Complex Functions (1B)

Young Won Lim
12/15/12

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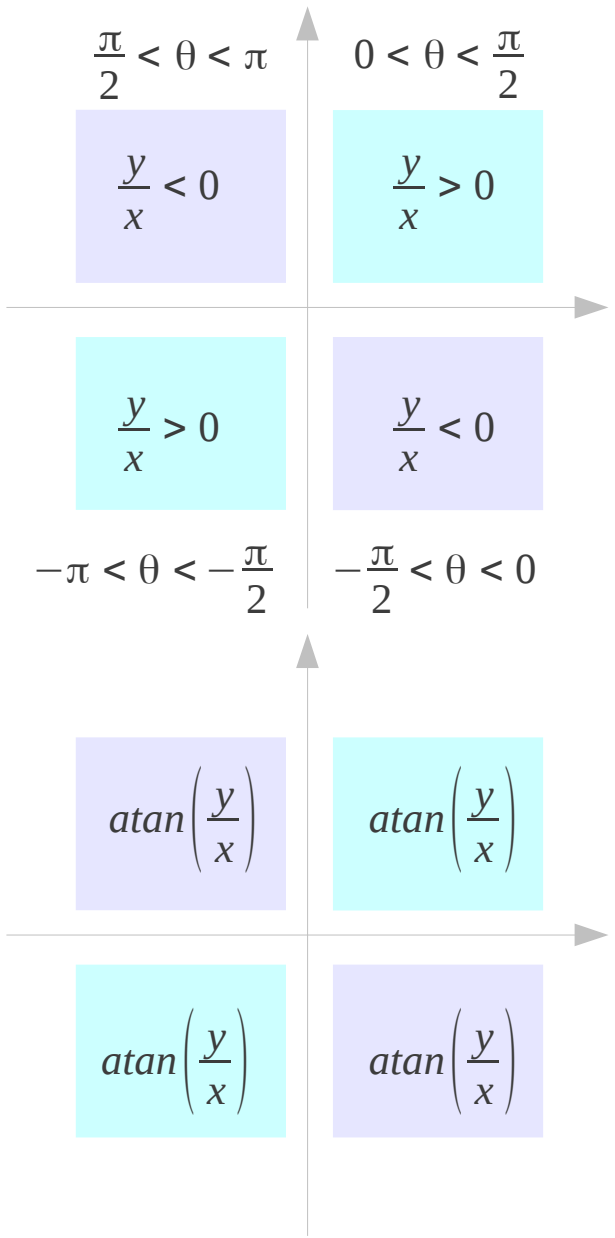
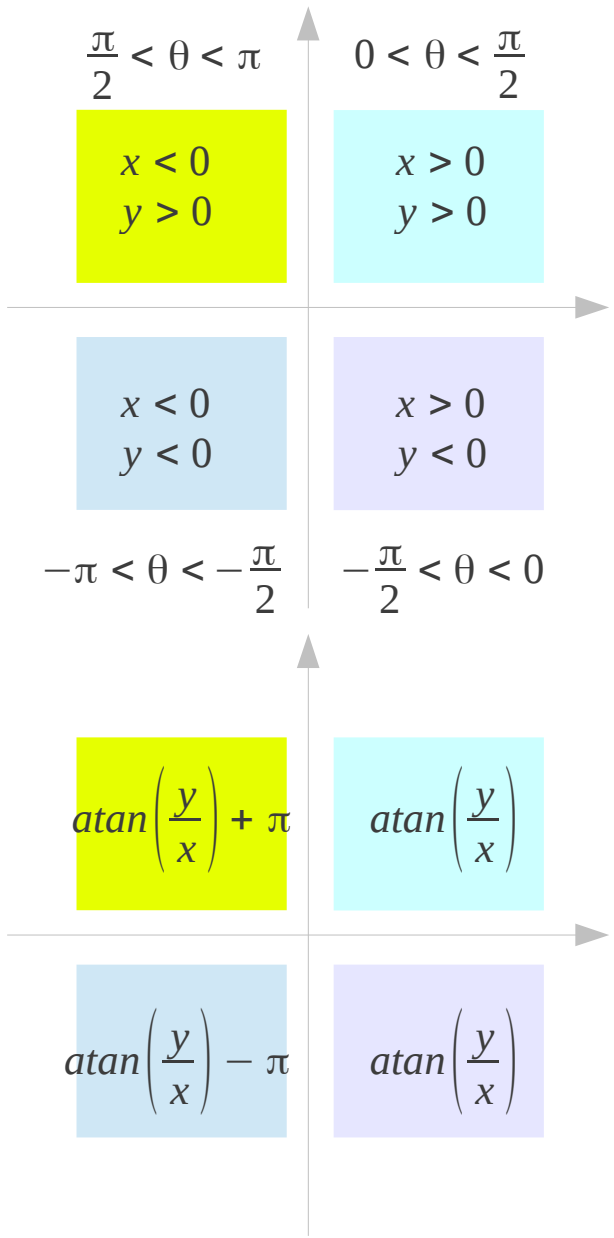
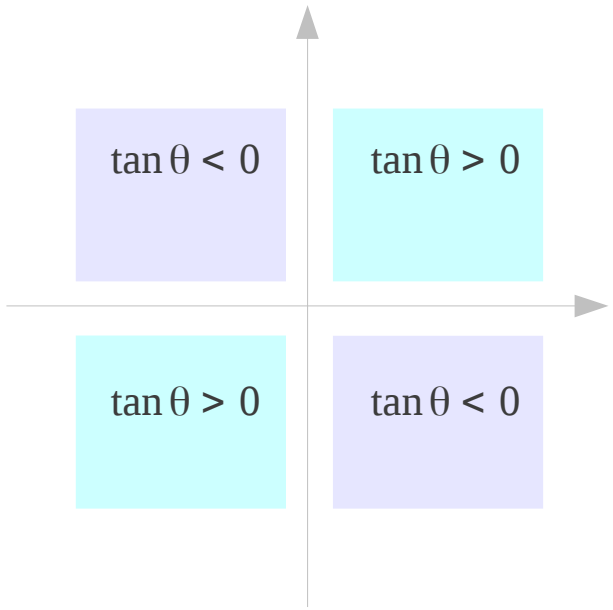
Young Won Lim
12/15/12

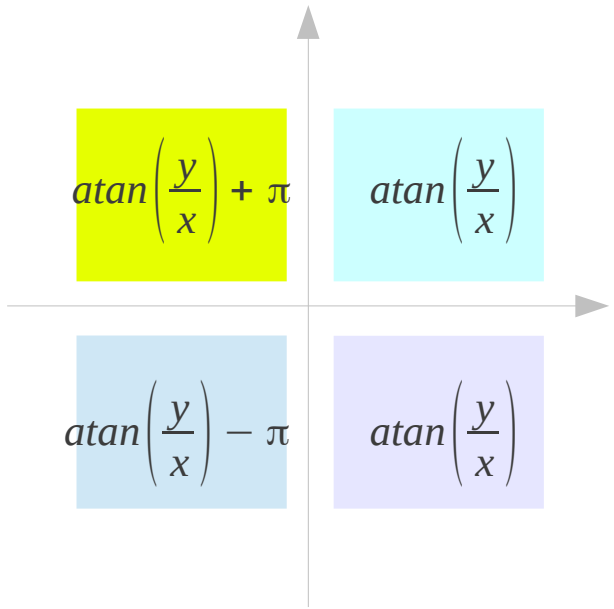
Argument of a Complex Number (1)



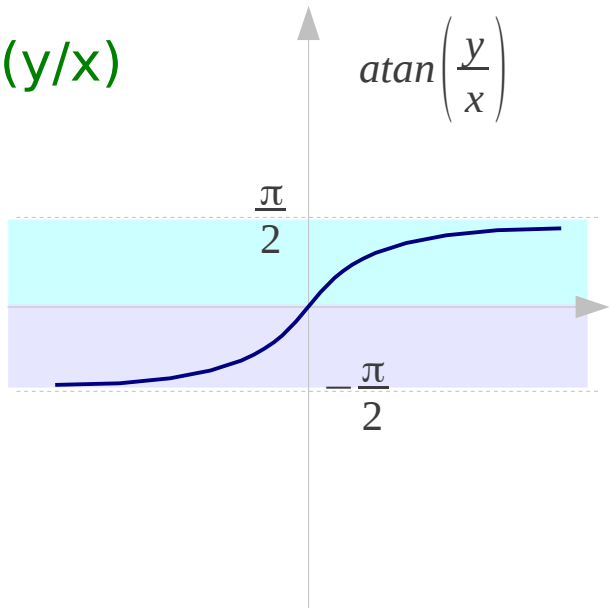


Argument of a Complex Number (2)

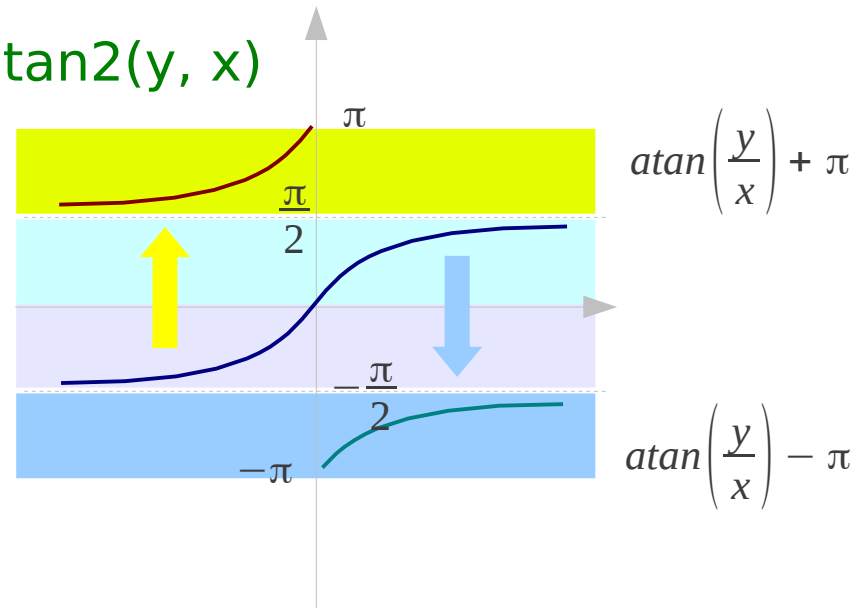




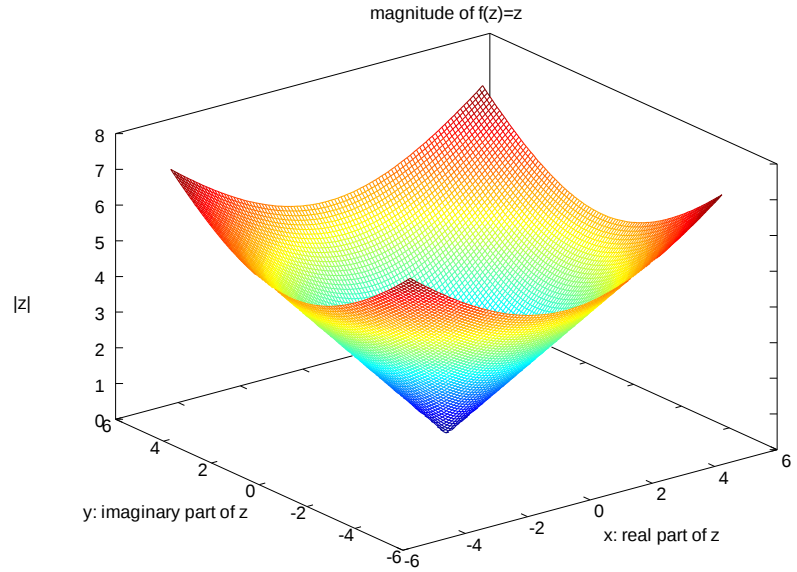
atan(y/x)

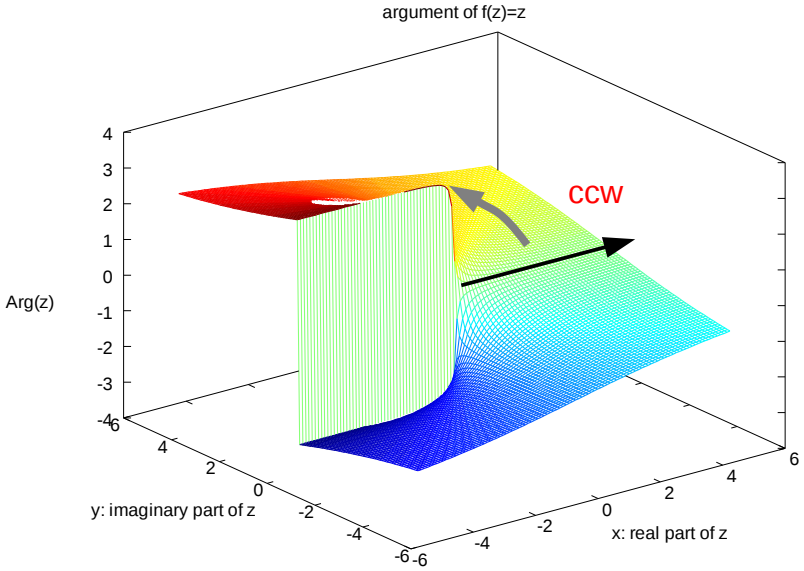


atan2(y, x)



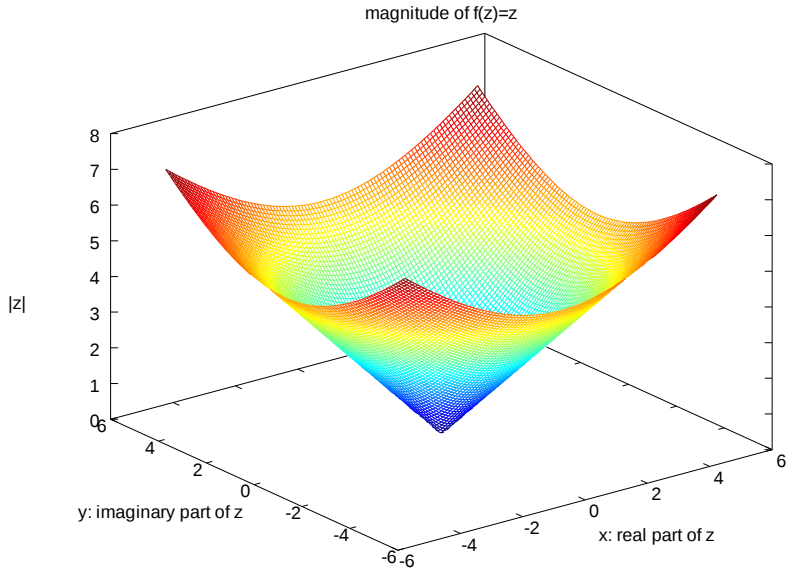
f(z)=z

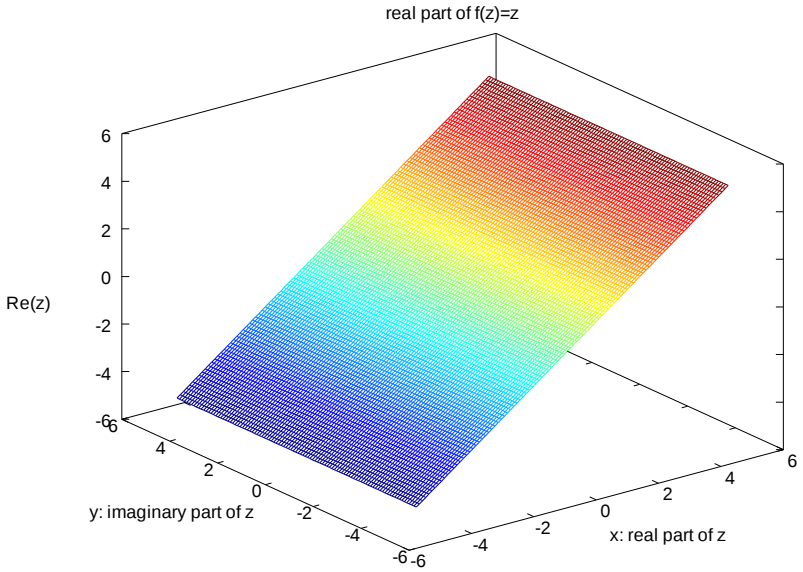


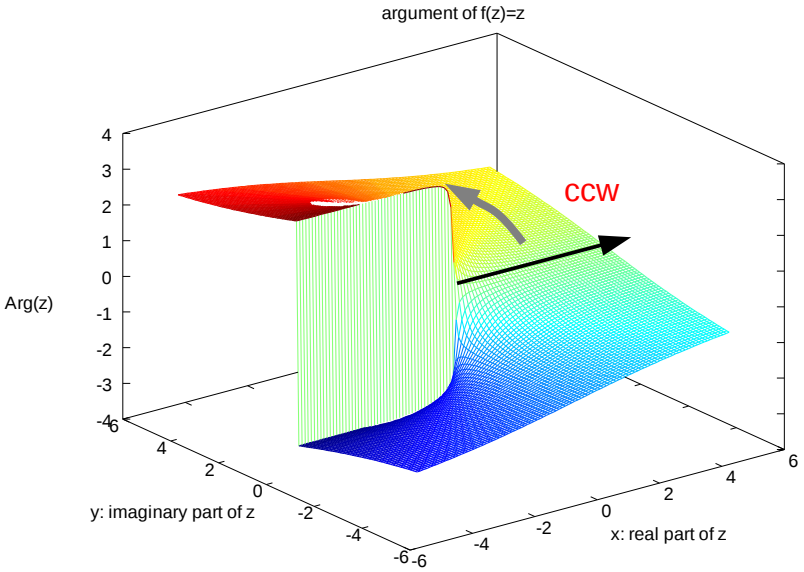


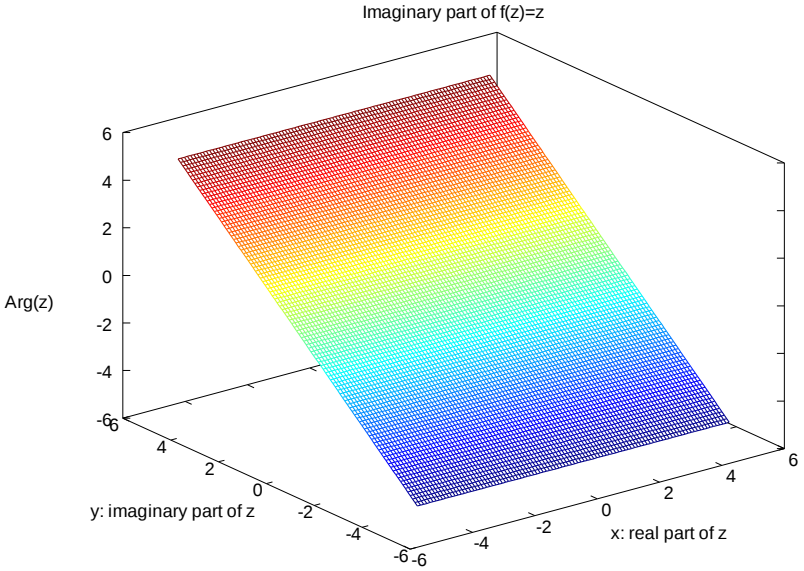

```
%-------------------------------------------------------------------------
% Plot f(z) = z^2
% Licensing: This code is distributed under the GNU LGPL license.
% Modified: 2012.11.23
% Author: Young W. Lim
%-------------------------------------------------------------------------

x = linspace(-5, +5, 100);
y = linspace(-5, +5, 100);
[xx yy] = meshgrid(x, y);

z = xx + i* yy;


mesh(xx, yy, abs(z))
title("magnitude of f(z)=z");
xlabel("x: real part of z");
ylabel("y: imaginary part of z");
zlabel("|z|");
print -demf z.mag.emf

pause

mesh(xx, yy, arg(z))
title("argument of f(z)=z");
xlabel("x: real part of z");
ylabel("y: imaginary part of z");
zlabel("Arg(z)");
print -demf z.arg.emf
```

f(z)=z


magnitude of f(z)=z
|z|
y: imaginary part of z
x: real part of z


real part of f(z)=z
Re(z)
y: imaginary part of z
x: real part of z


argument of f(z)=z
Arg(z)
CCW
y: imaginary part of z
x: real part of z


Imaginary part of f(z)=z
Arg(z)
y: imaginary part of z
x: real part of z

$f(z) = z^2$

$$z = x + iy$$

$$f(z) = u(x,y) + iv(x,y)$$

$$f(z) = z^2 = (x+iy)^2 = (x^2 + i2xy - y^2)$$

$$= (x^2 - y^2) + i(2xy)$$

$$u(x,y) = (x^2 - y^2) \qquad v(x,y) = (2xy)$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

$$\frac{\partial u}{\partial x} = 2x \qquad \frac{\partial v}{\partial x} = 2y$$

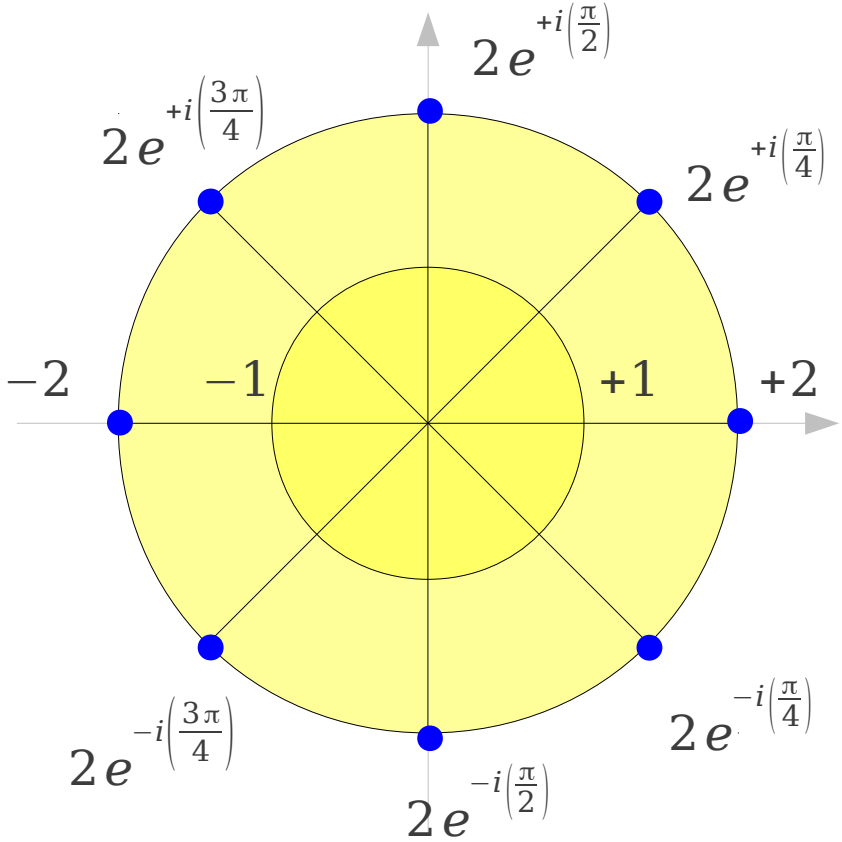
$$\frac{\partial u}{\partial y} = -2y \qquad \frac{\partial v}{\partial y} = 2x$$

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = 2x + i2y$$

$$f'(z) = -i\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} = i2y + 2x$$

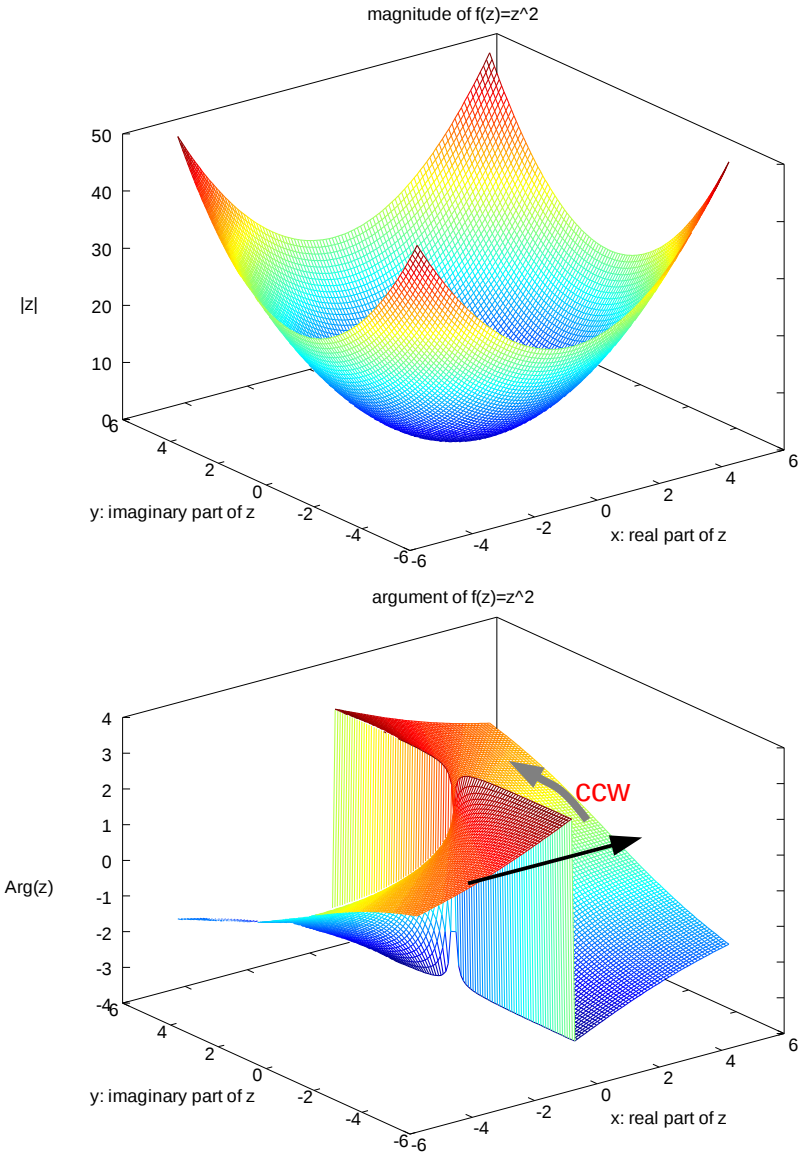
$$f'(z) = 2z = 2(x+iy)$$

Complex Numbers



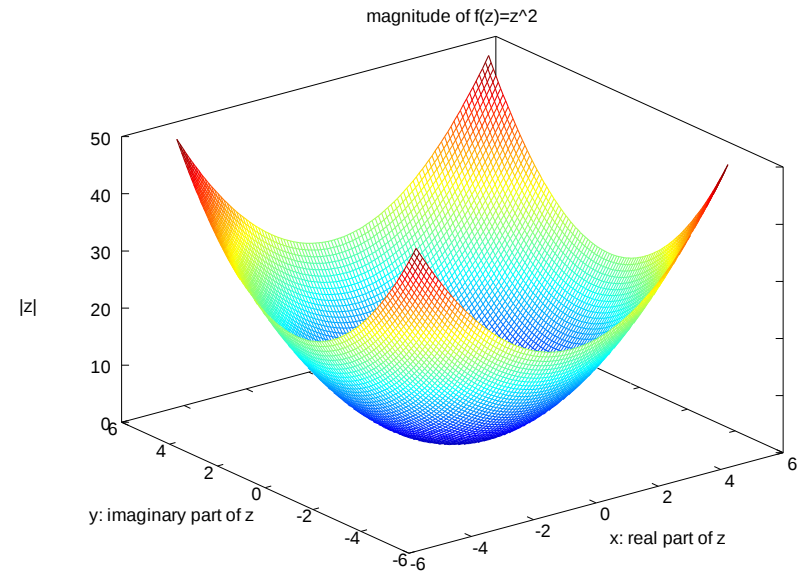
polar	rectangular
$+2$	$+2$
$2e^{+i\left(\frac{\pi}{4}\right)}$	$\sqrt{2}(+1+i)$
$2e^{+i\left(\frac{\pi}{2}\right)}$	$2i$
$2e^{+i\left(\frac{3\pi}{4}\right)}$	$\sqrt{2}(-1+i)$
-2	-2
$2e^{-i\left(\frac{3\pi}{4}\right)}$	$\sqrt{2}(-1-i)$
$2e^{-i\left(\frac{\pi}{2}\right)}$	$-2i$
$2e^{-i\left(\frac{\pi}{4}\right)}$	$\sqrt{2}(+1-i)$

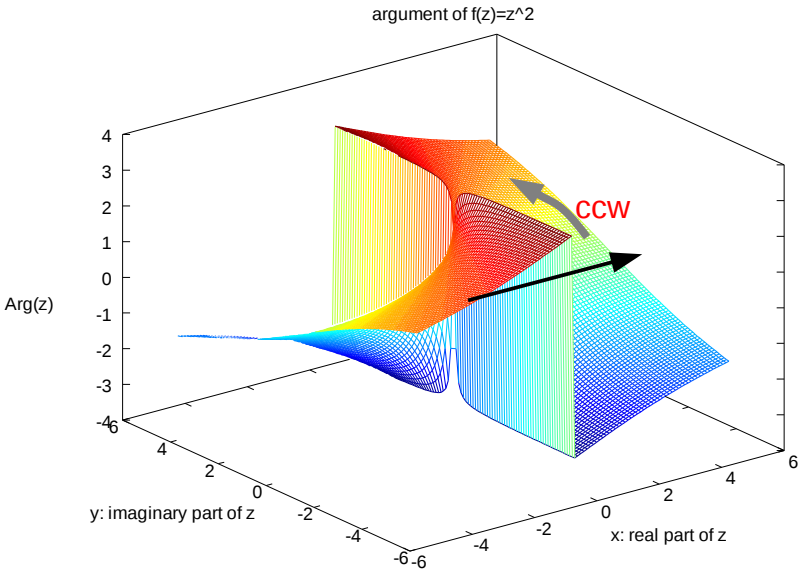
Complex Functions



f(z) =	z^2
$f(+2)$	4
$f\left(2e^{+i\left(\frac{\pi}{4}\right)}\right)$	$4e^{+i\left(\frac{\pi}{2}\right)}$
$f\left(2e^{+i\left(\frac{\pi}{2}\right)}\right)$	$4e^{+i(\pi)}$
$f\left(2e^{+i\left(\frac{3\pi}{4}\right)}\right)$	$4e^{-i\left(\frac{\pi}{2}\right)}$
$f(-2)$	4
$f\left(2e^{-i\left(\frac{3\pi}{4}\right)}\right)$	$4e^{+i\left(\frac{\pi}{2}\right)}$
$f\left(2e^{-i\left(\frac{\pi}{2}\right)}\right)$	$4e^{-i(\pi)}$
$f\left(2e^{-i\left(\frac{\pi}{4}\right)}\right)$	$4e^{-i\left(\frac{\pi}{2}\right)}$

f(z)=z^2






```
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x = linspace(-5, +5, 100);
y = linspace(-5, +5, 100);
[xx yy] = meshgrid(x, y);

z = xx + i* yy;


mesh(xx, yy, abs(z))
title("magnitude of f(z)=z");
xlabel("x: real part of z");
ylabel("y: imaginary part of z");
zlabel("|z|");
print -demf z.mag.emf

pause

mesh(xx, yy, arg(z))
title("argument of f(z)=z");
xlabel("x: real part of z");
ylabel("y: imaginary part of z");
zlabel("Arg(z)");
print -demf z.arg.emf
```
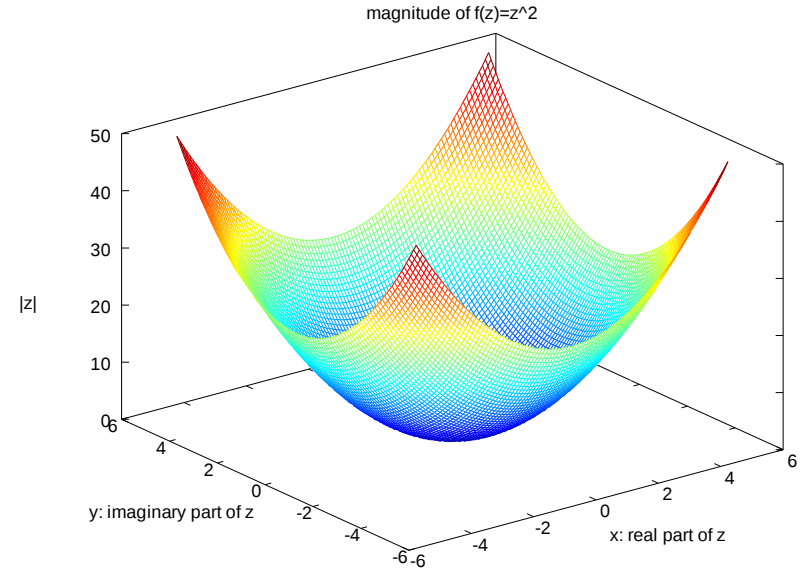
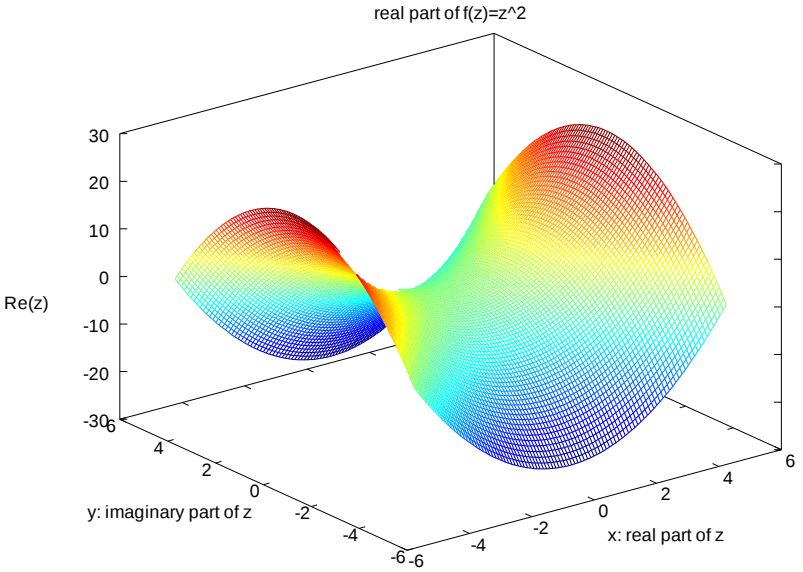
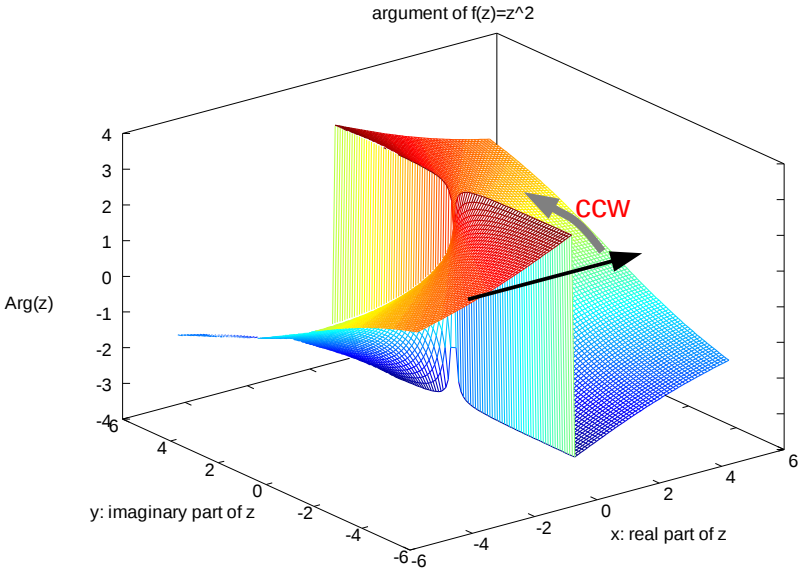
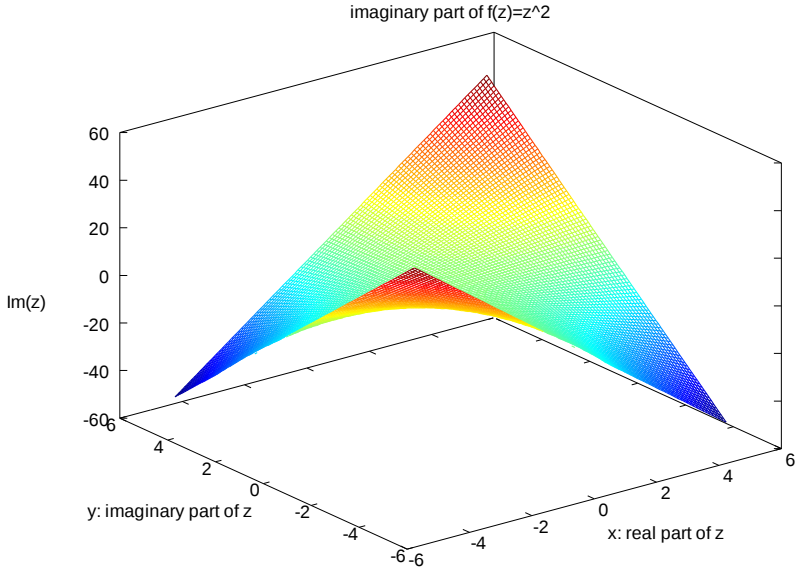

f(z)=z^2

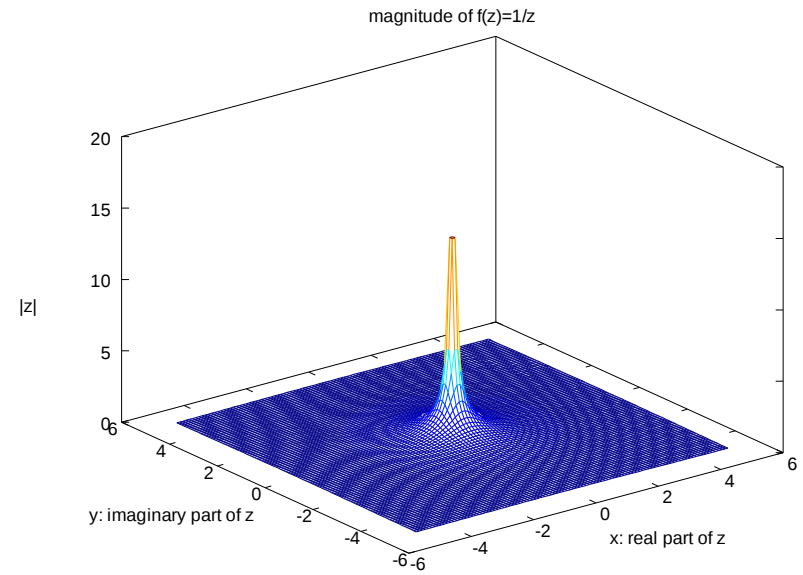

magnitude of f(z)=z^2
|z|
y: imaginary part of z
x: real part of z

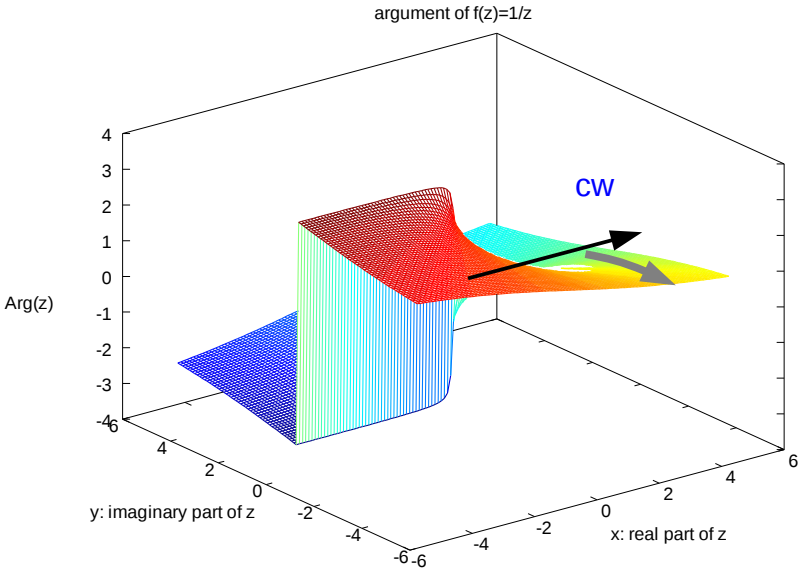

real part of f(z)=z^2
Re(z)
y: imaginary part of z
x: real part of z


argument of f(z)=z^2
Arg(z)
CCW
y: imaginary part of z
x: real part of z


imaginary part of f(z)=z^2
Im(z)
y: imaginary part of z
x: real part of z

f(z)=1/z






```
%-------------------------------------------------------------------------
% Plot f(z) = 1/z
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% Author: Young W. Lim
%-------------------------------------------------------------------------

x = linspace(-5, +5, 100);
y = linspace(-5, +5, 100);
[xx yy] = meshgrid(x, y);

z1 = xx + i* yy;
z = 1 ./ z1;


mesh(xx, yy, abs(z))
title("magnitude of f(z)=1/z");
xlabel("x: real part of z");
ylabel("y: imaginary part of z");
zlabel("|z|");
print -demf 1_z.mag.emf

pause

mesh(xx, yy, arg(z))
title("argument of f(z)=1/z");
xlabel("x: real part of z");
ylabel("y: imaginary part of z");
zlabel("Arg(z)");
print -demf 1_z.arg.emf
```
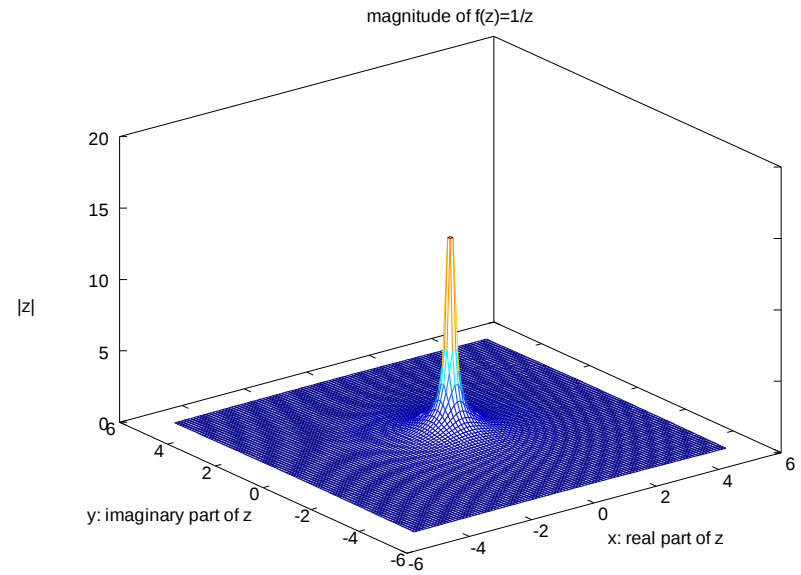
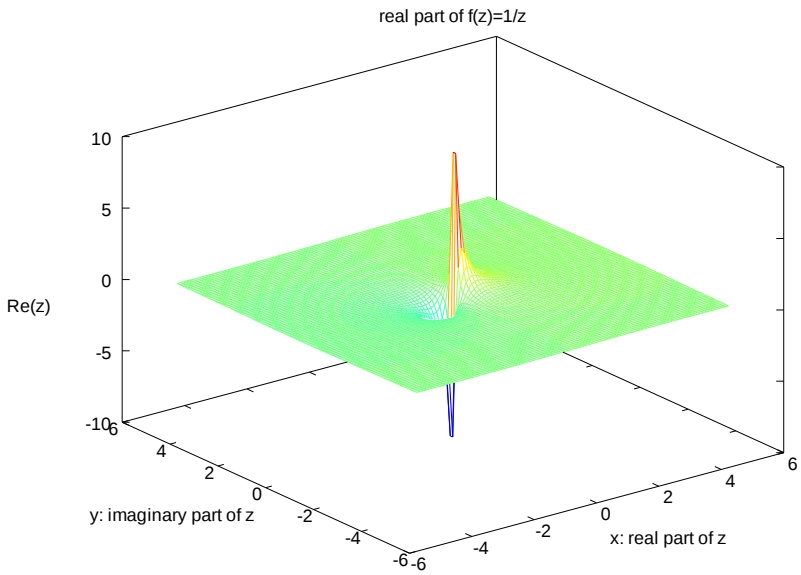
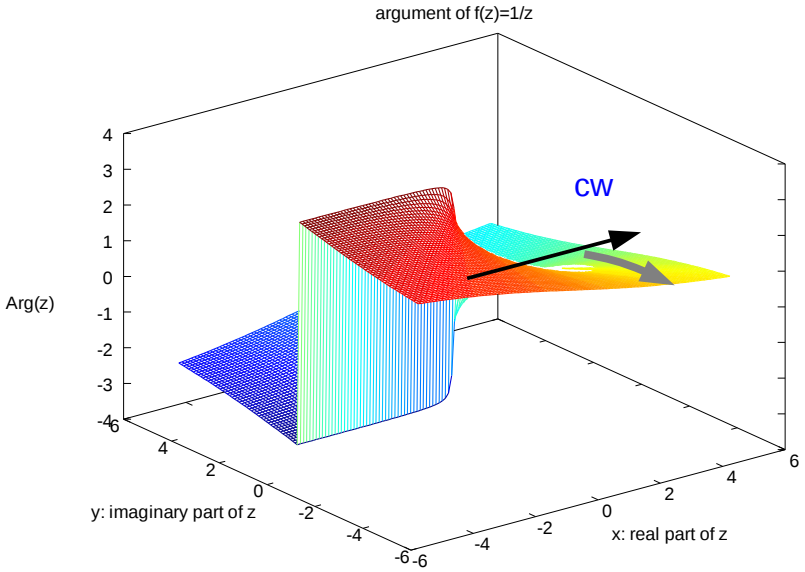
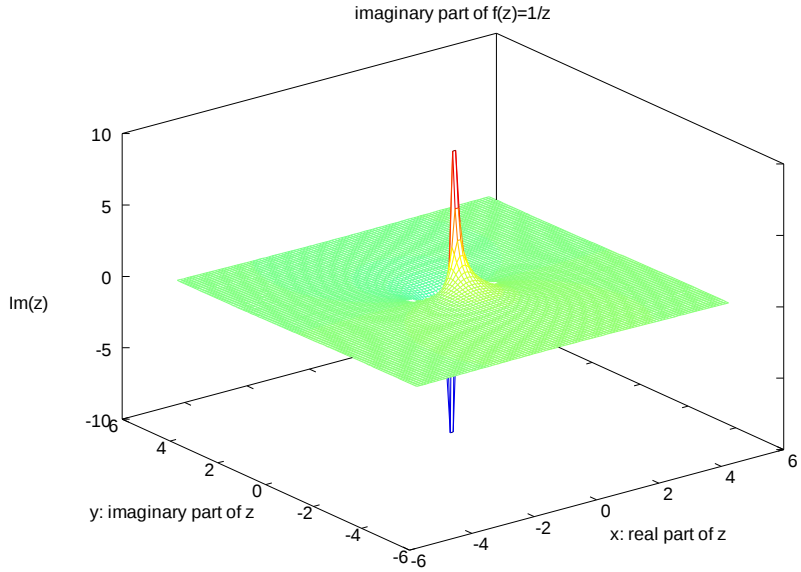

f(z)=1/z

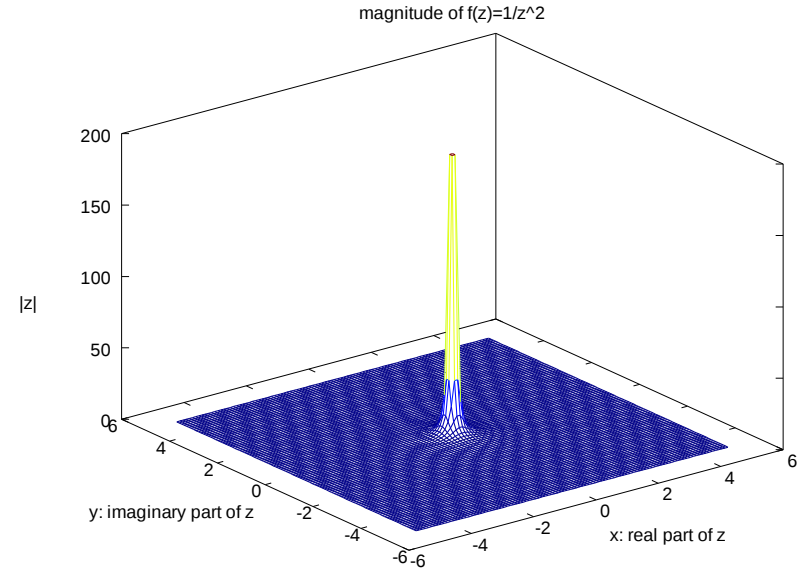

magnitude of f(z)=1/z
|z|
y: imaginary part of z
x: real part of z

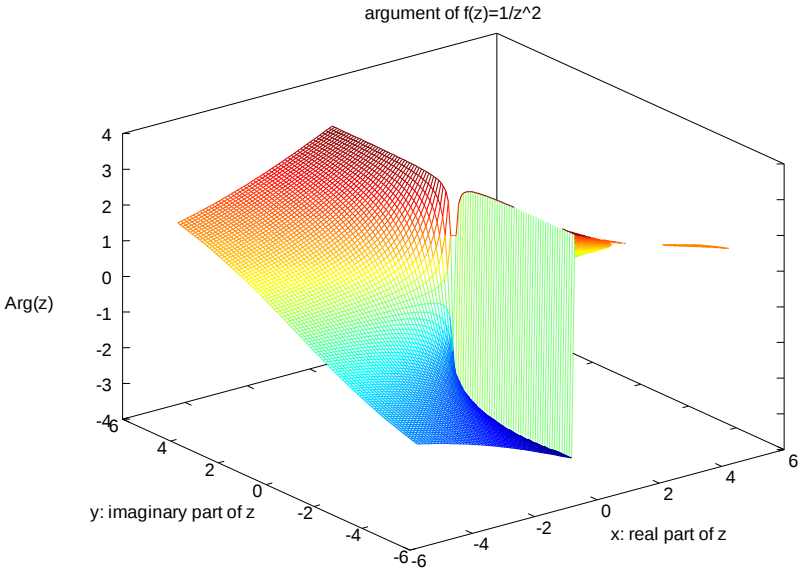

real part of f(z)=1/z
Re(z)
y: imaginary part of z
x: real part of z


argument of f(z)=1/z
Arg(z)
CW
y: imaginary part of z
x: real part of z


imaginary part of f(z)=1/z
Im(z)
y: imaginary part of z
x: real part of z

f(z)=1/z^2






```
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%-------------------------------------------------------------------------

x = linspace(-5, +5, 100);
y = linspace(-5, +5, 100);
[xx yy] = meshgrid(x, y);

z1 = xx + i* yy;
z2 = z1 .* z1;
z = 1 ./ z2;


mesh(xx, yy, abs(z))
title("magnitude of f(z)=1/z^2");
xlabel("x: real part of z");
ylabel("y: imaginary part of z");
zlabel("|z|");
print -demf 1_z2.mag.emf

pause

mesh(xx, yy, arg(z))
title("argument of f(z)=1/z^2");
xlabel("x: real part of z");
ylabel("y: imaginary part of z");
zlabel("Arg(z)");
print -demf 1_z2.arg.emf
```
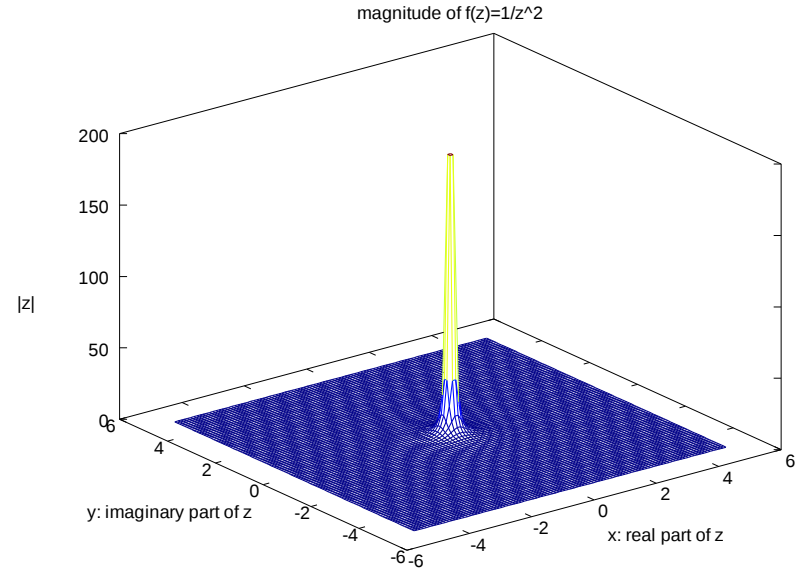
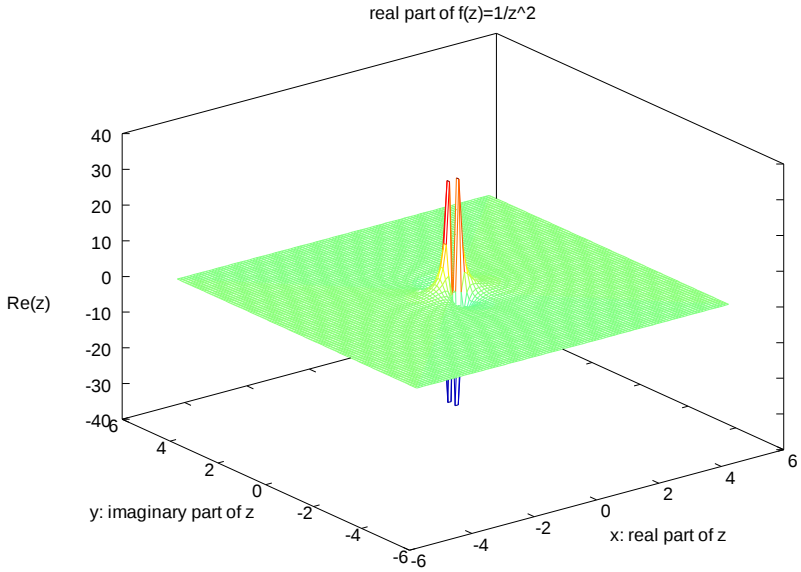
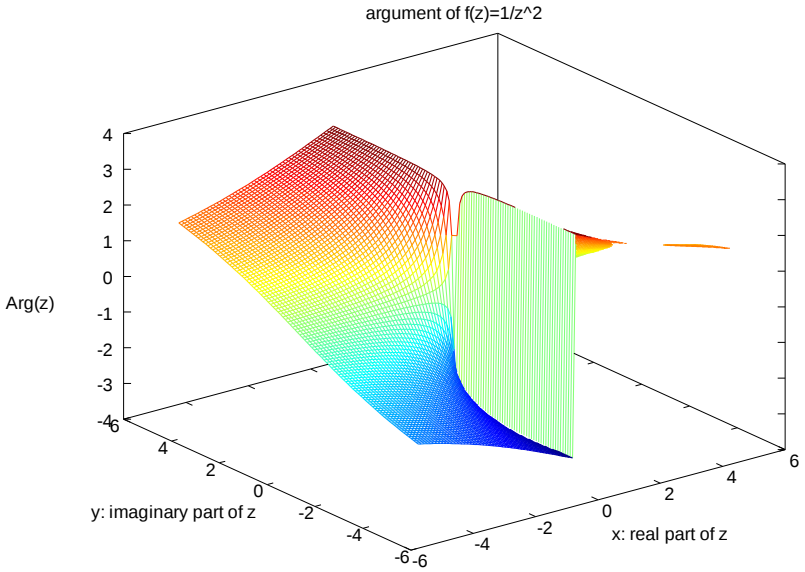
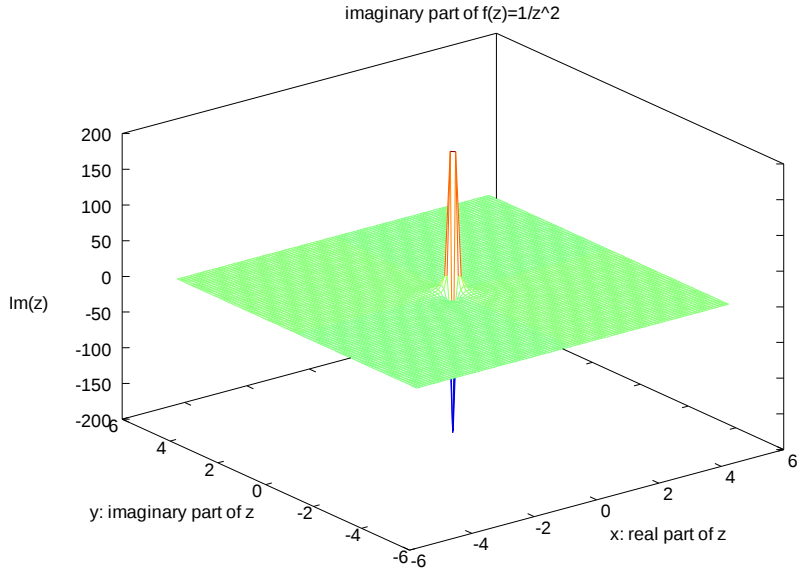

f(z)=1/z^2


magnitude of f(z)=1/z^2
|z|
200
150
100
50
0
6
4
2
0
-2
-4
-6
y: imaginary part of z
-6
-4
-2
0
2
4
6
x: real part of z


real part of f(z)=1/z^2
Re(z)
40
30
20
10
0
-10
-20
-30
-40
6
4
2
0
-2
-4
-6
y: imaginary part of z
-6
-4
-2
0
2
4
6
x: real part of z


argument of f(z)=1/z^2
Arg(z)
4
3
2
1
0
-1
-2
-3
-4
6
4
2
0
-2
-4
-6
y: imaginary part of z
-6
-4
-2
0
2
4
6
x: real part of z


imaginary part of f(z)=1/z^2
Im(z)
200
150
100
50
0
-50
-100
-150
-200
6
4
2
0
-2
-4
-6
y: imaginary part of z
-6
-4
-2
0
2
4
6
x: real part of z

Right Hand Rule

References

[1] http://en.wikipedia.org/
[2] http://planetmath.org/
[3] M.L. Boas, “Mathematical Methods in the Physical Sciences”
[4] D.G. Zill, “Advanced Engineering Mathematics”