

Trigonometry (4A)

- Trigonometric Identities
-

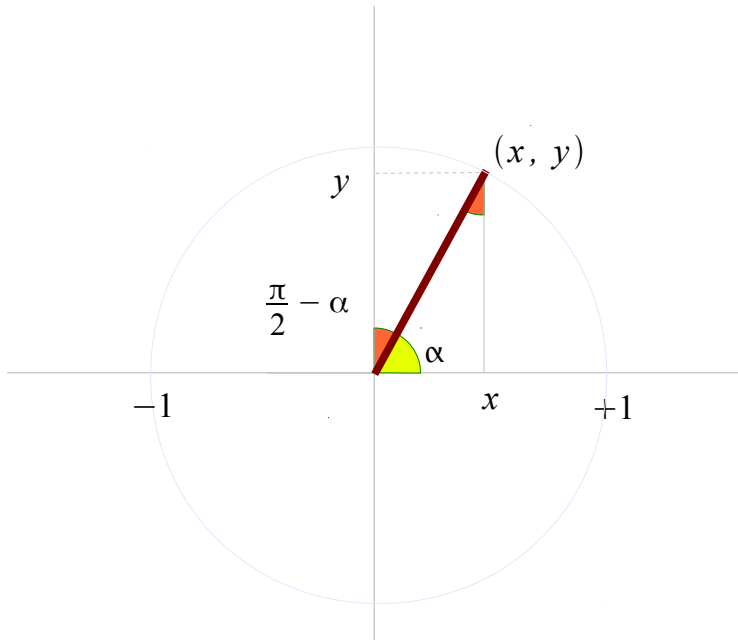
Copyright (c) 2011 Young W. Lim.

Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice and Octave.

Co-function Identities



$$\sin \alpha = y \Rightarrow \cos\left(\frac{\pi}{2} - \alpha\right)$$

$$\cos \alpha = x \Rightarrow \sin\left(\frac{\pi}{2} - \alpha\right)$$

$$\tan \alpha = y/x \Rightarrow \cot\left(\frac{\pi}{2} - \alpha\right)$$

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin \alpha$$

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos \alpha$$

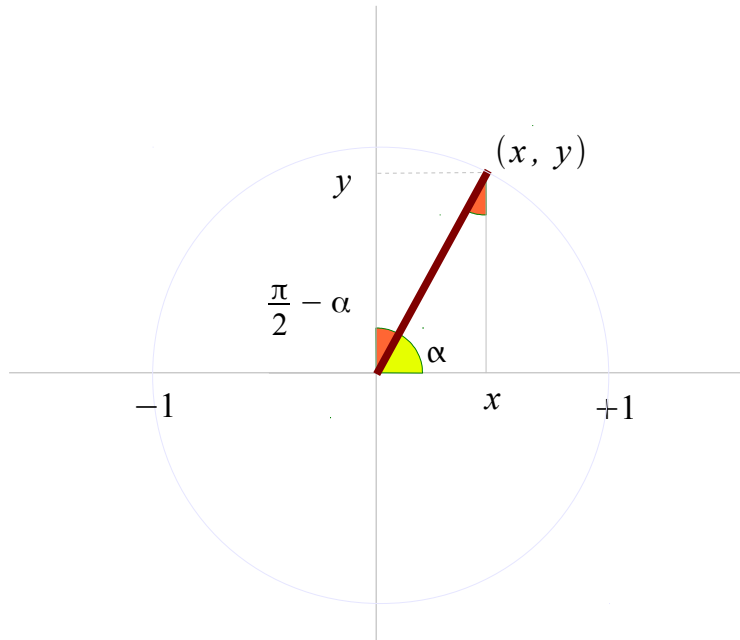
$$\tan\left(\frac{\pi}{2} - \alpha\right) = \cot \alpha$$

$$\cot\left(\frac{\pi}{2} - \alpha\right) = \tan \alpha$$

$$\csc\left(\frac{\pi}{2} - \alpha\right) = \sec \alpha$$

$$\sec\left(\frac{\pi}{2} - \alpha\right) = \csc \alpha$$

Angle Sum and Difference Identities (1)



$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\sin(60^\circ + 30^\circ) = 1$$

$$\begin{array}{l} \sin(60^\circ) = \frac{\sqrt{3}}{2} \\ \cos(60^\circ) = \frac{1}{2} \end{array} \quad \times \quad \begin{array}{l} \sin(30^\circ) = \frac{1}{2} \\ \cos(30^\circ) = \frac{\sqrt{3}}{2} \end{array}$$

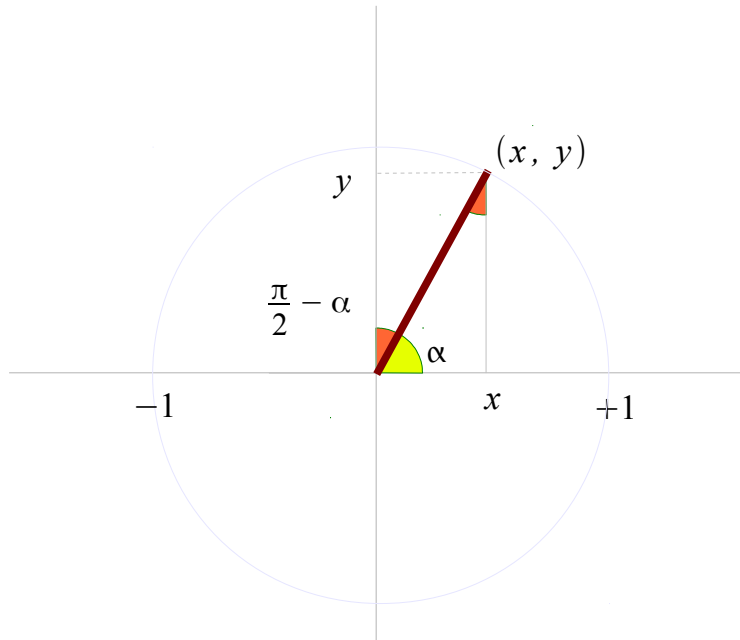
$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2} = 1$$

$$\sin(60^\circ - 30^\circ) = \frac{1}{2}$$

$$\begin{array}{l} \sin(60^\circ) = \frac{\sqrt{3}}{2} \\ \cos(60^\circ) = \frac{1}{2} \end{array} \quad \times \quad \begin{array}{l} \sin(30^\circ) = \frac{1}{2} \\ \cos(30^\circ) = \frac{\sqrt{3}}{2} \end{array}$$

$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$$

Angle Sum and Difference Identities (2)



$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$$

$$\cos(30^\circ + 60^\circ) = 0$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2} \quad \text{—————} \quad \sin(30^\circ) = \frac{1}{2}$$

$$\cos(60^\circ) = \frac{1}{2} \quad \text{—————} \quad \cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{2} \cdot \frac{1}{2} - \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = 0$$

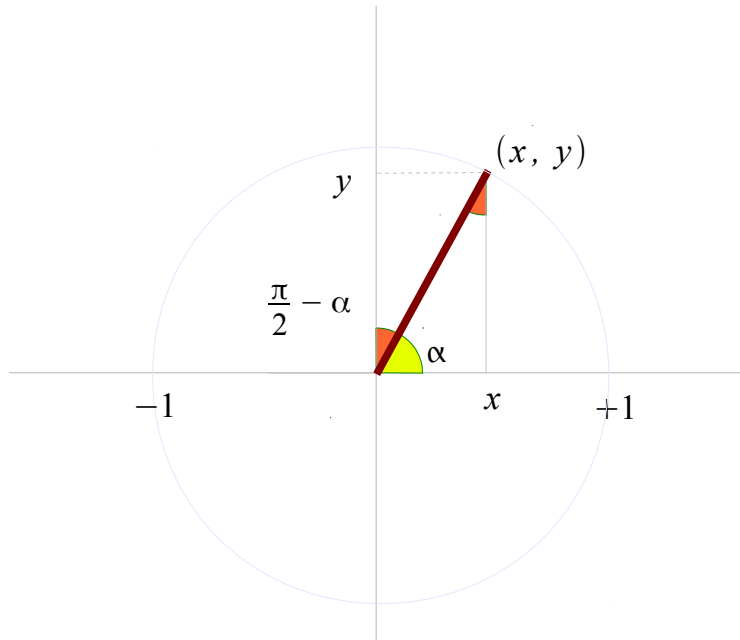
$$\cos(30^\circ - 60^\circ) = \frac{\sqrt{3}}{2}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2} \quad \text{—————} \quad \sin(30^\circ) = \frac{1}{2}$$

$$\cos(60^\circ) = \frac{1}{2} \quad \text{—————} \quad \cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

Angle Sum and Difference Identities (3)



$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$$

$$\tan(30^\circ + 60^\circ) = +\infty$$

$$\tan(30^\circ) = \frac{1}{\sqrt{3}} \quad \underline{\hspace{2cm}} \quad \tan(60^\circ) = \sqrt{3}$$

$$\tan(60^\circ) = \sqrt{3} \quad \underline{\hspace{2cm}} \quad \tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\frac{\frac{1}{\sqrt{3}} + \sqrt{3}}{1 - \frac{1}{\sqrt{3}} \cdot \sqrt{3}} = +\infty$$

$$\tan(30^\circ - 60^\circ) = -\frac{1}{\sqrt{3}}$$

$$\tan(30^\circ) = \frac{1}{\sqrt{3}} \quad \underline{\hspace{2cm}} \quad \tan(60^\circ) = \sqrt{3}$$

$$\tan(60^\circ) = \sqrt{3} \quad \underline{\hspace{2cm}} \quad \tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\frac{\frac{1}{\sqrt{3}} - \sqrt{3}}{1 + \frac{1}{\sqrt{3}} \cdot \sqrt{3}} = -\frac{1}{\sqrt{3}}$$

Angle Sum and Difference Identities (4)

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\begin{array}{ccc} + \sin \alpha & \times & \sin \beta \\ + \cos \alpha & \times & \cos \beta \end{array}$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\begin{array}{ccc} + \sin \alpha & \times & \sin \beta \\ - \cos \alpha & \times & \cos \beta \end{array}$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\begin{array}{ccc} - \sin \alpha & \text{---} & \sin \beta \\ + \cos \alpha & \text{---} & \cos \beta \end{array}$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\begin{array}{ccc} + \sin \alpha & \text{---} & \sin \beta \\ + \cos \alpha & \text{---} & \cos \beta \end{array}$$

$$\frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} = \frac{\sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta}{\cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta}$$

$$\frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)} = \frac{\sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta}{\cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta}$$

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$$

Product to Sum (1)

$$+ \sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$+ \sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2 \sin \alpha \cdot \cos \beta$$

$$\sin \alpha \cdot \cos \beta = \frac{1}{2} \{ \sin(\alpha + \beta) + \sin(\alpha - \beta) \}$$

$$+ \cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$+ \cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2 \cos \alpha \cdot \cos \beta$$

$$\cos \alpha \cdot \cos \beta = \frac{1}{2} \{ +\cos(\alpha + \beta) + \cos(\alpha - \beta) \}$$

$$+ \sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$- \sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha + \beta) - \sin(\alpha - \beta) = 2 \cos \alpha \sin \beta$$

$$\cos \alpha \cdot \sin \beta = \frac{1}{2} \{ \sin(\alpha + \beta) - \sin(\alpha - \beta) \}$$

$$- \cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$+ \cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) - \cos(\alpha - \beta) = 2 \sin \alpha \sin \beta$$

$$\sin \alpha \cdot \sin \beta = \frac{1}{2} \{ -\cos(\alpha + \beta) + \cos(\alpha - \beta) \}$$

Product to Sum (2)

$$\text{Green}(\alpha + \beta) = \text{Yellow}\alpha \cdot \text{Teal}\beta + \text{Teal}\alpha \text{Yellow}\beta$$

$$\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta$$

$$\text{Green}(\alpha - \beta) = \text{Teal}\alpha \cdot \text{Teal}\beta - \text{Yellow}\alpha \text{Yellow}\beta$$

$$\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$\text{Grey}\alpha \cdot \text{Orange}\beta = \frac{1}{2}\{\text{Yellow}(\alpha + \beta) + \text{Yellow}(\alpha - \beta)\}$$

$$\sin\alpha \cdot \cos\beta = \frac{1}{2}\{+\sin(\alpha + \beta) + \sin(\alpha - \beta)\}$$

$$\cos\alpha \cdot \sin\beta = \frac{1}{2}\{+\sin(\alpha + \beta) - \sin(\alpha - \beta)\}$$

$$\text{Blue}\alpha \cdot \text{Blue}\beta = \frac{1}{2}\{\text{Teal}(\alpha + \beta) + \text{Teal}(\alpha - \beta)\}$$

$$\cos\alpha \cdot \cos\beta = \frac{1}{2}\{+\cos(\alpha + \beta) + \cos(\alpha - \beta)\}$$

$$\sin\alpha \cdot \sin\beta = \frac{1}{2}\{-\cos(\alpha + \beta) + \cos(\alpha - \beta)\}$$

References

- [1] <http://en.wikipedia.org/>
- [2] <http://planetmath.org/>
- [3] Blitzer, R. "Algebra & Trigonometry." 3rd ed, Prentice Hall
- [4] Smith, R. T., Minton, R. B. "Calculus: Concepts & Connections," Mc Graw Hill
- [5] 홍성대, "기본/실력 수학의 정석,"성지출판