

Time Domain Techniques (3B)

for Noisy Signals

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Signal Detrending / Spike Removal

Remove low frequency noise

a constant
a line
a long period sinusoid

least square fitting a low frequency function $tr[i]$

subtracting the trend from the measurements $y[i] = x[i] - tr[i]$

example)

removing the dc component

$$y[i] = x[i] - tr[i] = x[i] - \frac{1}{N} \sum_{i=0}^{N-1} x[i-1]$$

removing spikes

$$y[i] = x[i]$$

if $|x[i] - y[i-1]| < threshold$

$$y[i] = \frac{x[i-1] + x[i+1]}{2}$$

else

Stacking

Remove background noise

Assume noise has a zero mean

Measuring multiple times (ensemble)

Averaging across the ensemble

Reduces the noise level in the averaged signal
Increases the SNR of the correlated component (signal)

$$E[X[i]] \rightarrow x[i]$$

$$E[N[i]] \rightarrow 0$$

$$E[N^2[i]] \rightarrow \frac{\sigma_n}{\sqrt{M}} \quad SNR \propto \sqrt{M}$$

Moving Average Filter

L-point Running Averager

$$y[n] = \frac{1}{L} \sum_{k=0}^{L-1} x[n-k] = \frac{1}{L} \{x[n] + x[n-1] + \dots + x[n-L+1]\}$$

$$Y(e^{j\hat{\omega}}) = \frac{1}{L} \sum_{k=0}^{L-1} e^{-j\hat{\omega}k} X(e^{j\hat{\omega}}) = \frac{1}{L} \{X(e^{j\hat{\omega}}) + X(e^{j\hat{\omega}})e^{-j\hat{\omega}1} + \dots + X(e^{j\hat{\omega}})e^{-j\hat{\omega}(L-1)}\}$$

$$H(e^{j\hat{\omega}}) = \frac{1}{L} \sum_{k=0}^{L-1} e^{j\hat{\omega}k}$$

$$\begin{aligned} &= \frac{1}{L} \{1 + e^{-j\hat{\omega}1} + \dots + e^{-j\hat{\omega}(L-1)}\} = \frac{1}{L} \left(\frac{1 - e^{-j\hat{\omega}L}}{1 - e^{-j\hat{\omega}}} \right) \\ &= \frac{1}{L} \left(\frac{e^{-j\hat{\omega}L/2}}{e^{-j\hat{\omega}/2}} \right) \left(\frac{e^{+j\hat{\omega}L/2} - e^{-j\hat{\omega}L/2}}{e^{+j\hat{\omega}/2} - e^{-j\hat{\omega}/2}} \right) = \frac{1}{L} \left(\frac{e^{+j\hat{\omega}L/2} - e^{-j\hat{\omega}L/2}}{e^{+j\hat{\omega}/2} - e^{-j\hat{\omega}/2}} \right) e^{-j\hat{\omega}(L-1)/2} \\ &= \left(\frac{\sin(\hat{\omega}L/2)}{L \sin(\hat{\omega}/2)} \right) e^{-j\hat{\omega}(L-1)/2} \end{aligned}$$

Dirichlet Function

$$H(e^{j\hat{\omega}}) = D_L(e^{j\hat{\omega}}) e^{-j\hat{\omega}(L-1)/2}$$

$$D_L(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}L/2)}{L \sin(\hat{\omega}/2)}$$

Correlation: Identifying Similarities

References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] J.C. Santamarina, D Fratta, "Discrete Signals and Inverse Problems: An Introduction for Engineers and Scientists", 2005