Elementary Matrix

Young Won Lim
09/06/2012

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice and Octave.

Gauss-Jordan Elimination

Forward Phase - Gaussian Elimination

$$\left(\begin{array}{ccc|c} +2 & +1 & -1 & +8 \\ -3 & -1 & +2 & -11 \\ -2 & +1 & +2 & -3 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ -3 & -1 & +2 & -11 \\ -2 & +1 & +2 & -3 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ \boxed{0} & +1/2 & +1/2 & +1 \\ \boxed{0} & +2 & +1 & +5 \end{array}\right) \Rightarrow$$

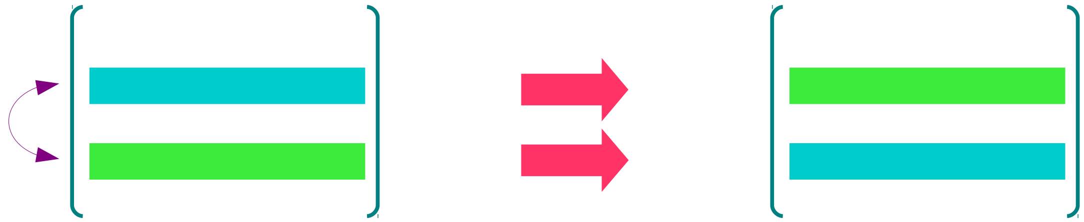
$$\left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & +2 & +1 & +5 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & \boxed{0} & -1 & +1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & 0 & +1 & -1 \end{array}\right) \Rightarrow$$

Backward Phase

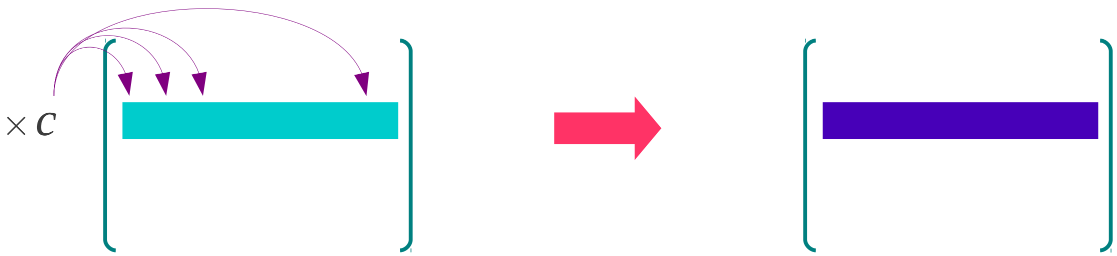
$$\left(\begin{array}{ccc|c} +1 & +1/2 & \boxed{-1/2} & +4 \\ 0 & +1 & \boxed{+1} & +2 \\ 0 & 0 & +1 & -1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & +1/2 & \boxed{0} & +7/2 \\ 0 & +1 & \boxed{0} & +3 \\ 0 & 0 & +1 & -1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & \boxed{0} & 0 & +2 \\ 0 & +1 & 0 & +3 \\ 0 & 0 & +1 & -1 \end{array}\right)$$

Elementary Row Operation

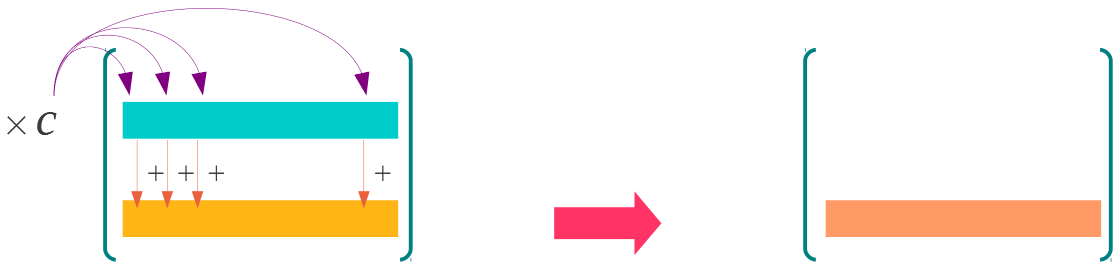
Interchange two rows



Multiply a row by a nonzero constant



Add a multiple of one row to another

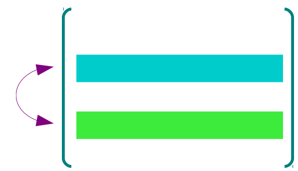


Elementary Matrix

Identity Matrix

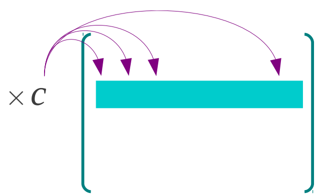
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Interchange two rows



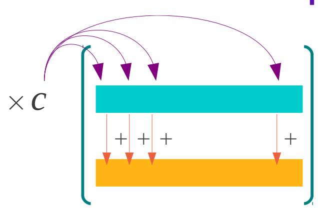
$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Multiply a row by a nonzero constant



$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Add a multiple of one row to another



$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Multiplication by an Elementary Matrix

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \begin{bmatrix} 3 & 6 & 9 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 \\ 8 & 13 & 18 \\ 7 & 8 & 9 \end{bmatrix}$$

Gauss-Jordan Elimination – Step 1

$$+2x_1 + x_2 - x_3 = 8 \qquad (L_1)$$
$$-3x_1 - x_2 + 2x_3 = -11 \qquad (L_2)$$
$$-2x_1 + x_2 + 2x_3 = -3 \qquad (L_3)$$

$$\left(\begin{array}{ccc|c} +2 & +1 & -1 & +8 \\ -3 & -1 & +2 & -11 \\ -2 & +1 & +2 & -3 \end{array}\right)$$

$$\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{array}{ccc|c} +2 & +1 & -1 & +8 \\ -3 & -1 & +2 & -11 \\ -2 & +1 & +2 & -3 \end{array}\right)$$

$$+1x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 = 4 \qquad (\tfrac{1}{2} \times L_1)$$
$$-3x_1 - x_2 + 2x_3 = -11 \qquad (L_2)$$
$$-2x_1 + x_2 + 2x_3 = -3 \qquad (L_3)$$

$$\left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ -3 & -1 & +2 & -11 \\ -2 & +1 & +2 & -3 \end{array}\right)$$

Gauss-Jordan Elimination – Step 2

$$+1x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 = +4 \qquad (L_1)$$
$$-3x_1 - x_2 + 2x_3 = -11 \qquad (L_2)$$
$$-2x_1 + x_2 + 2x_3 = -3 \qquad (L_3)$$

$$\left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ \boxed{-3} & -1 & +2 & -11 \\ \boxed{-2} & +1 & +2 & -3 \end{array}\right)$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ \boxed{-3} & -1 & +2 & -11 \\ \boxed{-2} & +1 & +2 & -3 \end{array}\right)$$

$$+1x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 = +4 \qquad (L_1)$$
$$0x_1 + \tfrac{1}{2}x_2 + \tfrac{1}{2}x_3 = +1 \qquad (\boxed{3 \times L_1} + L_2)$$
$$0x_1 + 2x_2 + 1x_3 = +5 \qquad (\boxed{2 \times L_1} + L_3)$$

$$\left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ \boxed{0} & +1/2 & +1/2 & +1 \\ \boxed{0} & +2 & +1 & +5 \end{array}\right)$$

Gauss-Jordan Elimination – Step 3

$$+1x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 = +4 \qquad (L_1)$$
$$0x_1 + \tfrac{1}{2}x_2 + \tfrac{1}{2}x_3 = +1 \qquad (L_2)$$
$$0x_1 + 2x_2 + 1x_3 = +5 \qquad (L_3)$$

$$\left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1/2 & +1/2 & +1 \\ 0 & +2 & +1 & +5 \end{array}\right)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1/2 & +1/2 & +1 \\ 0 & +2 & +1 & +5 \end{array}\right)$$

$$+1x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 = +4 \qquad (L_1)$$
$$0x_1 + 1x_2 + 1x_3 = +2 \qquad (2 \times L_2)$$
$$0x_1 + 2x_2 + 1x_3 = +5 \qquad (L_3)$$

$$\left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & +2 & +1 & +5 \end{array}\right)$$

Gauss-Jordan Elimination – Step 4

$$+1x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 \;=\; +4 \qquad (L_1)$$

$$0x_1 + 1x_2 + 1x_3 \;=\; +2 \qquad (L_2)$$

$$0x_1 + 2x_2 + 1x_3 \;=\; +5 \qquad (L_3)$$

$$\left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & \boxed{+2} & +1 & +5 \end{array}\right)$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & \boxed{+2} & +1 & +5 \end{array}\right)$$

$$+1x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 \;=\; +4 \qquad (L_1)$$

$$0x_1 + 1x_2 + 1x_3 \;=\; +2 \qquad (L_2)$$

$$0x_1 + 0x_2 - 1x_3 \;=\; +1 \qquad (\boxed{-2 \times L_2} + L_3)$$

$$\left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & \boxed{0} & -1 & +1 \end{array}\right)$$

Gauss-Jordan Elimination – Step 5

$$+1x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 = +4 \qquad (L_1)$$
$$0x_1 + 1x_2 + 1x_3 = +2 \qquad (L_2)$$
$$0x_1 + 0x_2 - 1x_3 = +1 \qquad (L_3)$$

$$\left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & 0 & -1 & +1 \end{array}\right)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & 0 & -1 & +1 \end{array}\right)$$

$$+1x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 = +4 \qquad (L_1)$$
$$0x_1 + 1x_2 + 1x_3 = +2 \qquad (L_2)$$
$$0x_1 + 0x_2 + 1x_3 = -1 \qquad (-1 \times L_3)$$

$$\left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & 0 & +1 & -1 \end{array}\right)$$

Young Won Lim
09/06/2012

Forward Phase

$$\left(\begin{array}{ccc|c} +2 & +1 & -1 & +8 \\ -3 & -1 & +2 & -11 \\ -2 & +1 & +2 & -3 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ -3 & -1 & +2 & -11 \\ -2 & +1 & +2 & -3 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ \boxed{0} & +1/2 & +1/2 & +1 \\ \boxed{0} & +2 & +1 & +5 \end{array}\right) \Rightarrow$$

$$\left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & +2 & +1 & +5 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & \boxed{0} & -1 & +1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & 0 & +1 & -1 \end{array}\right) \Rightarrow$$

Forward Phase - Gaussian Elimination

Gauss-Jordan Elimination – Step 6

$$+1x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 = +4 \qquad (L_1)$$
$$0x_1 + 1x_2 + 1x_3 = +2 \qquad (L_2)$$
$$0x_1 + 0x_2 + 1x_3 = -1 \qquad (L_3)$$

$$\left(\begin{array}{ccc|c} +1 & +1/2 & \boxed{-1/2} & +4 \\ 0 & +1 & \boxed{+1} & +2 \\ 0 & 0 & +1 & -1 \end{array}\right)$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \left(\begin{array}{ccc|c} +1 & +1/2 & \boxed{-1/2} & +4 \\ 0 & +1 & \boxed{+1} & +2 \\ 0 & 0 & +1 & -1 \end{array}\right)$$

$$+1x_1 + \tfrac{1}{2}x_2 + 0x_3 = +\tfrac{7}{2} \qquad \left(+\tfrac{1}{2} \times L_3 + L_1\right)$$
$$0x_1 + 1x_2 + 0x_3 = +3 \qquad \left(-1 \times L_3 + L_2\right)$$
$$0x_1 + 0x_2 + 1x_3 = -1 \qquad (L_3)$$

$$\left(\begin{array}{ccc|c} +1 & +1/2 & \boxed{0} & +7/2 \\ 0 & +1 & \boxed{0} & +3 \\ 0 & 0 & +1 & -1 \end{array}\right)$$

Gauss-Jordan Elimination – Step 7

$$+1\,x_1 + \tfrac{1}{2}x_2 + 0\,x_3 = +\tfrac{7}{2} \qquad (L_1)$$
$$0\,x_1 + 1\,x_2 + 0\,x_3 = +3 \qquad (L_2)$$
$$0\,x_1 + 0\,x_2 + 1\,x_3 = -1 \qquad (L_3)$$

$$\left(\begin{array}{ccc|c} +1 & \boxed{+1/2} & 0 & +7/2 \\ 0 & +1 & 0 & +3 \\ 0 & 0 & +1 & -1 \end{array}\right)$$

$$\begin{bmatrix} 1 & 0 & +1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{array}{ccc|c} +1 & \boxed{+1/2} & 0 & +7/2 \\ 0 & +1 & 0 & +3 \\ 0 & 0 & +1 & -1 \end{array}\right)$$

$$+1\,x_1 + 0\,x_2 - 0\,x_3 = +2 \qquad (+1 \times L_3 + L_1)$$
$$0\,x_1 + 1\,x_2 + 0\,x_3 = +3 \qquad (L_2)$$
$$0\,x_1 + 0\,x_2 + 1\,x_3 = -1 \qquad (L_3)$$

$$\left(\begin{array}{ccc|c} +1 & \boxed{0} & 0 & +2 \\ 0 & +1 & 0 & +3 \\ 0 & 0 & +1 & -1 \end{array}\right)$$

Backward Phase

$$\left(\begin{array}{ccc|c} +1 & +1/2 & \boxed{-1/2} & +4 \\ 0 & +1 & \boxed{+1} & +2 \\ 0 & 0 & +1 & -1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & +1/2 & \boxed{0} & +7/2 \\ 0 & +1 & \boxed{0} & +3 \\ 0 & 0 & +1 & -1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & \boxed{0} & 0 & +2 \\ 0 & +1 & 0 & +3 \\ 0 & 0 & +1 & -1 \end{array}\right) \Rightarrow$$

Gauss-Jordan Elimination

Forward Phase - Gaussian Elimination

$$\left(\begin{array}{ccc|c} +2 & +1 & -1 & +8 \\ -3 & -1 & +2 & -11 \\ -2 & +1 & +2 & -3 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ -3 & -1 & +2 & -11 \\ -2 & +1 & +2 & -3 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ \boxed{0} & +1/2 & +1/2 & +1 \\ \boxed{0} & +2 & +1 & +5 \end{array}\right) \Rightarrow$$

$$\left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & +2 & +1 & +5 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & \boxed{0} & -1 & +1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & 0 & +1 & -1 \end{array}\right) \Rightarrow$$

Backward Phase

$$\left(\begin{array}{ccc|c} +1 & +1/2 & \boxed{-1/2} & +4 \\ 0 & +1 & \boxed{+1} & +2 \\ 0 & 0 & +1 & -1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & +1/2 & \boxed{0} & +7/2 \\ 0 & +1 & \boxed{0} & +3 \\ 0 & 0 & +1 & -1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & \boxed{0} & 0 & +2 \\ 0 & +1 & 0 & +3 \\ 0 & 0 & +1 & -1 \end{array}\right) \Rightarrow$$

Pulse

Pulse

References

[1] http://en.wikipedia.org/
[2] Anton & Busby, “Contemporary Linear Algebra”