2}(t), \ldots\ldots, v_{i,D}(t)\}$ corresponding to the ith target vector $\vec{X}_1(t)$ by the following scheme $\vec{V}_1(t) = \vec{X}_{r1}(t) + F * (\vec{X}_{r2}(t) - \vec{X}_{r3}(t))$ where r_1, r_2 and r_3 are distinct random integers in the range [1, NP]

b. **Crossover**:

Generate trial vector $\vec{U}_i(t) = \{u_{i,1}(t), u_{i,2}(t), \ldots, u_{i,D}(t)\}$ for the ith target vector $\vec{X}_1(t)$ by binomial crossover as

$$\begin{aligned} \vec{u}_{i,j}(t) &= \vec{v}_{i,j}(t)\ if\ rand\,(0,1) < Cr \\ &= \vec{x}_{i,j}(t)\ otherwise. \end{aligned}$$

c. **Selection**:

Evaluate the trial vector $\vec{U}_i(t)$

if$f(\vec{U}_i(t)) \leq f(\vec{X}_i(t))$,

then$vecX_i(t+1) = vecU_i(t)$

$f(\vec{X}_i(t+1)) =$

$f(vecU_i(t))$

end if

end for

d. Increase the counter value t = t + 1.

end while

The parameters used in the algorithm namely scaling factor "F" and crossover rate "Cr" should be initialized before calling the "while" loop. The terminate condition can be defined in many ways, a few of which include: 1) fixing the number of iterations N, 2) when best fitness of population does not change appreciably over successive iterations, and 3) either of 1) and 2), whichever occurs earlier.

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