Trigonometry (4A)

- Trigonometric Identities
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5/27/12

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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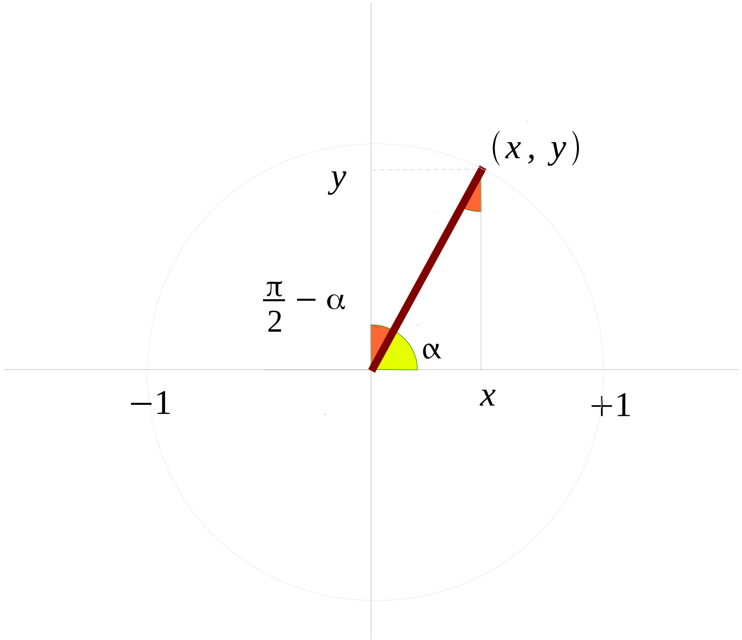
Co-function Identities



$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$$

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$$

$$\tan\left(\frac{\pi}{2} - \alpha\right) = \cot\alpha$$

$$\cot\left(\frac{\pi}{2} - \alpha\right) = \tan\alpha$$

$$csc\left(\frac{\pi}{2} - \alpha\right) = sec\,\alpha$$

$$sec\left(\frac{\pi}{2} - \alpha\right) = sec\,\alpha$$

$$\sin\alpha = y \quad \Rightarrow \quad \cos\left(\frac{\pi}{2} - \alpha\right)$$

$$\cos\alpha = x \quad \Rightarrow \quad \sin\left(\frac{\pi}{2} - \alpha\right)$$

$$\tan\alpha = y/x \Rightarrow \cot\left(\frac{\pi}{2} - \alpha\right)$$

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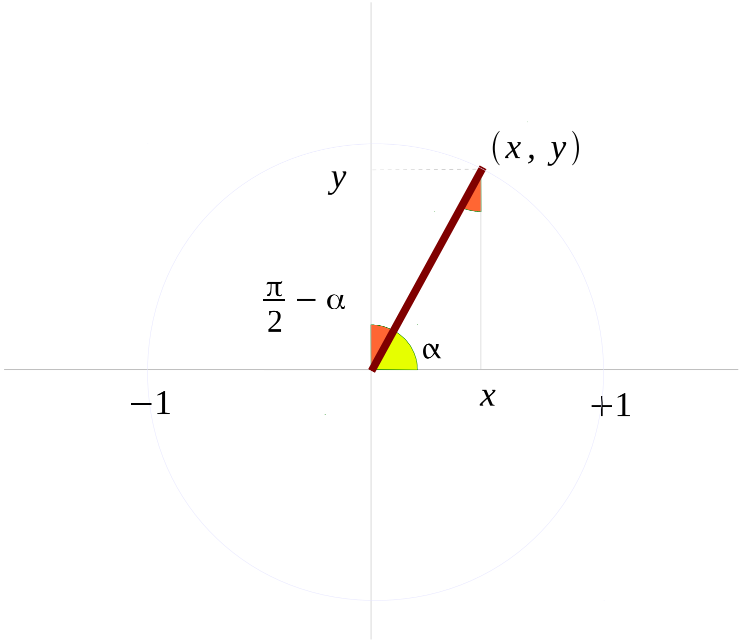
Angle Sum and Difference Identities (1)



$$\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta$$

$$\sin(60^\circ + 30^\circ) = 1$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2} \qquad \sin(30^\circ) = \frac{1}{2}$$

$$\cos(60^\circ) = \frac{1}{2} \qquad \cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\cdot\frac{1}{2} = 1$$

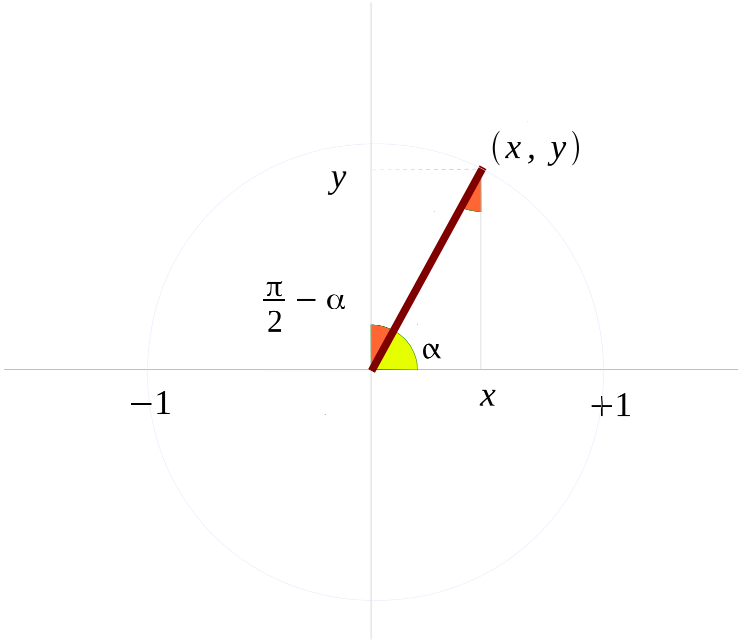
$$\sin(60^\circ - 30^\circ) = \frac{1}{2}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2} \qquad \sin(30^\circ) = \frac{1}{2}$$

$$\cos(60^\circ) = \frac{1}{2} \qquad \cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{2}$$

Angle Sum and Difference Identities (2)



$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$$

$$\cos(30^\circ + 60^\circ) = 0$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2} \qquad \sin(30^\circ) = \frac{1}{2}$$

$$\cos(60^\circ) = \frac{1}{2} \qquad \cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{2} \cdot \frac{1}{2} - \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = 0$$

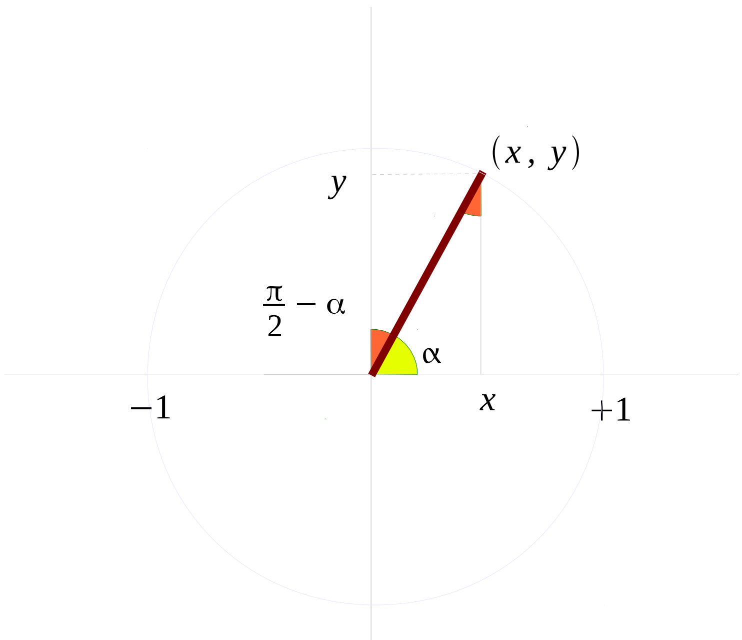
$$\cos(30^\circ - 60^\circ) = \frac{\sqrt{3}}{2}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2} \qquad \sin(30^\circ) = \frac{1}{2}$$

$$\cos(60^\circ) = \frac{1}{2} \qquad \cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

Angle Sum and Difference Identities (3)



$$\tan(\alpha+\beta) = \frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\tan(\beta)}$$

$$\tan(\alpha-\beta) = \frac{\tan(\alpha)-\tan(\beta)}{1+\tan(\alpha)\tan(\beta)}$$

$$\tan(30^\circ+60^\circ) = +\infty$$

$$\tan(30^\circ) = \frac{1}{\sqrt{3}} \qquad \tan(60^\circ) = \sqrt{3}$$

$$\tan(60^\circ) = \sqrt{3} \qquad \tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\frac{\frac{1}{\sqrt{3}}+\sqrt{3}}{1-\frac{1}{\sqrt{3}}\cdot\sqrt{3}} = +\infty$$

$$\tan(30^\circ-60^\circ) = -\frac{1}{\sqrt{3}}$$

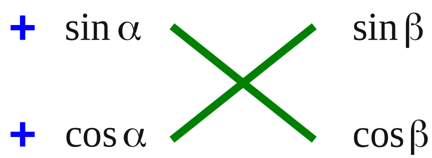
$$\tan(30^\circ) = \frac{1}{\sqrt{3}} \qquad \tan(60^\circ) = \sqrt{3}$$

$$\tan(60^\circ) = \sqrt{3} \qquad \tan(30^\circ) = \frac{1}{\sqrt{3}}$$

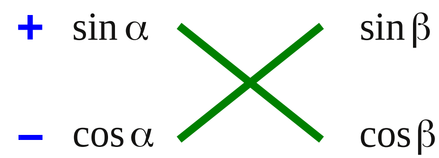
$$\frac{\frac{1}{\sqrt{3}}-\sqrt{3}}{1+\frac{1}{\sqrt{3}}\cdot\sqrt{3}} = -\frac{1}{\sqrt{3}}$$

Angle Sum and Difference Identities (4)

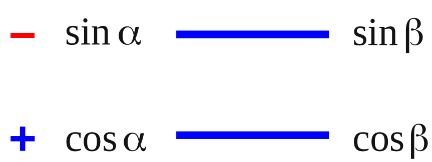
$$\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta$$



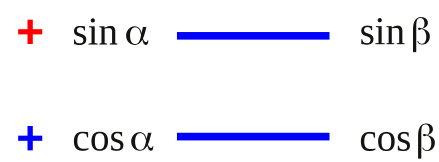
$$\sin(\alpha-\beta) = \sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta$$



$$\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta$$



$$\cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta$$



$$\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta}{\cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta}$$

$$\frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{\sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta}{\cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta}$$

$$\tan(\alpha+\beta) = \frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\tan(\beta)}$$

$$\tan(\alpha-\beta) = \frac{\tan(\alpha)-\tan(\beta)}{1+\tan(\alpha)\tan(\beta)}$$

Product to Sum (1)

$$+\ \sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta$$
$$+\ \sin(\alpha-\beta) = \sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta$$
$$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cdot\cos\beta$$

$$\sin\alpha\cdot\cos\beta = \tfrac{1}{2}\{\sin(\alpha+\beta) + \sin(\alpha-\beta)\}$$

$$+\ \sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta$$
$$-\ \sin(\alpha-\beta) = \sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta$$
$$\sin(\alpha+\beta) - \sin(\alpha-\beta) = 2\cos\alpha\sin\beta$$

$$\cos\alpha\cdot\sin\beta = \tfrac{1}{2}\{\sin(\alpha+\beta) - \sin(\alpha-\beta)\}$$

$$+\ \cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta$$
$$+\ \cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta$$
$$\cos(\alpha+\beta) + \cos(\alpha-\beta) = 2\cos\alpha\cdot\cos\beta$$

$$\cos\alpha\cdot\cos\beta = \tfrac{1}{2}\{+\cos(\alpha+\beta) + \cos(\alpha-\beta)\}$$

$$-\ \cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta$$
$$+\ \cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta$$
$$\cos(\alpha+\beta) - \cos(\alpha-\beta) = 2\sin\alpha\sin\beta$$

$$\sin\alpha\cdot\sin\beta = \tfrac{1}{2}\{-\cos(\alpha+\beta) + \cos(\alpha-\beta)\}$$

Product to Sum (2)

$$\square(\alpha + \beta) = \square\,\alpha \cdot \square\,\beta + \square\,\alpha \cdot \square\,\beta$$

$$\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta$$

$$\square(\alpha - \beta) = \square\,\alpha \cdot \square\,\beta - \square\,\alpha \cdot \square\,\beta$$

$$\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$\square\,\alpha \cdot \square\,\beta = \tfrac{1}{2}\{\square(\alpha + \beta) + \square(\alpha - \beta)\}$$

$$\sin\alpha \cdot \cos\beta = \tfrac{1}{2}\{+\sin(\alpha + \beta) + \sin(\alpha - \beta)\}$$

$$\cos\alpha \cdot \sin\beta = \tfrac{1}{2}\{+\sin(\alpha + \beta) - \sin(\alpha - \beta)\}$$

$$\square\,\alpha \cdot \square\,\beta = \tfrac{1}{2}\{\square(\alpha + \beta) + \square(\alpha - \beta)\}$$

$$\cos\alpha \cdot \cos\beta = \tfrac{1}{2}\{+\cos(\alpha + \beta) + \cos(\alpha - \beta)\}$$

$$\sin\alpha \cdot \sin\beta = \tfrac{1}{2}\{-\cos(\alpha + \beta) + \cos(\alpha - \beta)\}$$

Sum to Product

$$\square\,(\alpha + \beta) = \square\,\alpha \cdot \square\,\beta + \square\,\alpha\ \square\,\beta$$

$$\mathbf{sin}(\alpha \pm \beta) = \mathbf{sin}\,\alpha \cdot \cos\beta \pm \cos\alpha \cdot \mathbf{sin}\,\beta$$

$$\mathbf{cos}(\alpha \pm \beta) = \mathbf{cos}\,\alpha \cdot \mathbf{cos}\,\beta \mp \sin\alpha \cdot \sin\beta$$

Product to Sum

$$\square\,\alpha \cdot \square\,\beta = \frac{1}{2}\{\square(\alpha+\beta) + \square(\alpha-\beta)\}$$

$$\sin\alpha \cdot \cos\beta = \frac{1}{2}\{\sin(\alpha+\beta) + \sin(\alpha-\beta)\}$$

$$\cos\alpha \cdot \sin\beta = \frac{1}{2}\{+\sin(\alpha+\beta) - \sin(\alpha-\beta)\}$$

$$\underline{\cos\alpha \cdot \cos\beta} = \frac{1}{2}\{+\cos(\alpha+\beta) + \cos(\alpha-\beta)\}$$

$$\underline{\sin\alpha \cdot \sin\beta} = \frac{1}{2}\{-\cos(\alpha+\beta) + \cos(\alpha-\beta)\}$$

References

[1] http://en.wikipedia.org/
[2] http://planetmath.org/
[3] Blitzer, R. “Algebra & Trigonometry.” 3rd ed, Prentice Hall
[4] Smith, R. T., Minton, R. B. “Calculus: Concepts & Connections,” Mc Graw Hill
[5] 홍성대, “기본/실력 수학의 정석,”성지출판