

Trigonometry (4A)

- Trigonometric Identities
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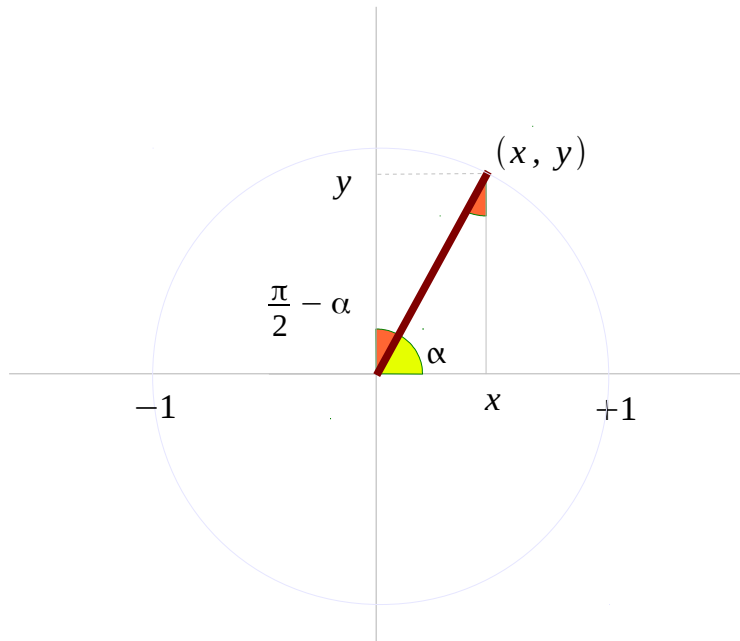
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Co-function Identities



$$\sin \alpha = y \Rightarrow \cos\left(\frac{\pi}{2} - \alpha\right)$$

$$\cos \alpha = x \Rightarrow \sin\left(\frac{\pi}{2} - \alpha\right)$$

$$\tan \alpha = y/x \Rightarrow \cot\left(\frac{\pi}{2} - \alpha\right)$$

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin \alpha$$

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos \alpha$$

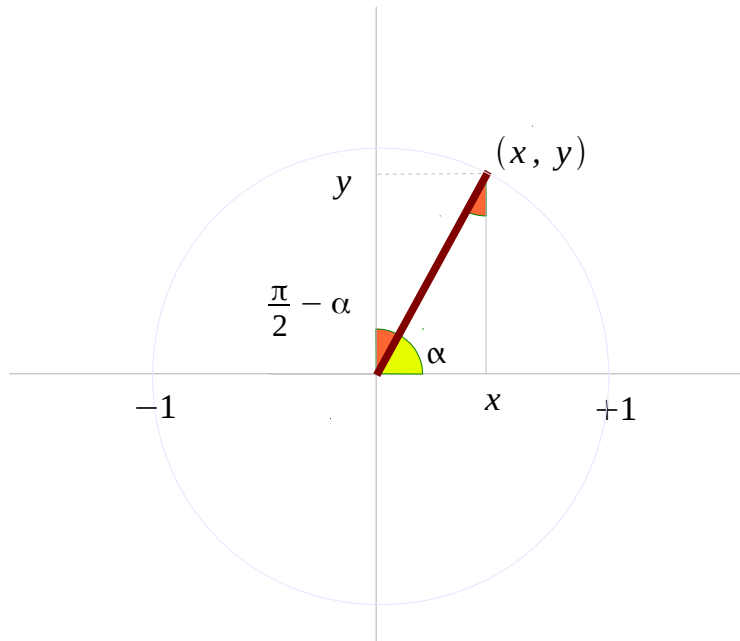
$$\tan\left(\frac{\pi}{2} - \alpha\right) = \cot \alpha$$

$$\cot\left(\frac{\pi}{2} - \alpha\right) = \tan \alpha$$

$$\csc\left(\frac{\pi}{2} - \alpha\right) = \sec \alpha$$

$$\sec\left(\frac{\pi}{2} - \alpha\right) = \csc \alpha$$

Angle Sum and Difference Identities (1)



$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\sin(60^\circ + 30^\circ) = 1$$

$$\begin{array}{l} \sin(60^\circ) = \frac{\sqrt{3}}{2} \\ \cos(60^\circ) = \frac{1}{2} \end{array} \quad \times \quad \begin{array}{l} \sin(30^\circ) = \frac{1}{2} \\ \cos(30^\circ) = \frac{\sqrt{3}}{2} \end{array}$$

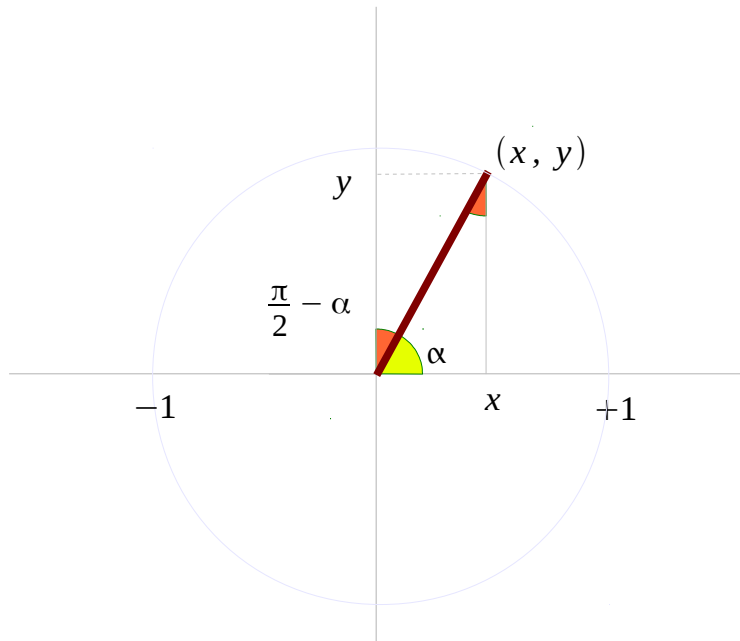
$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2} = 1$$

$$\sin(60^\circ - 30^\circ) = \frac{1}{2}$$

$$\begin{array}{l} \sin(60^\circ) = \frac{\sqrt{3}}{2} \\ \cos(60^\circ) = \frac{1}{2} \end{array} \quad \times \quad \begin{array}{l} \sin(30^\circ) = \frac{1}{2} \\ \cos(30^\circ) = \frac{\sqrt{3}}{2} \end{array}$$

$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$$

Angle Sum and Difference Identities (2)



$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos(30^\circ + 60^\circ) = 0$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2} \quad \text{—————} \quad \sin(30^\circ) = \frac{1}{2}$$

$$\cos(60^\circ) = \frac{1}{2} \quad \text{—————} \quad \cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{2} \cdot \frac{1}{2} - \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = 0$$

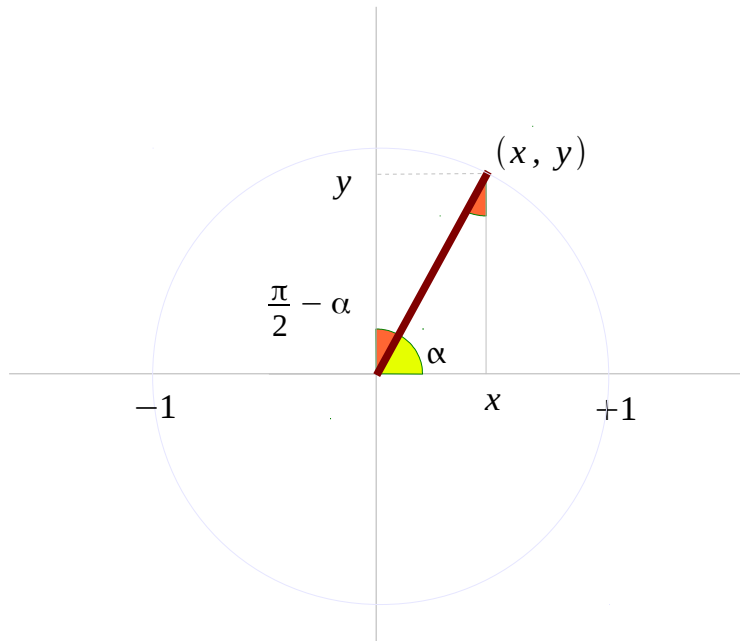
$$\cos(30^\circ - 60^\circ) = \frac{\sqrt{3}}{2}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2} \quad \text{—————} \quad \sin(30^\circ) = \frac{1}{2}$$

$$\cos(60^\circ) = \frac{1}{2} \quad \text{—————} \quad \cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

Angle Sum and Difference Identities (3)



$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$$

$$\tan(30^\circ + 60^\circ) = +\infty$$

$$\tan(30^\circ) = \frac{1}{\sqrt{3}} \quad \underline{\hspace{2cm}} \quad \tan(60^\circ) = \sqrt{3}$$

$$\tan(60^\circ) = \sqrt{3} \quad \underline{\hspace{2cm}} \quad \tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\frac{\frac{1}{\sqrt{3}} + \sqrt{3}}{1 - \frac{1}{\sqrt{3}} \cdot \sqrt{3}} = +\infty$$

$$\tan(30^\circ - 60^\circ) = -\frac{1}{\sqrt{3}}$$

$$\tan(30^\circ) = \frac{1}{\sqrt{3}} \quad \underline{\hspace{2cm}} \quad \tan(60^\circ) = \sqrt{3}$$

$$\tan(60^\circ) = \sqrt{3} \quad \underline{\hspace{2cm}} \quad \tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\frac{\frac{1}{\sqrt{3}} - \sqrt{3}}{1 + \frac{1}{\sqrt{3}} \cdot \sqrt{3}} = -\frac{1}{\sqrt{3}}$$

Angle Sum and Difference Identities (4)

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\begin{array}{ccc} + \sin \alpha & \times & \sin \beta \\ + \cos \alpha & \times & \cos \beta \end{array}$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\begin{array}{ccc} + \sin \alpha & \times & \sin \beta \\ - \cos \alpha & \times & \cos \beta \end{array}$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\begin{array}{ccc} - \sin \alpha & \text{---} & \sin \beta \\ + \cos \alpha & \text{---} & \cos \beta \end{array}$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\begin{array}{ccc} + \sin \alpha & \text{---} & \sin \beta \\ + \cos \alpha & \text{---} & \cos \beta \end{array}$$

$$\frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} = \frac{\sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta}{\cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta}$$

$$\frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)} = \frac{\sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta}{\cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta}$$

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$$

Product to Sum (1)

$$+ \sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$+ \sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2 \sin \alpha \cdot \cos \beta$$

$$\sin \alpha \cdot \cos \beta = \frac{1}{2} \{ \sin(\alpha + \beta) + \sin(\alpha - \beta) \}$$

$$+ \cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$+ \cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2 \cos \alpha \cdot \cos \beta$$

$$\cos \alpha \cdot \cos \beta = \frac{1}{2} \{ +\cos(\alpha + \beta) + \cos(\alpha - \beta) \}$$

$$+ \sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$- \sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha + \beta) - \sin(\alpha - \beta) = 2 \cos \alpha \sin \beta$$

$$\cos \alpha \cdot \sin \beta = \frac{1}{2} \{ \sin(\alpha + \beta) - \sin(\alpha - \beta) \}$$

$$- \cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$+ \cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) - \cos(\alpha - \beta) = 2 \sin \alpha \sin \beta$$

$$\sin \alpha \cdot \sin \beta = \frac{1}{2} \{ -\cos(\alpha + \beta) + \cos(\alpha - \beta) \}$$

Product to Sum (2)

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\sin \alpha \cdot \cos \beta = \frac{1}{2} \{ \sin(\alpha + \beta) + \sin(\alpha - \beta) \}$$

$$\sin \alpha \cdot \cos \beta = \frac{1}{2} \{ +\sin(\alpha + \beta) + \sin(\alpha - \beta) \}$$

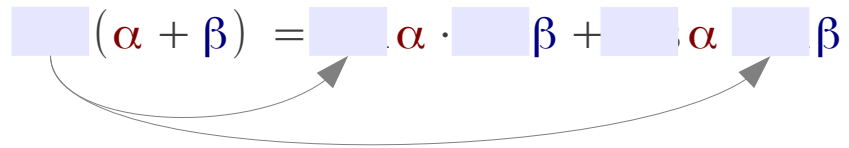
$$\cos \alpha \cdot \sin \beta = \frac{1}{2} \{ +\sin(\alpha + \beta) - \sin(\alpha - \beta) \}$$

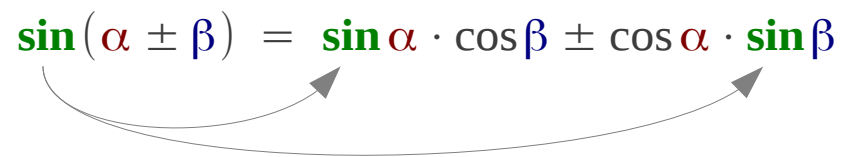
$$\cos \alpha \cdot \cos \beta = \frac{1}{2} \{ \cos(\alpha + \beta) + \cos(\alpha - \beta) \}$$

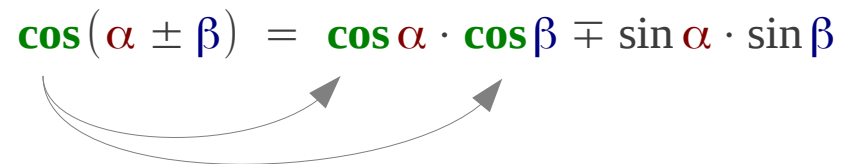
$$\cos \alpha \cdot \cos \beta = \frac{1}{2} \{ +\cos(\alpha + \beta) + \cos(\alpha - \beta) \}$$

$$\sin \alpha \cdot \sin \beta = \frac{1}{2} \{ -\cos(\alpha + \beta) + \cos(\alpha - \beta) \}$$

Sum to Product

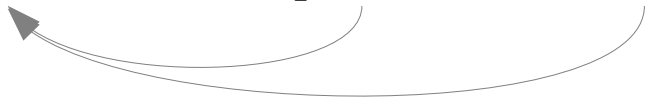
$$\square(\alpha + \beta) = \square \alpha \cdot \square \beta + \square \alpha \square \beta$$


$$\sin(\alpha \pm \beta) = \sin \alpha \cdot \cos \beta \pm \cos \alpha \cdot \sin \beta$$


$$\cos(\alpha \pm \beta) = \cos \alpha \cdot \cos \beta \mp \sin \alpha \cdot \sin \beta$$


Product to Sum

$$\square \alpha \cdot \square \beta = \frac{1}{2} \{ \square (\alpha + \beta) + \square (\alpha - \beta) \}$$

$$\sin \alpha \cdot \cos \beta = \frac{1}{2} \{ \sin (\alpha + \beta) + \sin (\alpha - \beta) \}$$


$$\cos \alpha \cdot \sin \beta = \frac{1}{2} \{ +\sin (\alpha + \beta) - \sin (\alpha - \beta) \}$$


$$\underline{\cos \alpha \cdot \cos \beta} = \frac{1}{2} \{ +\cos (\alpha + \beta) + \cos (\alpha - \beta) \}$$


$$\underline{\sin \alpha \cdot \sin \beta} = \frac{1}{2} \{ -\cos (\alpha + \beta) + \cos (\alpha - \beta) \}$$


References

- [1] <http://en.wikipedia.org/>
- [2] <http://planetmath.org/>
- [3] Blitzer, R. "Algebra & Trigonometry." 3rd ed, Prentice Hall
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- [5] 홍성대, "기본/실력 수학의 정석," 성지출판