

# Formatting (2A)

---

Copyright (c) 2012 Young W. Lim.

Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

Please send corrections (or suggestions) to [youngwlim@hotmail.com](mailto:youngwlim@hotmail.com).

This document was produced by using OpenOffice and Octave.

# Formatting and Source Coding

## **Formatting**

Make the source signal compatible with digital processing

## **Transmit Formatting**

A transformation from source information to **digital symbols**

## **Source Coding**

Formatting + Data Compression

## **Baseband Signal**

From DC up to some finite frequency ( $<$  a few MHz)

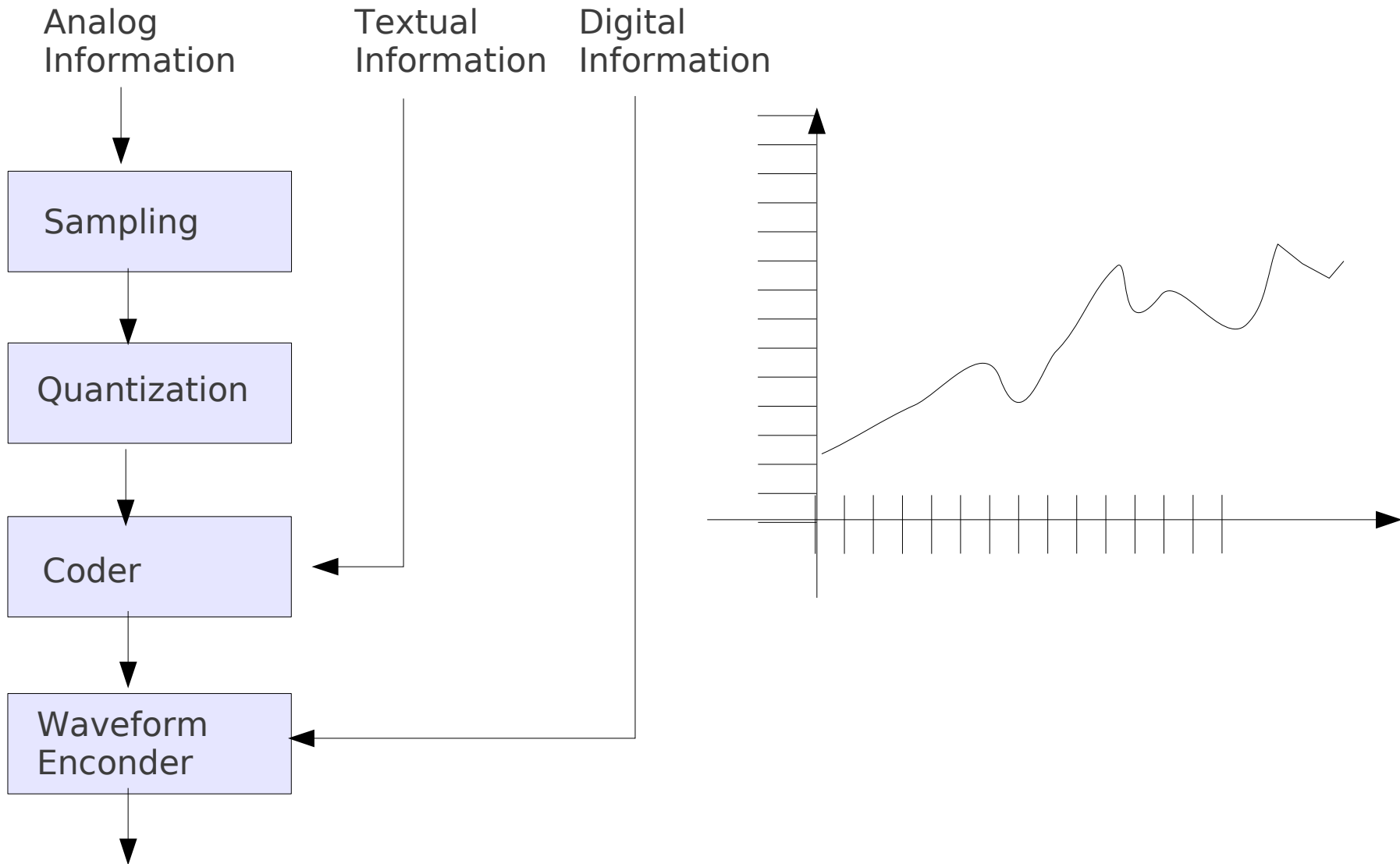
Transmitted over the cable

Not appropriate to transmit over long distance  $\rightarrow$  Bandpass Mod

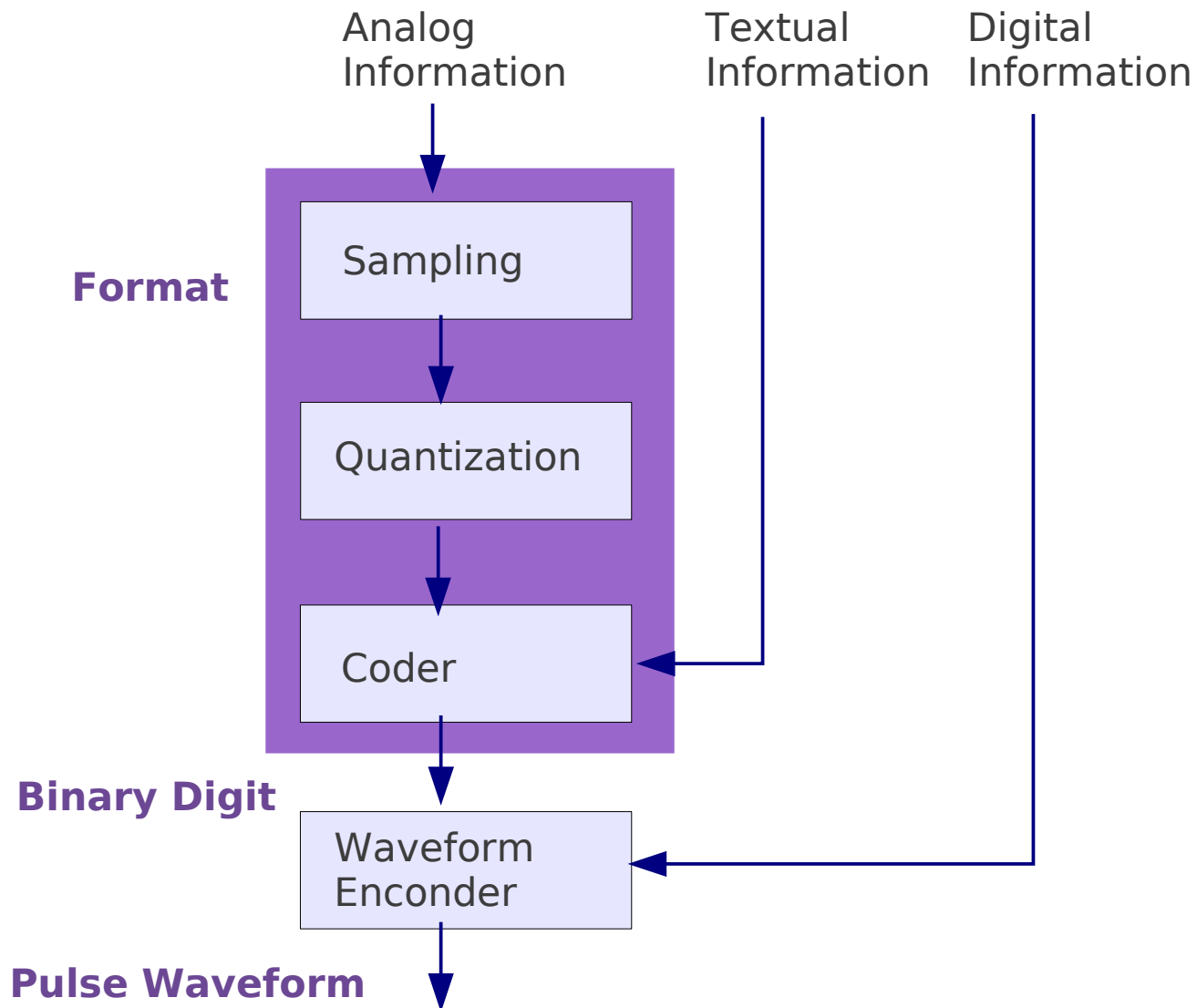
## **Pulse (Baseband) Modulation**

Pulse waveforms are assigned that represent formatted symbols

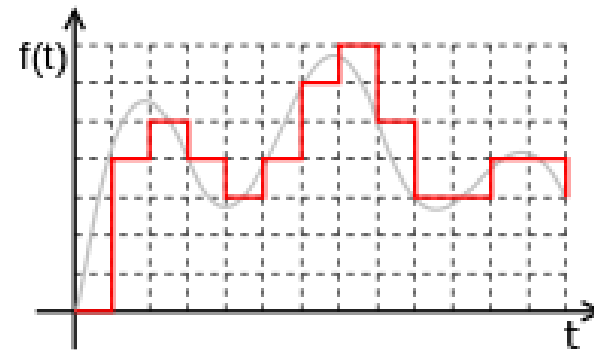
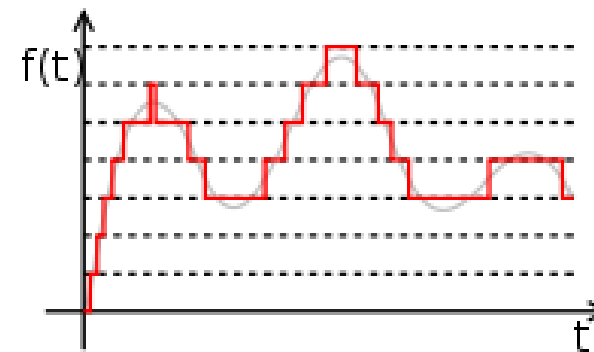
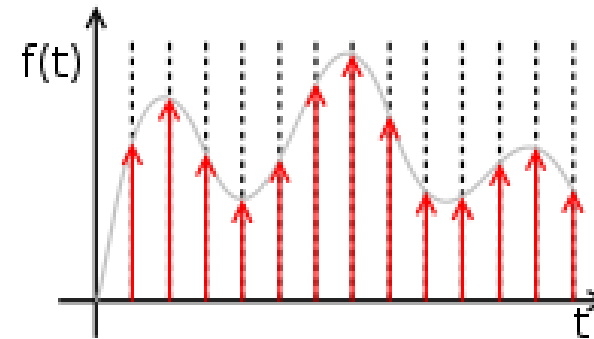
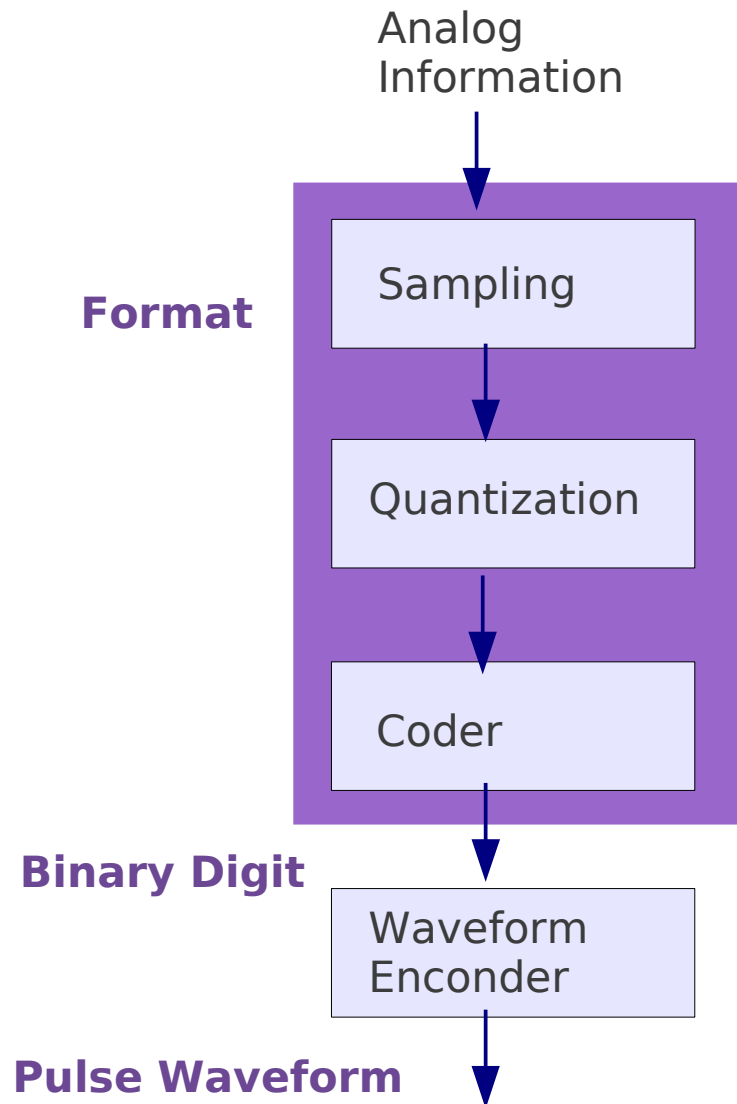
# Energy and Power Spectral Densities (2)



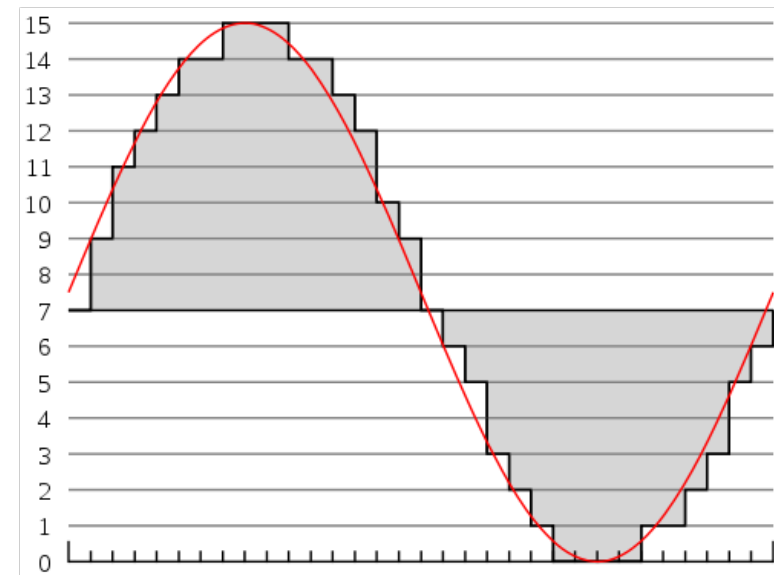
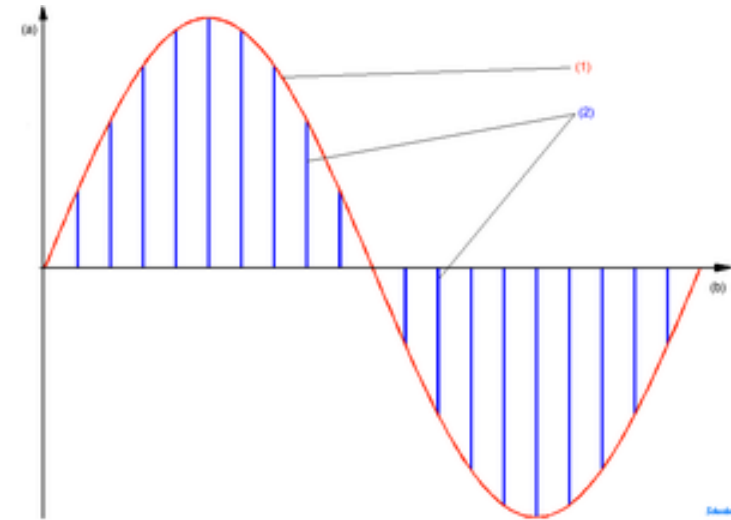
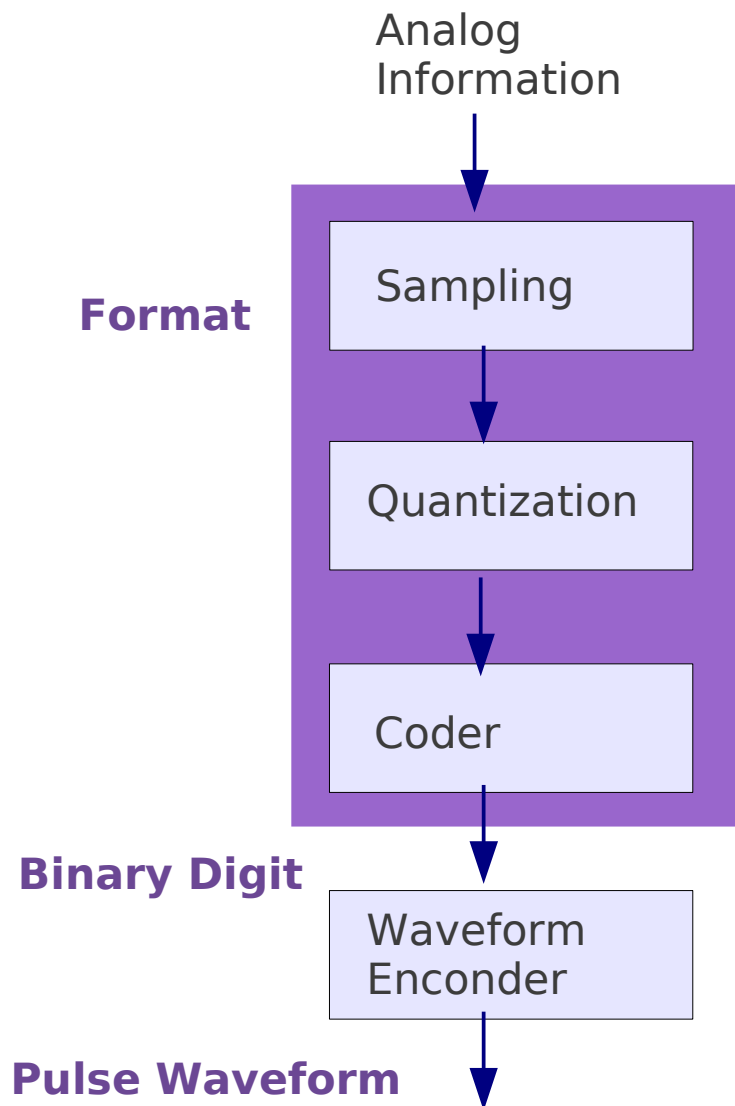
# Baseband Signal



# Sampling and Quantization



# PAM and PCM



# 8-ary Symbol

T H I N K

Message

001010 000100 100100 011100 110100

6-bit ASCII

001 010 000 100 100 100 011 100 110 100

1 2 0 4 4 4 3 4 6 4

8-ary digits  
(symbols)

$s_1(t)$   $s_2(t)$   $s_0(t)$   $s_4(t)$   $s_4(t)$   $s_4(t)$   $s_3(t)$   $s_4(t)$   $s_6(t)$   $s_4(t)$

8-ary waveform



# Binary Symbol

T H I N K

Message

001010 000100 100100 011100 110100

6-bit ASCII

0 0 1 0 1 0 0 0 0 1 0 0 1 0 0 0 1 1 1 0 0 1 1 0 1 0 0

0 0 1 0 1 0 0 0 0 1 0 0 1 0 0 0 1 1 1 0 0 1 1

binary digits  
(symbols)

$s_0(t)$   $s_0(t)$   $s_1(t)$   $s_0(t)$   $s_1(t)$   $s_0(t)$   $s_0(t)$   $s_0(t)$   $s_0(t)$   $s_0(t)$   $s_1(t)$   $s_0(t)$   $s_0(t)$

binary waveform

# Impulse Sampling

Impulse train

$$x_{\delta}(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s) \quad \longleftrightarrow \quad X_{\delta}(f) = \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} \delta(f - nf_s)$$

Shifting property

$$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$$

$$\begin{aligned} x_s(t) &= x(t)x_{\delta}(t) & \longleftrightarrow & & X_s(f) &= X(f) * X_{\delta}(f) \\ &= \sum_{n=-\infty}^{+\infty} x(t)\delta(t - nT_s) & & & &= X(f) * \left[ \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} \delta(f - nf_s) \right] \\ &= \sum_{n=-\infty}^{+\infty} x(nT_s)\delta(t - nT_s) & & & &= \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} X(f - nf_s) \end{aligned}$$

# Natural Sampling

Pulse train

$$x_p(t) = \sum_{n=-\infty}^{+\infty} c_n e^{j2\pi n f_s t} \quad \longleftrightarrow \quad c_n = \frac{1}{T_s} \text{sinc}\left(\frac{nT_s}{T_s}\right)$$

$$x_s(t) = x(t) x_p(t) \quad \longleftrightarrow \quad X_s(f) = X(f) * X_p(f)$$

$$= x(t) \sum_{n=-\infty}^{+\infty} c_n e^{j2\pi f_s t}$$

$$= \sum_{n=-\infty}^{+\infty} c_n [x(t) e^{j2\pi f_s t}]$$



$$= \sum_{n=-\infty}^{+\infty} c_n X(f - n f_s)$$

# Sample and Hold

Sampled Pulse train

$$x_p(t) = \sum_{n=-\infty}^{+\infty} c_n e^{j2\pi n f_s t} \iff c_n = \frac{1}{T_s} \text{sinc}\left(\frac{nT_s}{T_s}\right)$$

$$x_s(t) = p(t) * [x(t) x_\delta(t)] \iff X_s(f) = X(f) * X_p(f)$$

$$\begin{aligned} &= p(t) * \left[ x(t) \sum_{n=-\infty}^{+\infty} \delta(t - nT_s) \right] \iff \\ &= P(f) \left[ X(f) * \left[ \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} \delta(f - nf_s) \right] \right] \\ &= P(f) \left[ \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} X(f - nf_s) \right] \end{aligned}$$

# Sampling Theorem

## Uniform Sampling Theorem

A bandlimited signal having no spectral components above  $f_m$  Hz can be determined uniquely by values sampled at *uniform intervals* of  $T_s$  seconds

$$T_s \leq \frac{1}{2f_m}$$

*Upper limit of  $T_s$*

$$f_s = \frac{1}{T_s}$$

$$f_s \geq 2f_m$$

*Lower limit of  $f_s$*

*Nyquist Criterion*

*Nyquist Rate  $f_s = 2f_m$*

# Autocorrelation of Energy and Power Signals

---

# Ensemble Average

---

# WSS (Wide Sense Stationary)

---



# Autocorrelation of Random and Power Signals

---

# Time Averaging and Ergodicity

---

# Autocorrelation of Random and Power Signals

---

# Time Averaging and Ergodicity

---

## References

- [1] <http://en.wikipedia.org/>
- [2] <http://planetmath.org/>
- [3] M.L. Boas, “Mathematical Methods in the Physical Sciences”