CLTI Correlation (2A)

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Correlation

How signals move
relative to each other

Positively correlated	the same direction
	Average of product > product of averages
Negatively correlated	the opposite direction
	Average of product < product of averages
Uncorrelated	

Correlation Function for Energy Signals

$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t)\, y^*(t+\tau)\, dt = \int_{-\infty}^{+\infty} x(t-\tau)\, y^*(t)\, dt$$

Both real

$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t)\, y(t+\tau)\, dt = \int_{-\infty}^{+\infty} x(t-\tau)\, y(t)\, dt$$

Correlation and Convolution

Both real

$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t)\, y(t+\tau)\, dt = \int_{-\infty}^{+\infty} x(t-\tau)\, y(t)\, dt$$

Convolution

$$x(t) * y(t) = \int_{-\infty}^{+\infty} x(t-\tau)\, y(\tau)\, d\tau$$

$$R_{xy}(\tau) = x(-\tau) * y(\tau)$$

$$x(-t) \Leftrightarrow X^*(f)$$

$$R_{xy}(\tau) \Leftrightarrow X^*(f)\, Y(f)$$

Correlation for Power Signals

$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t)\,y^*(t+\tau)\,dt = \int_{-\infty}^{+\infty} x(t-\tau)\,y^*(t)\,dt$$

Energy Signal

$$R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_T x(t)\,y^*(t+\tau)\,dt = \lim_{T\to\infty} \frac{1}{T}\int_T x(t-\tau)\,y^*(t)\,dt$$

Power Signal

Both real

$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t)\,y(t+\tau)\,dt = \int_{-\infty}^{+\infty} x(t-\tau)\,y(t)\,dt$$

Energy Signal

$$R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_T x(t)\,y(t+\tau)\,dt = \lim_{T\to\infty} \frac{1}{T}\int_T x(t-\tau)\,y(t)\,dt$$

Power Signal

$$R_{xy}(\tau) = \frac{1}{T}\int_T x(t)\,y(t+\tau)\,dt$$

Periodic Power Signal

Correlation for Periodic Power Signals

$$R_{xy}(\tau) = \frac{1}{T}\int_T x(t)\, y(t+\tau)\, dt$$

Periodic Power Signal

$$R_{xy}(\tau) = \frac{1}{T}[x(-\tau) \circledast y(\tau)]$$

$$R_{xy}(\tau) \quad \overset{\text{CTFS}}{\Longleftrightarrow} \quad X^*[k]Y[k]$$

Circular Convolution

$$x(t) * y(t) \quad \overset{\text{CTFS}}{\Longleftrightarrow} \quad T\, X[k]Y[k]$$

$$x[n] * y[n] \quad \overset{\text{CTFS}}{\Longleftrightarrow} \quad N_0 Y[k]X[k]$$

$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t)\, y(t+\tau)\, dt$$

$$R_{xy}(\tau) = \frac{1}{T}\int_T x(t)\, y(t+\tau)\, dt$$

Correlation for Power & Energy Signals

One signal – a power signal
The other – an energy signal

Use the Energy Signal Version

$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t)\, y(t+\tau)\, dt$$

Autocorrelation

$$R_{xx}(\tau) = \int_{-\infty}^{+\infty} x(t)x(t+\tau)\,dt$$

Energy Signal

$$R_{xx}(0) = \int_{-\infty}^{+\infty} x^2(t)\,dt$$

total signal energy

$$R_{xx}(\tau) = \frac{1}{T}\int_T x(t)x(t+\tau)\,dt$$

Power Signal

$$R_{xx}(0) = \lim_{T\to\infty}\frac{1}{T}\int_{-\infty}^{+\infty} x^2(t)\,dt$$

average signal power

References

[1] http://en.wikipedia.org/
[2] M.J. Roberts, Signals and Systems,