Upsampling (5B)

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9/24/12

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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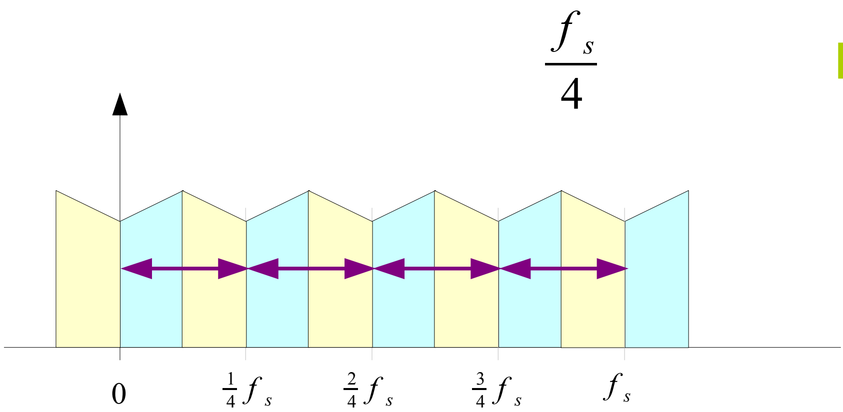
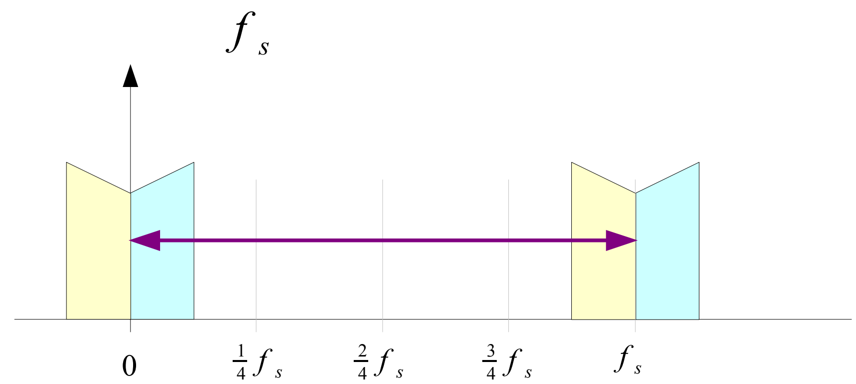
Band-limited Signal



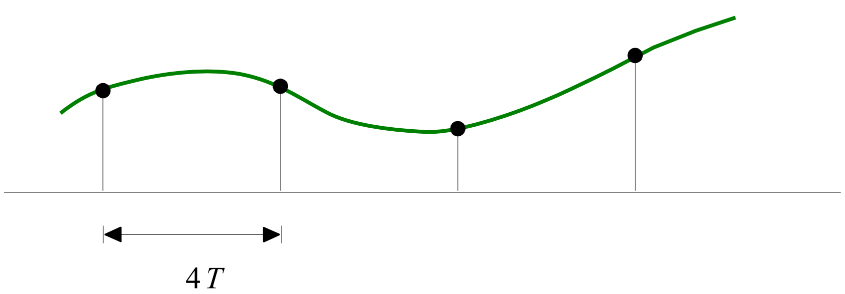


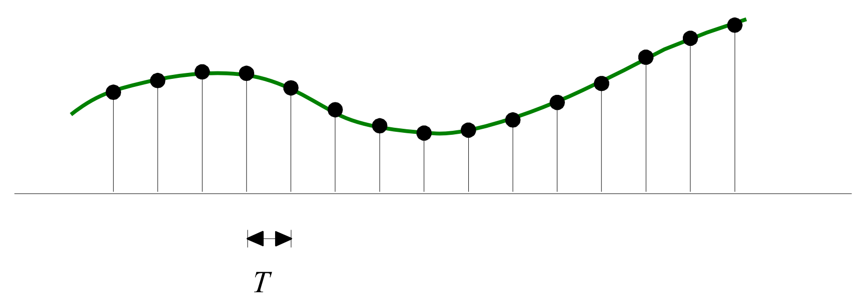
Sampling Frequency $\frac{1}{4}f_s$

Sampling Time $T = \frac{4}{f_s}$

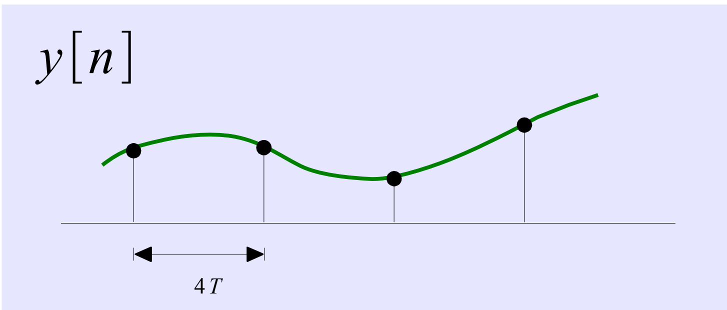
Sampling Frequency $f'_s = f_s$

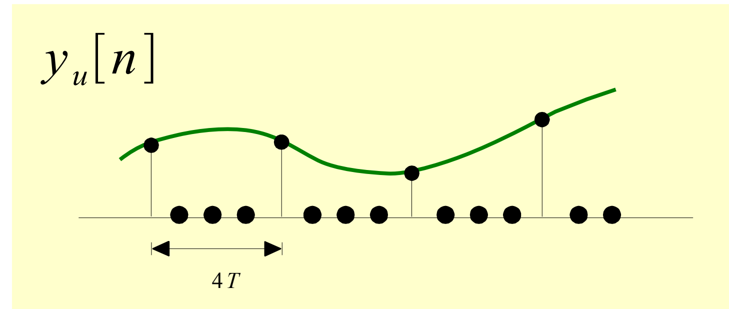
Sampling Time $T' = \frac{1}{f_s}$

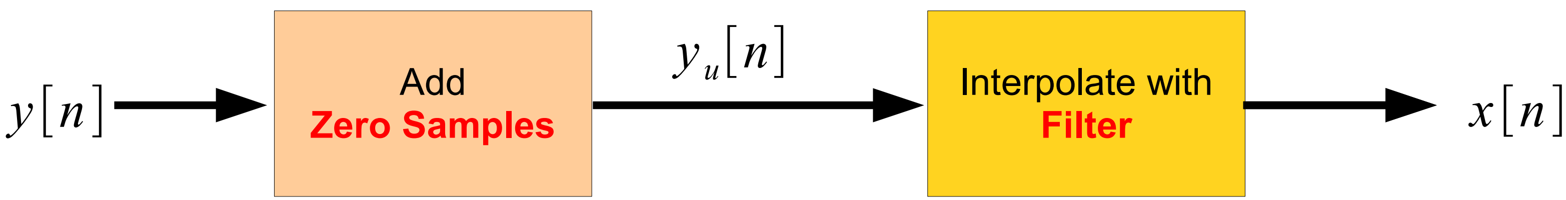




Time Sequence

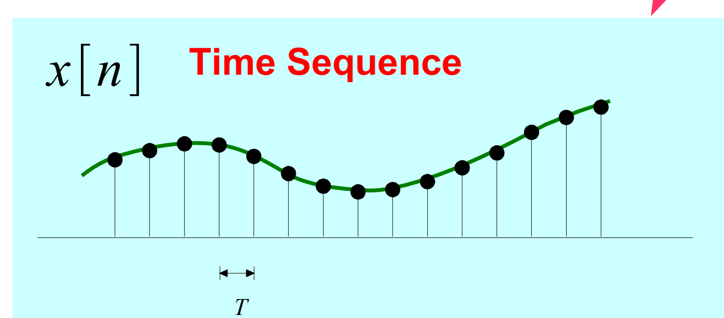






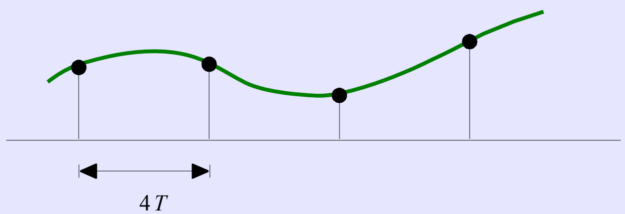
Ideal Sampling

T **Sampling Period**



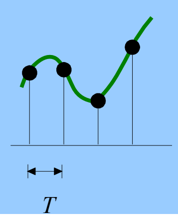
Normalized Radian Frequency

$y[n]$ **Time Sequence** f_H



|| **The Same Time Sequence**

$y[n]$ **Time Sequence** $4f_H$

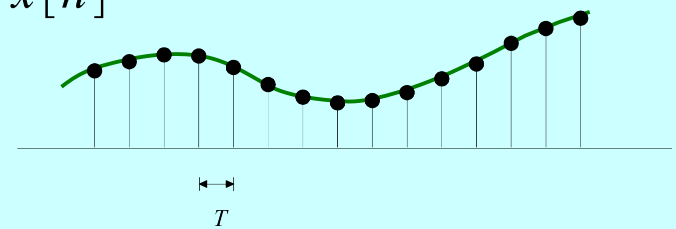


The Same Normalized Radian Frequency

The Highest Frequency: $f_H, \quad 4f_H$

$$\frac{f_H}{1/4T} = f_H \cdot 4T \qquad \frac{4f_H}{1/T} = f_H \cdot 4T$$

$x[n]$



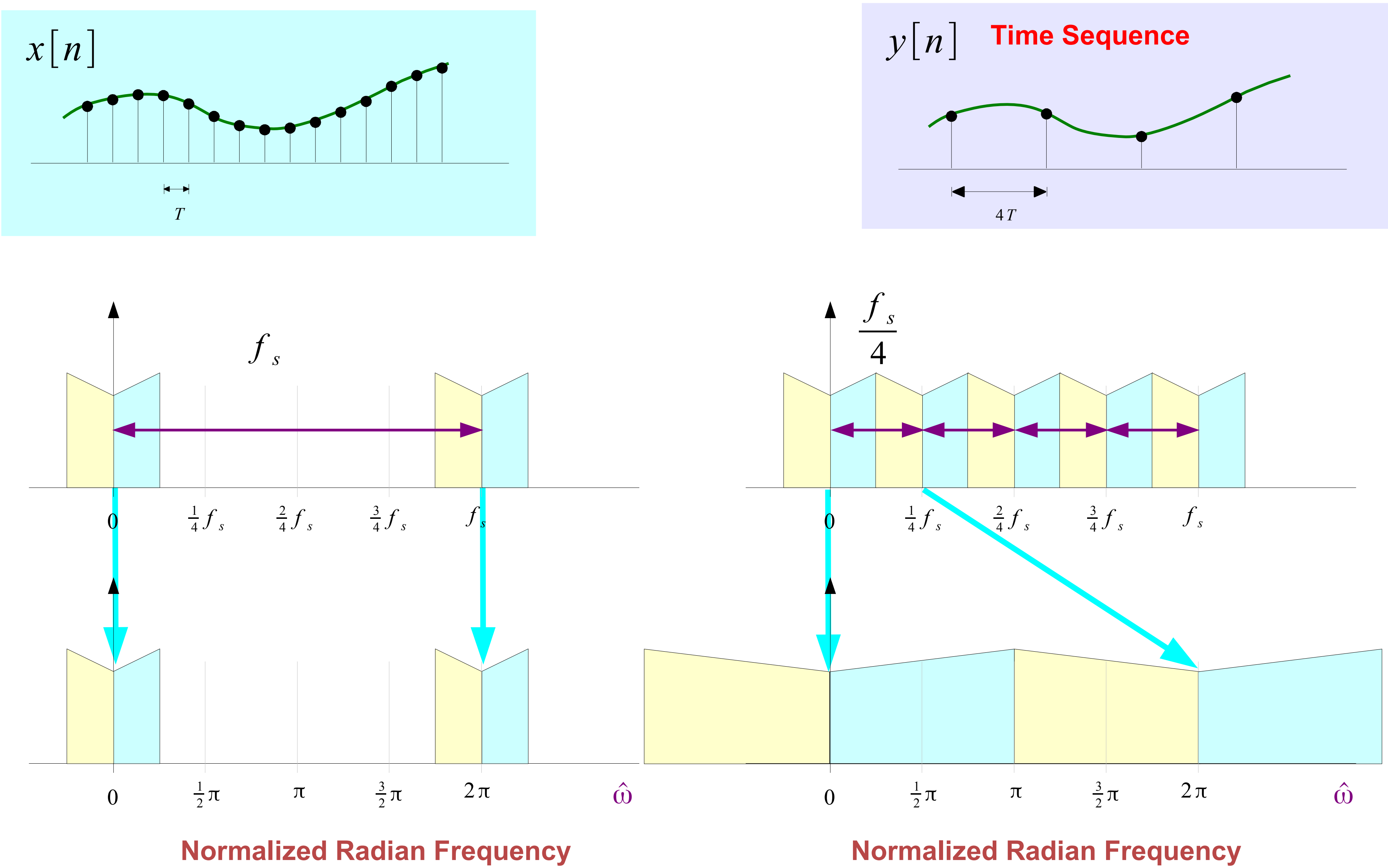
$$\hat{\omega} = \omega \cdot T_s = \frac{\omega}{1/T_s}$$

$$\hat{\omega} = \frac{\omega}{f_s} = 2\pi \frac{f}{f_s}$$

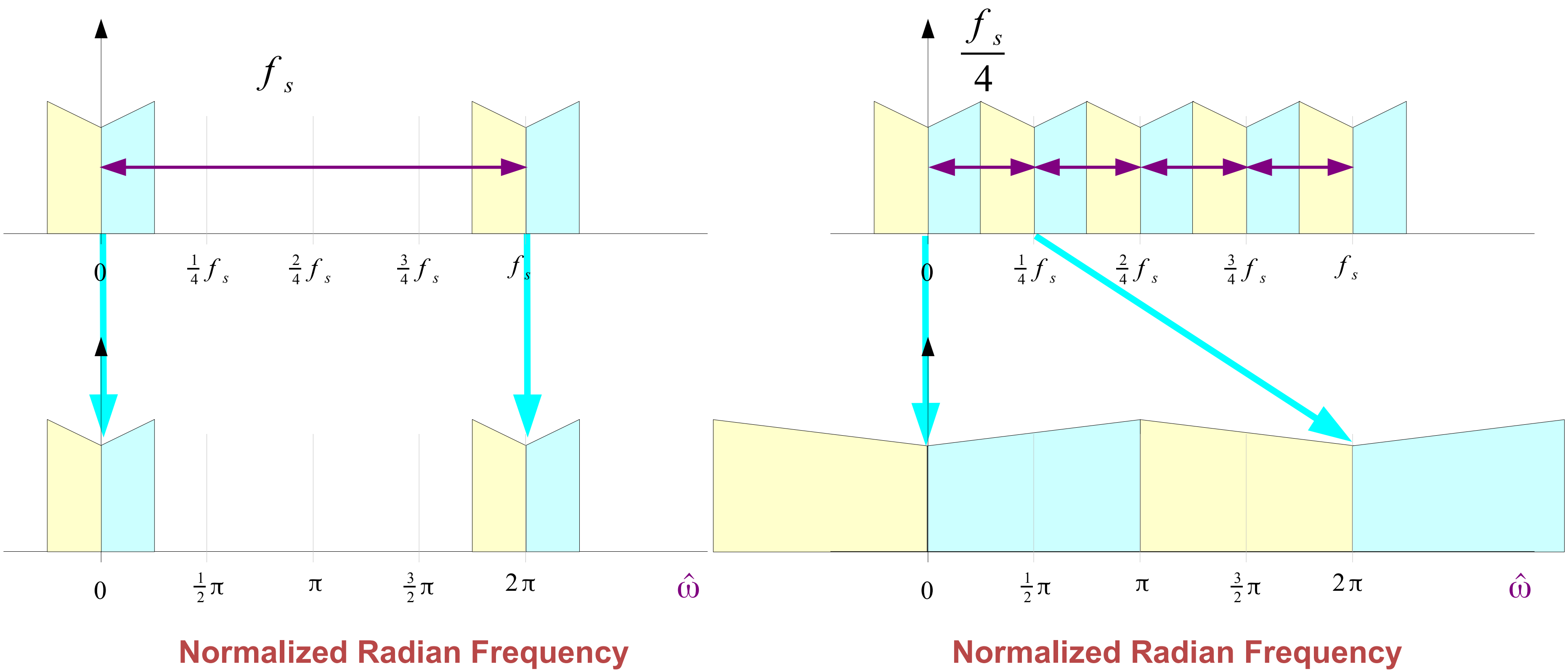
Normalized to f_s

Normalized Radian Frequency

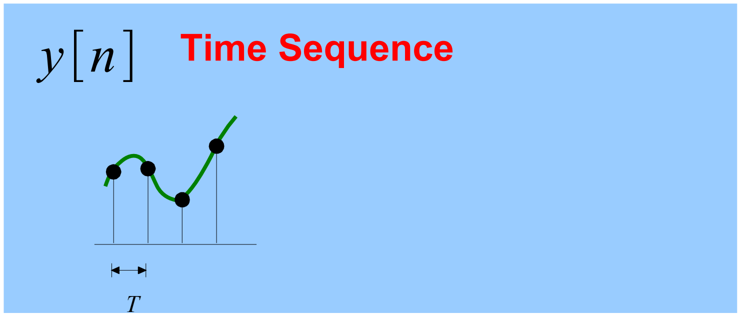
Adding Zero Samples

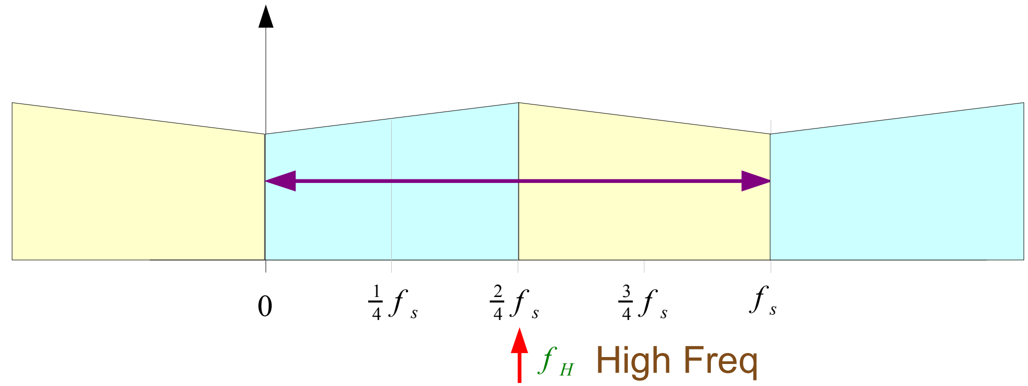


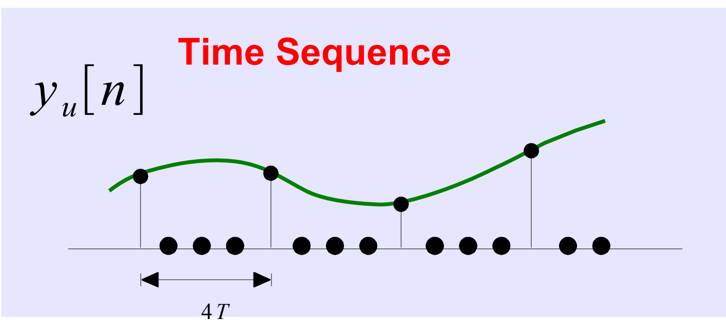
Adding Zero Samples

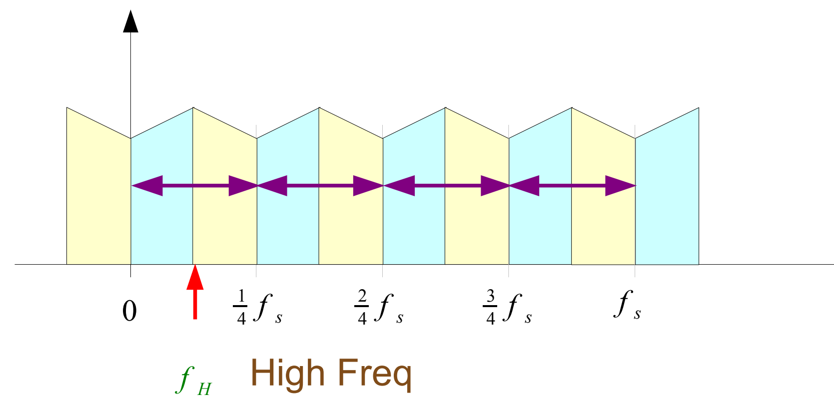


Time Sequence Spectrum in Linear Frequency

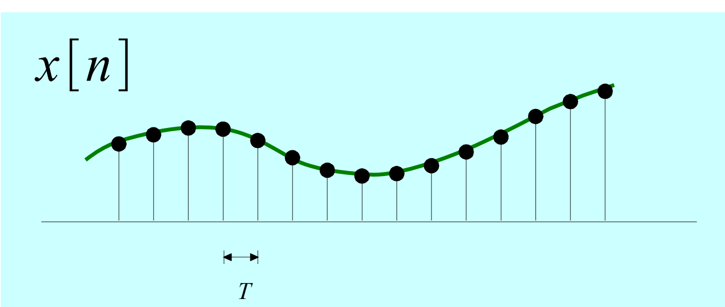


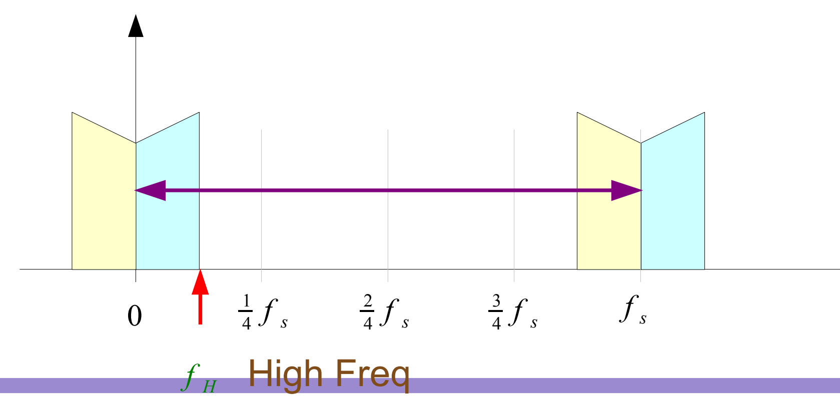




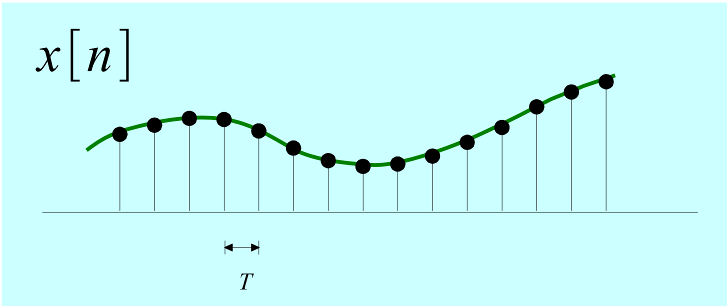


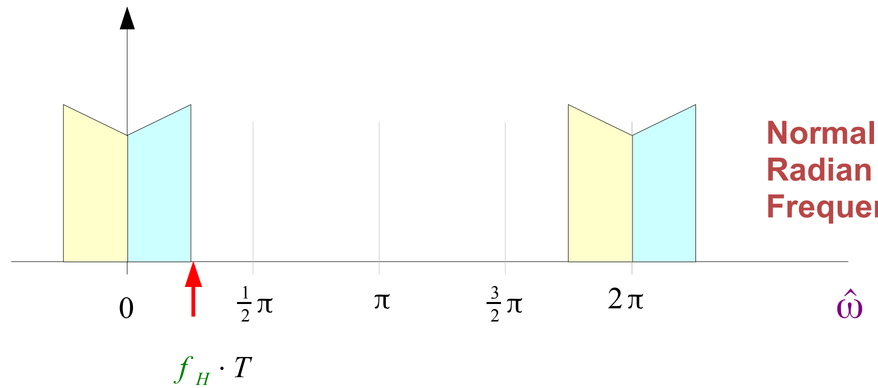
The Same Time Sequence

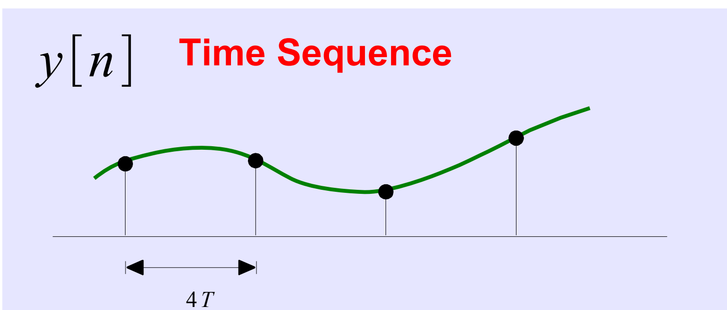


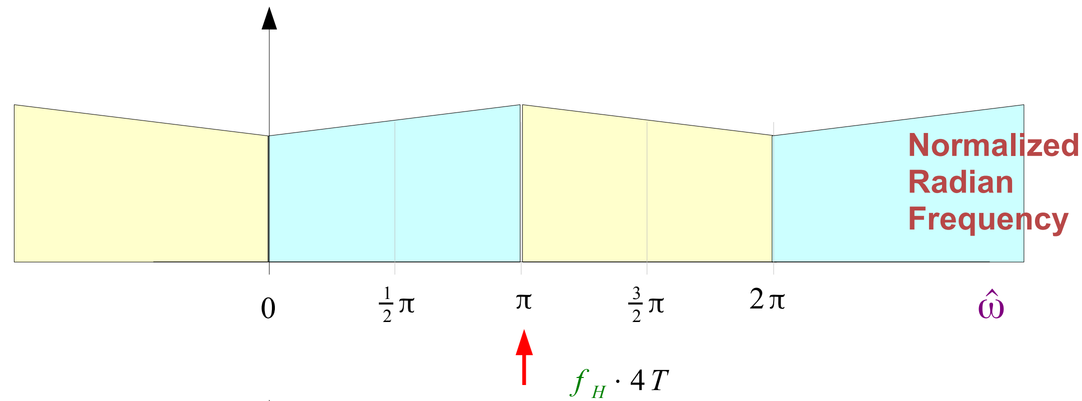


Time Sequence Spectrum in Normalized Frequency

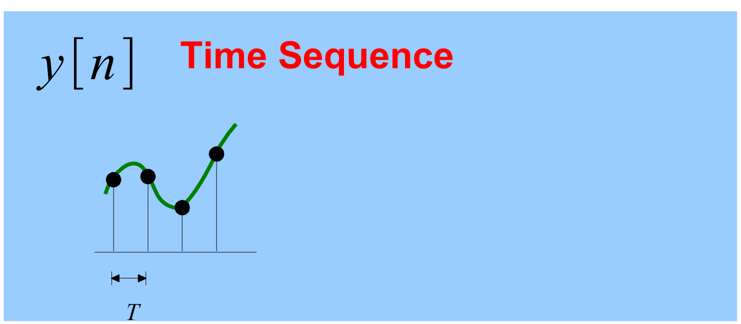


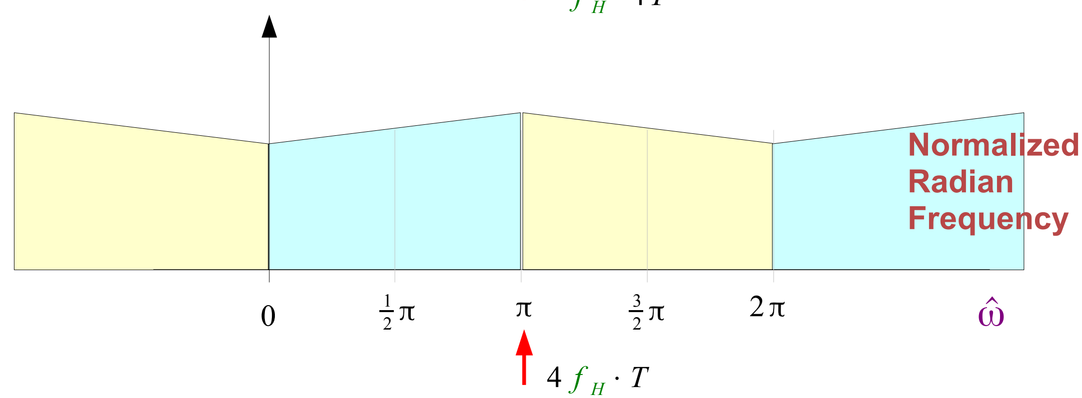




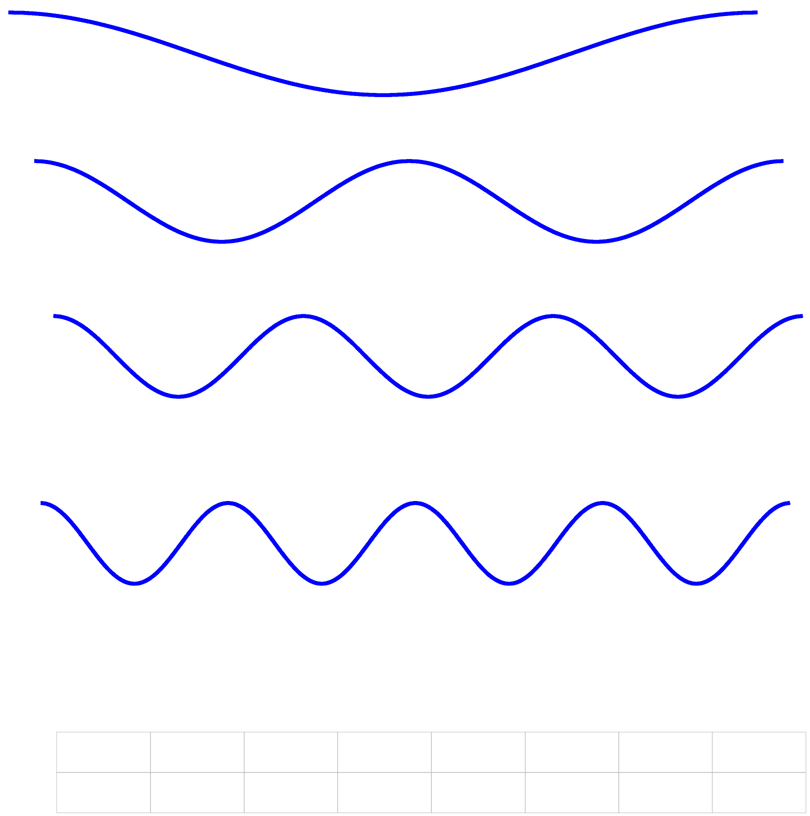


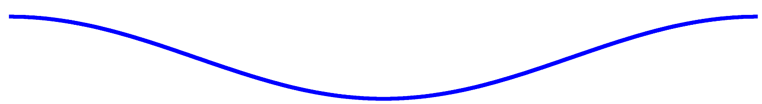
= The Same Time Sequence



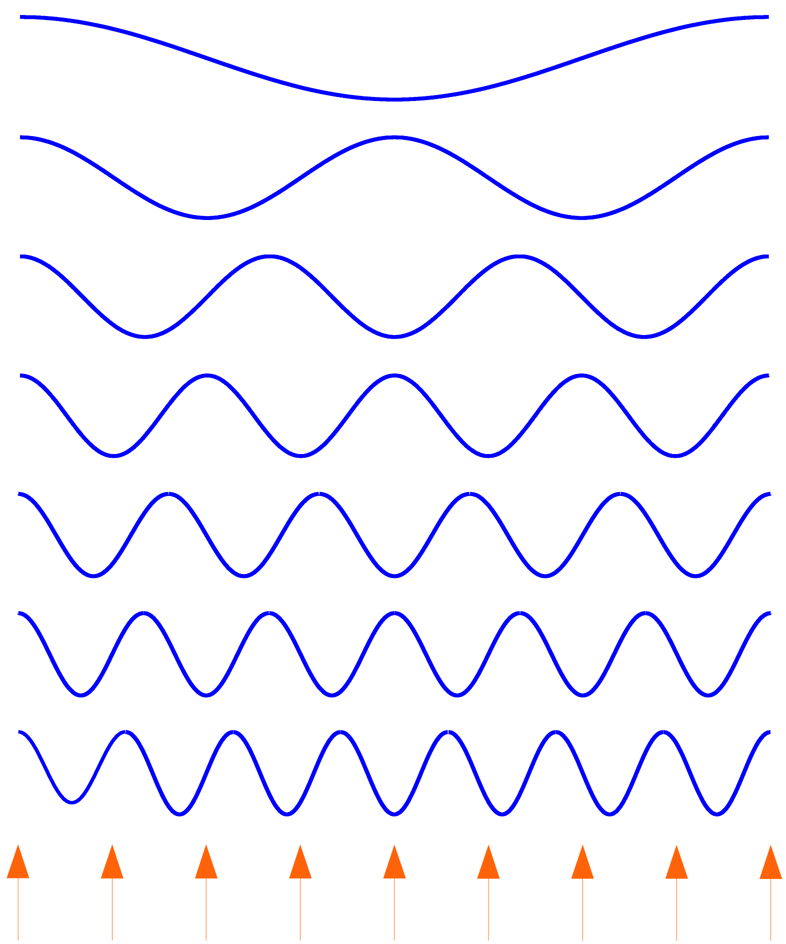


Measuring Rotation Rate





Signals with Harmonic Frequencies (1)



1 Hz
1 cycle / sec

$$\cos(1\cdot 2\pi t) = \frac{e^{+j(1\cdot 2\pi)t} + e^{-j(1\cdot 2\pi)t}}{2}$$

2 Hz
2 cycles / sec

$$\cos(2\cdot 2\pi t) = \frac{e^{+j(2\cdot 2\pi)t} + e^{-j(2\cdot 2\pi)t}}{2}$$

3 Hz
3 cycles / sec

$$\cos(3\cdot 2\pi t) = \frac{e^{+j(3\cdot 2\pi)t} + e^{-j(3\cdot 2\pi)t}}{2}$$

4 Hz
4 cycles / sec

$$\cos(4\cdot 2\pi t) = \frac{e^{+j(4\cdot 2\pi)t} + e^{-j(4\cdot 2\pi)t}}{2}$$

5 Hz
5 cycles / sec

$$\cos(5\cdot 2\pi t) = \frac{e^{+j(5\cdot 2\pi)t} + e^{-j(5\cdot 2\pi)t}}{2}$$

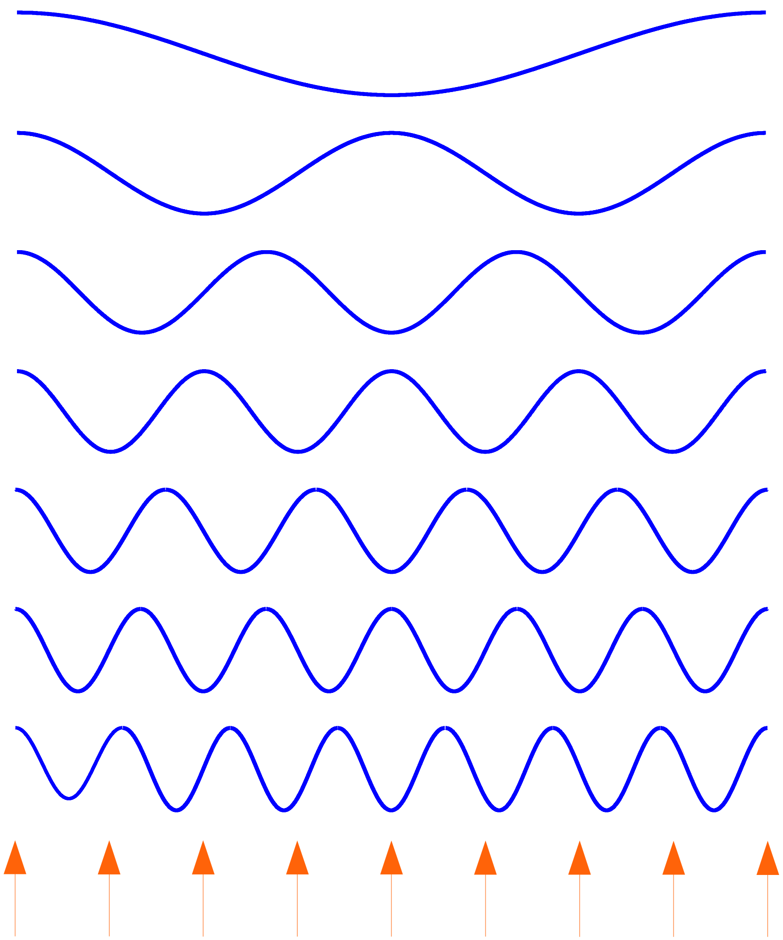
6 Hz
6 cycles / sec

$$\cos(6\cdot 2\pi t) = \frac{e^{+j(6\cdot 2\pi)t} + e^{-j(6\cdot 2\pi)t}}{2}$$

7 Hz
7 cycles / sec

$$\cos(7\cdot 2\pi t) = \frac{e^{+j(7\cdot 2\pi)t} + e^{-j(7\cdot 2\pi)t}}{2}$$

Signals with Harmonic Frequencies (2)



1 Hz
1 cycle / sec

2 Hz
2 cycles / sec

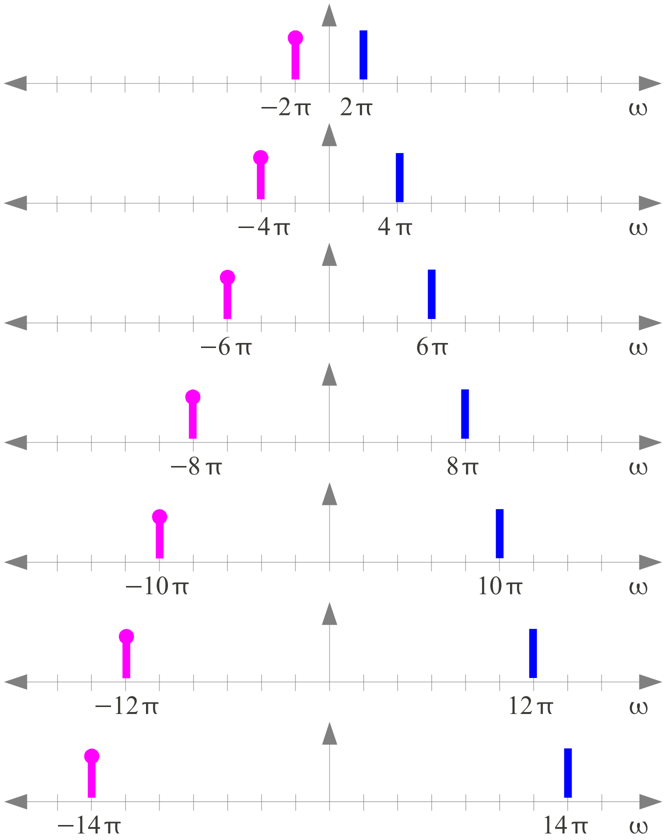
3 Hz
3 cycles / sec

4 Hz
4 cycles / sec

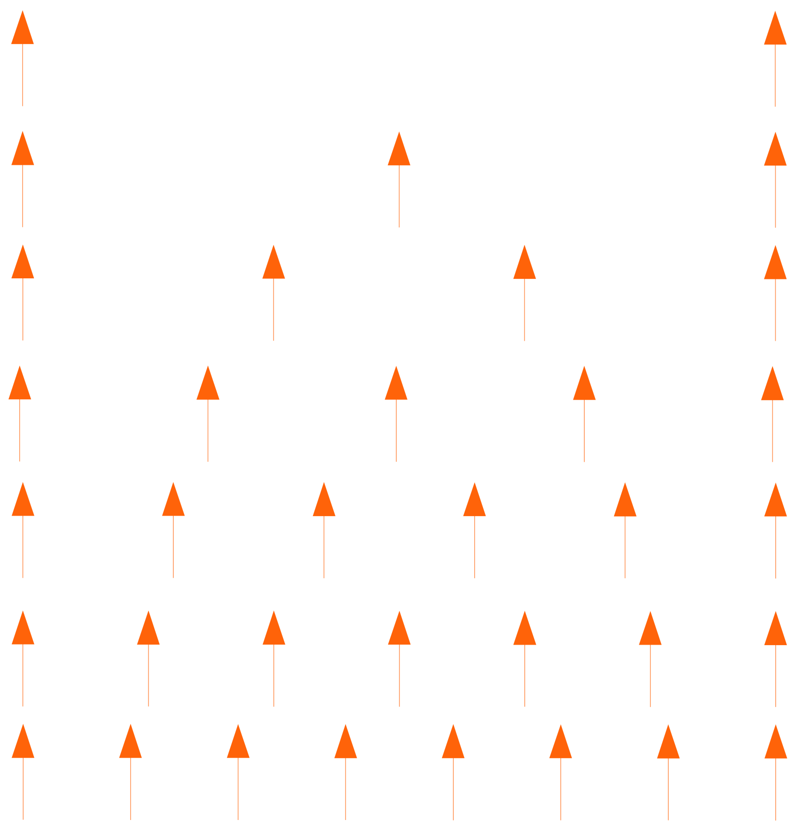
5 Hz
5 cycles / sec

6 Hz
6 cycles / sec

7 Hz
7 cycles / sec



Sampling Frequency



1 Hz
1 sample / sec

2 Hz
2 samples / sec

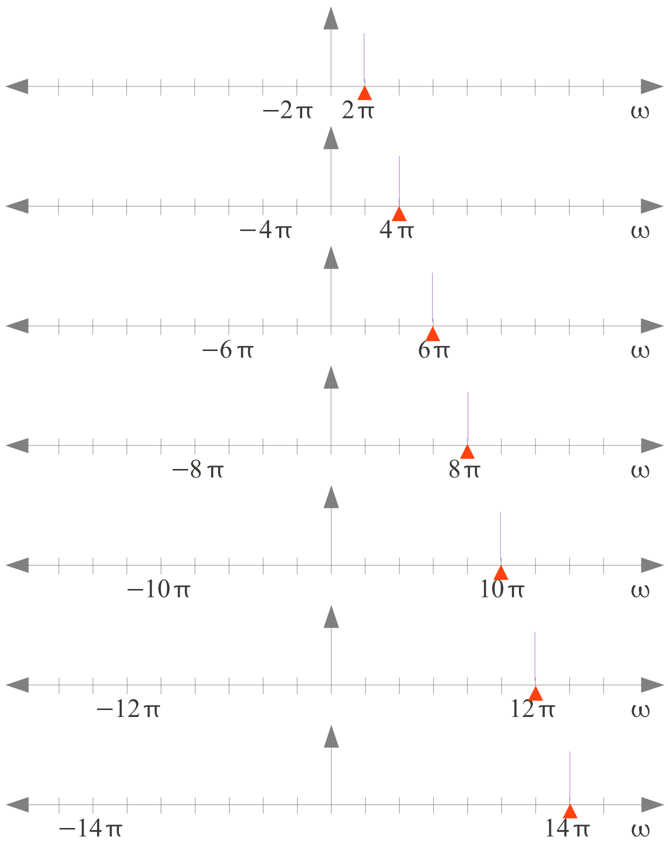
3 Hz
3 samples / sec

4 Hz
4 samples / sec

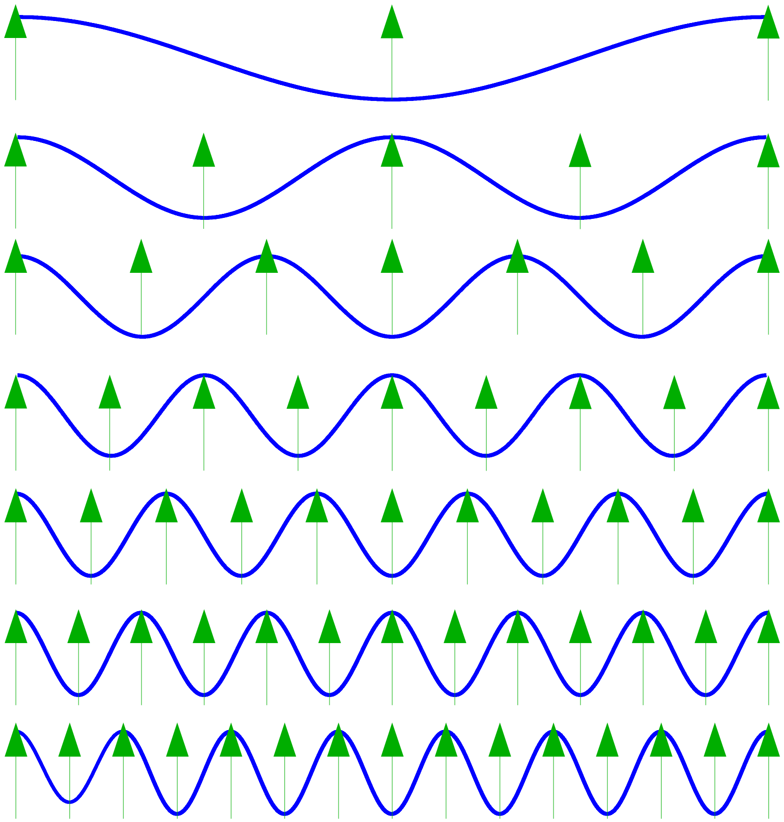
5 Hz
5 samples / sec

6 Hz
6 samples / sec

7 Hz
7 samples / sec

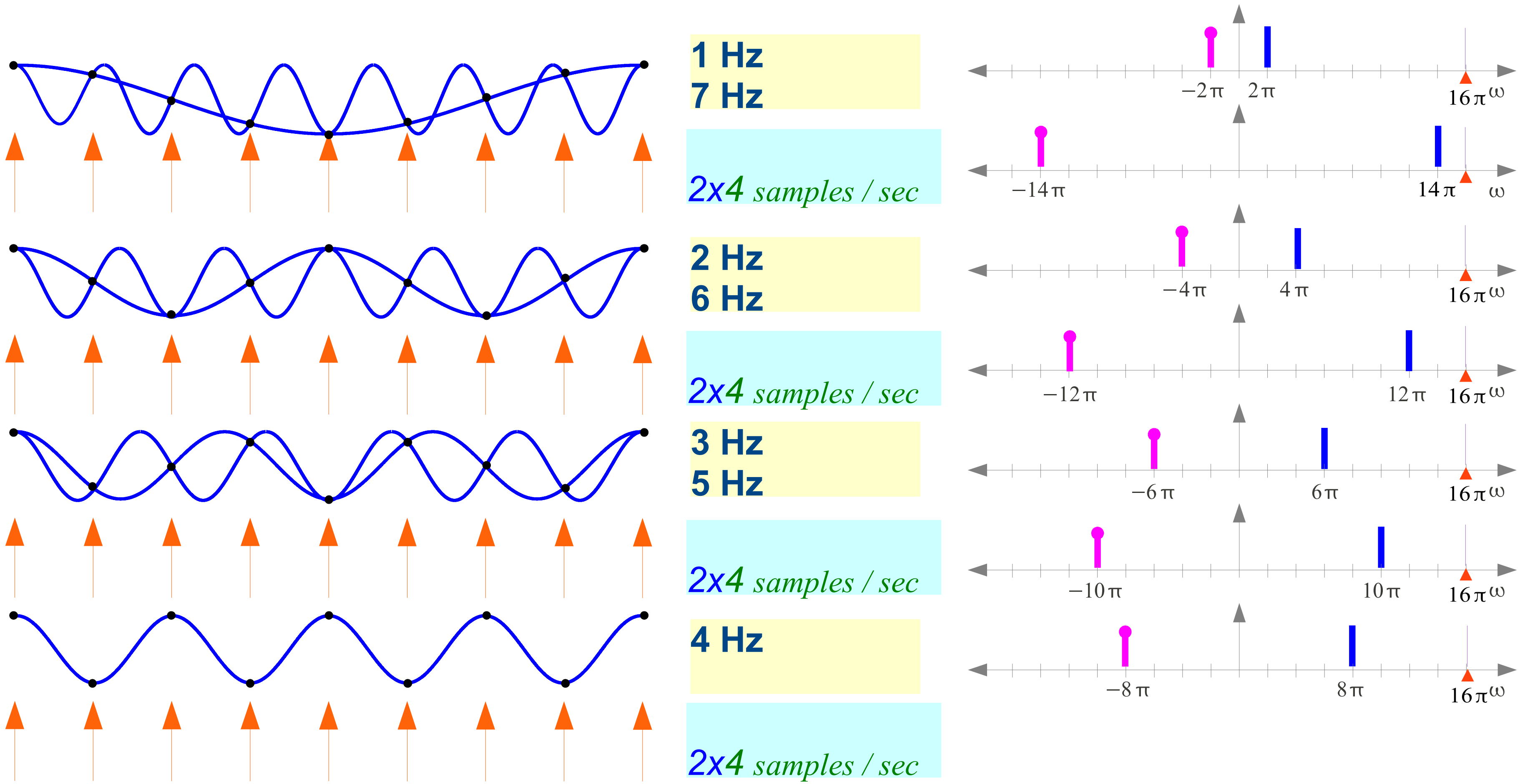


Nyquist Frequency



1 Hz *1 cycle / sec*	*2x1 sample / sec*
2 Hz *2 cycles / sec*	*2x2 samples / sec*
3 Hz *3 cycles / sec*	*2x3 samples / sec*
4 Hz *4 cycles / sec*	*2x4 samples / sec*
5 Hz *5 cycles / sec*	*2x5 samples / sec*
6 Hz *6 cycles / sec*	*2x6 samples / sec*
7 Hz *7 cycles / sec*	*2x7 samples / sec*

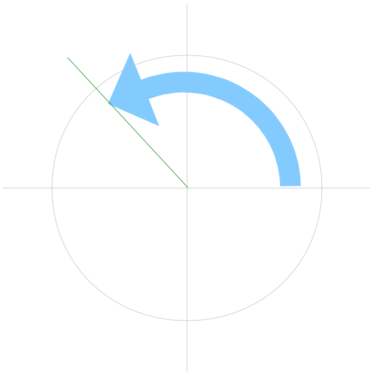
Aliasing



1 Hz
7 Hz
2x4 samples / sec
2 Hz
6 Hz
2x4 samples / sec
3 Hz
5 Hz
2x4 samples / sec
4 Hz
2x4 samples / sec
−2π
2π
16π
ω
−14π
14π
ω
−4π
4π
16π
ω
−12π
12π
16π
ω
−6π
6π
16π
ω
−10π
10π
16π
ω
−8π
8π
16π
ω

Sampling

$$\omega_s = 2\pi f_s \ (rad/sec)$$



$$\omega_1 = 2\pi f_1$$

$$\omega_1 = \frac{\omega_s}{2} \ (rad/sec)$$

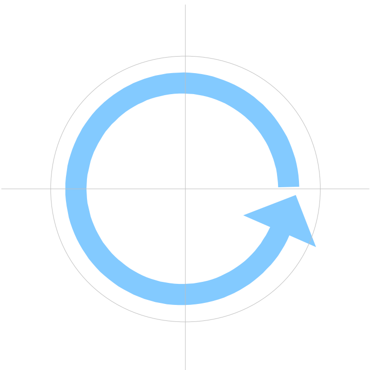
$$f_1 = \frac{f_s}{2} \ (rad/sec)$$

$$\omega_2 = 2\pi f_2$$

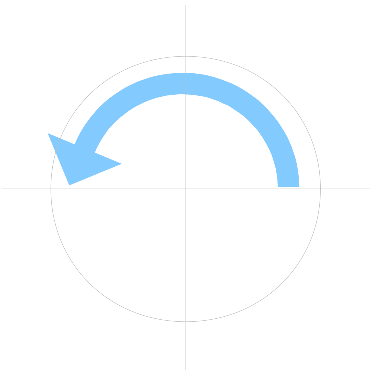
$$\omega_2 = -\frac{\omega_s}{2} \ (rad/sec)$$

$$f_2 = -\frac{f_s}{2} \ (rad/sec)$$

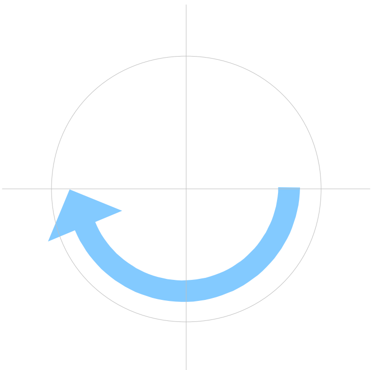
$$2\pi \ (rad) \ / \ T_s \ (sec)$$



$$\pi \ (rad) \ / \ T_s \ (sec)$$

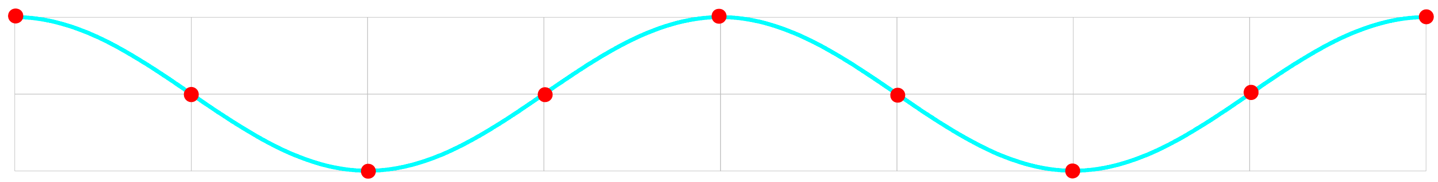


$$-\pi \ (rad) \ / \ T_s \ (sec)$$

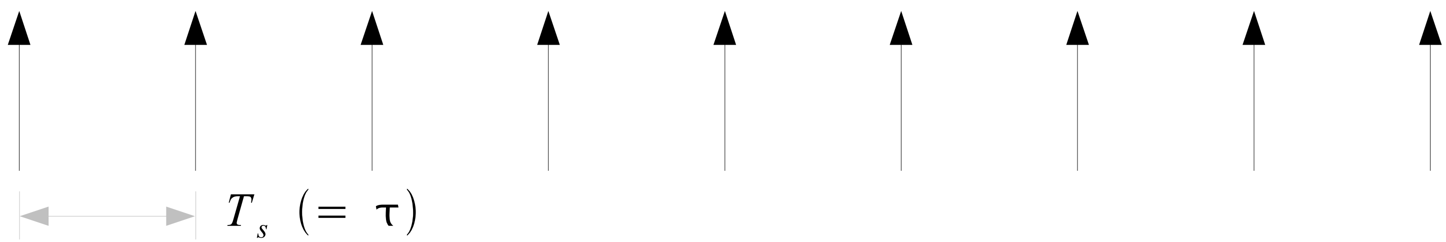


Sampling

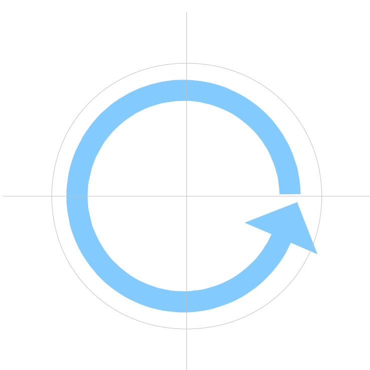
$\omega_1 = 2\pi f_1 \;(rad/sec)$



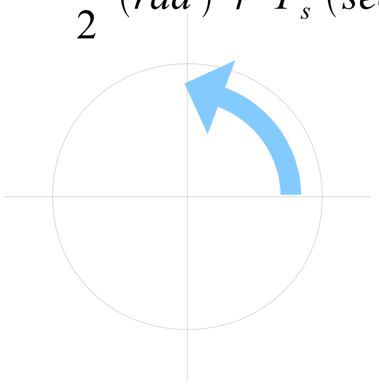
$\omega_s = 2\pi f_s \;(rad/sec)$



$2\pi \;(rad) \;/\; T_s \;(sec)$



$\frac{\pi}{2} \;(rad) \;/\; T_s \;(sec)$



For the period of T_s

Angular displacement $\frac{\pi}{2} \;(rad)$

$$\hat{\omega} = \omega \cdot T_s \;(rad)$$
$$= 2\pi f_1 \cdot T_s \;(rad)$$
$$= 2\pi \frac{f_s}{4} \cdot T_s \;(rad)$$
$$= \frac{\pi}{2} \;(rad)$$

Angular Frequencies in Sampling

continuous-time signals

Signal Frequency

$$f_0 = \frac{1}{T_0}$$

Signal Angular Frequency

$$\omega_0 = 2\pi f_0 \quad (rad/sec)$$

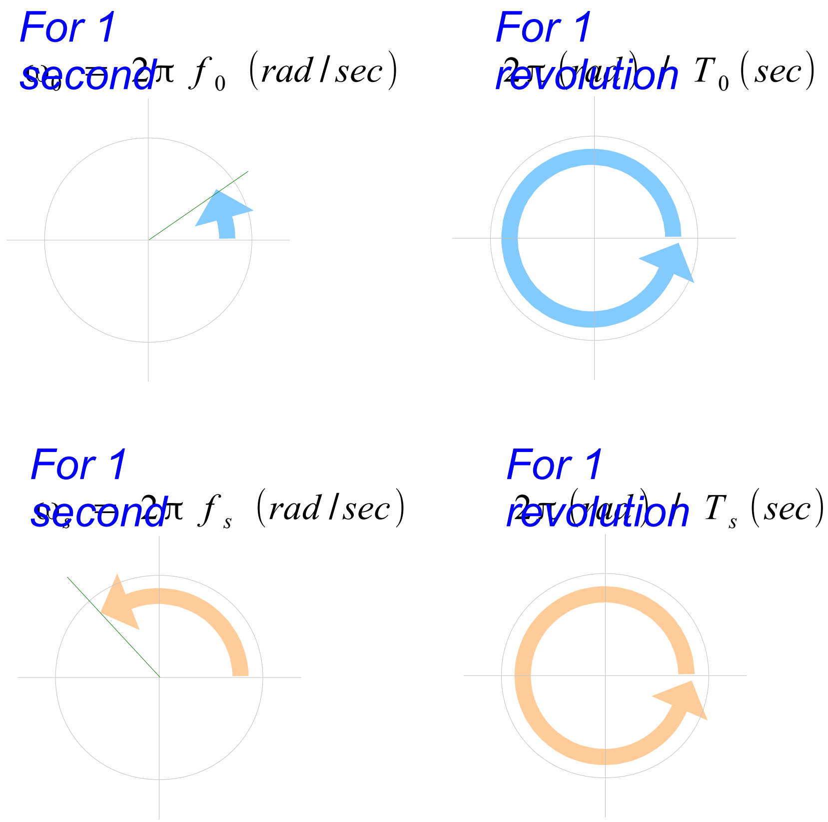
sampling sequence

Sampling Frequency

$$f_s = \frac{1}{T_s}$$

Sampling Angular Frequency

$$\omega_s = 2\pi f_s \quad (rad/sec)$$



References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A "graphical interpretation" of the DFT and FFT, by Steve Mann