Time Domain Techniques (3B)

for Noisy Signals

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Signal Detrending / Spike Removal

Remove low frequency noise

a constant
a line
a long period sinusoid

least square fitting a low frequency function $tr[i]$

subtracting the trend from the measurements $y[i] = x[i] - tr[i]$

example)

removing the dc component

$$y[i] = x[i] - tr[i] = x[i] - \frac{1}{N}\sum_{i=0}^{N-1} x[i-1]$$

removing spikes

$$y[i] = x[i] \qquad if \; |x[i] - y[i-1]| < threshold$$

$$y[i] = \frac{x[i-1] + x[i+1]}{2} \qquad else$$

Stacking

Remove background noise

Assume noise has a zero mean

Measuring multiple times (ensemble)

Averaging across the ensemble

Reduces the noise level in the averaged signal
Increases the SNR of the correlated component (signal)

$$\boldsymbol{E}[X[i]] \rightarrow x[i]$$

$$\boldsymbol{E}[N[i]] \rightarrow 0$$

$$\boldsymbol{E}[N^2[i]] \rightarrow \frac{\sigma_n}{\sqrt{M}} \qquad SNR \propto \sqrt{M}$$

Moving Average Filter

L-point Running Averager

$$y[n] = \frac{1}{L}\sum_{k=0}^{L-1} x[n-k] \qquad = \frac{1}{L}\{x[n] + x[n-1] + \cdots + x[n-L+1]\}$$

$$Y(e^{j\hat{\omega}}) = \frac{1}{L}\sum_{k=0}^{L-1} e^{-j\hat{\omega}k}\, X(e^{j\hat{\omega}}) \qquad = \frac{1}{L}\{X(e^{j\hat{\omega}}) + X(e^{j\hat{\omega}})e^{-j\hat{\omega}1} + \cdots + X(e^{j\hat{\omega}})e^{-j\hat{\omega}(L-1)}\}$$

$$H(e^{j\hat{\omega}}) = \frac{1}{L}\sum_{k=0}^{L-1} e^{j\hat{\omega}k} \qquad = \frac{1}{L}\{1 + e^{-j\hat{\omega}1} + \cdots + e^{-j\hat{\omega}(L-1)}\} \qquad = \frac{1}{L}\left(\frac{1-e^{-j\hat{\omega}L}}{1-e^{-j\hat{\omega}}}\right)$$

$$= \frac{1}{L}\left(\frac{e^{-j\hat{\omega}L/2}}{e^{-j\hat{\omega}/2}}\right)\left(\frac{e^{+j\hat{\omega}L/2} - e^{-j\hat{\omega}L/2}}{e^{+j\hat{\omega}/2} - e^{-j\hat{\omega}/2}}\right) \qquad = \frac{1}{L}\left(\frac{e^{+j\hat{\omega}L/2} - e^{-j\hat{\omega}L/2}}{e^{+j\hat{\omega}/2} - e^{-j\hat{\omega}/2}}\right) e^{-j\hat{\omega}(L-1)/2}$$

$$= \left(\frac{\sin(\hat{\omega}L/2)}{L\sin(\hat{\omega}/2)}\right) e^{-j\hat{\omega}(L-1)/2}$$

Dirichlet Function

$$H(e^{j\hat{\omega}}) = D_L(e^{j\hat{\omega}})\, e^{-j\hat{\omega}(L-1)/2} \qquad D_L(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}L/2)}{L\sin(\hat{\omega}/2)}$$

Correlation: Identifying Similarities

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] J.C. Santamarina, D Fratta, "Discrete Signals and Inverse Problems: An Introduction for Engineers and Scientists", 2005