DFT

- Discrete Fourier Transform

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CTFT and DFT

Continuous Time Fourier Transform

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \quad \Leftrightarrow \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

Discrete Fourier Transform

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn} \quad \Leftrightarrow \quad x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k]\, e^{+j(2\pi/N)kn}$$

From CTFT to DFT (1)

Continuous Time Fourier Transform

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \quad \Leftrightarrow \quad x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

$$t \rightarrow nT_s \qquad dt \rightarrow T_s \qquad \int \rightarrow \sum \qquad T_s \rightarrow 0$$

$$\hat{X}(j\omega) = \sum_{n=-\infty}^{+\infty} x(nT_s)\, e^{-j\omega nT_s} \cdot T_s \quad \Leftrightarrow \quad x(nT_s) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{X}(j\omega)\, e^{+j\omega nT_s}\, d\omega$$

$$\omega \rightarrow \omega_k \qquad 0 \le \omega_k < \frac{2\pi}{T_s} \qquad 0 \le k < N \qquad 0 \le n < L$$

$$\omega_k = \frac{2\pi}{T_s}\frac{k}{N}$$

$$\hat{X}(j\omega_k) = T_s \sum_{n=0}^{L-1} x[n]\, e^{-j\omega_k nT_s} \quad \Leftrightarrow \quad x[n] = \frac{1}{2\pi}\sum_{k=0}^{N-1} \hat{X}(j\omega_k)\, e^{+j\omega_k nT_s}\, \frac{2\pi}{T_s}\frac{1}{N}$$

From CTFT to DFT (2)

Continuous Time Fourier Transform

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \quad \Leftrightarrow \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

$$t \to n T_s \qquad dt \to T_s \qquad \int \to \sum \qquad T_s \to 0$$

$$\omega \to \omega_k \qquad 0 \le \omega_k < \frac{2\pi}{T_s} \qquad 0 \le k < N \qquad 0 \le n < L$$

$$\omega_k = \frac{2\pi}{T_s} \frac{k}{N}$$

$$\hat{X}(j\omega_k) = T_s \sum_{n=0}^{L-1} x[n]\, e^{-j\omega_k n T_s} \quad \Leftrightarrow \quad x[n] = \frac{1}{2\pi} \sum_{k=0}^{N-1} \hat{X}(j\omega_k)\, e^{+j\omega_k n T_s} \frac{2\pi}{T_s} \frac{1}{N}$$

$$\omega_k T_s \to \frac{2\pi}{N} k$$

$$\omega_k = \frac{2\pi}{T_s} \frac{k}{N}$$

$$\frac{1}{T_s} \hat{X}(j\omega_k) = \sum_{n=0}^{L-1} x[n]\, e^{-j\left(\frac{2\pi}{N}\right) k n} \quad \Leftrightarrow \quad x[n] = \sum_{k=0}^{N-1} \frac{1}{T_s} \hat{X}(j\omega_k)\, e^{+j\left(\frac{2\pi}{N}\right) k n}$$

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From CTFT to DFT (3)

Continuous Time Fourier Transform

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \quad \Leftrightarrow \quad x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

$$t \rightarrow nT_s \qquad dt \rightarrow T_s \qquad \int \rightarrow \sum \qquad T_s \rightarrow 0$$

$$\omega \rightarrow \omega_k \qquad 0 \le \omega_k < \frac{2\pi}{T_s} \qquad 0 \le k < N \qquad 0 \le n < L$$

$$\omega_k T_s \rightarrow \frac{2\pi}{N} k$$

$$\omega_k = \frac{2\pi}{T_s}\frac{k}{N}$$

$$\omega_k = \frac{2\pi}{T_s}\frac{k}{N}$$

$$\frac{1}{T_s}\hat{X}(j\omega_k) = \sum_{n=0}^{L-1} x[n]\, e^{-j(\frac{2\pi}{N})kn} \quad \Leftrightarrow \quad x[n] = \sum_{k=0}^{N-1} \frac{1}{T_s}\hat{X}(j\omega_k)\, e^{+j(\frac{2\pi}{N})kn}$$

From CTFT to DFT (4)

Continuous Time Fourier Transform

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \quad \Leftrightarrow \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

$$\omega_k T_s \rightarrow \frac{2\pi}{N} k$$

$$\omega_k = \frac{2\pi}{T_s} \frac{k}{N}$$

$$\frac{1}{T_s} \hat{X}(j\omega_k) = \sum_{n=0}^{L-1} x[n]\, e^{-j(\frac{2\pi}{N})kn} \quad \Leftrightarrow \quad x[n] = \sum_{k=0}^{N-1} \frac{1}{T_s} \hat{X}(j\omega_k)\, e^{+j(\frac{2\pi}{N})kn}$$

Discrete Fourier Transform

$$L = N$$

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn} \quad \Leftrightarrow \quad x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k]\, e^{+j(2\pi/N)kn}$$

DTFT and CTFT

$$X_s(j\omega) = \sum_{n=-\infty}^{+\infty} X_c(nT_s)\, e^{-j\omega n T_s}$$

$$= \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j\omega n T_s}$$

$$\Longleftrightarrow$$

$$X_s(j\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{+\infty} X_c(j(\omega - k\omega_s))$$

$$\omega_s = \frac{2\pi}{T_s}$$

$$X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j\hat{\omega} n}$$

DTFT of a sampled signal

$$X(e^{j\hat{\omega}}) \Big|_{\hat{\omega} = \omega T_s} = X(e^{j\omega T_s}) = \frac{1}{T_s} \sum_{k=-\infty}^{+\infty} X_c(j(\omega - k\omega_s))$$

$$= \frac{1}{T_s} \sum_{k=-\infty}^{+\infty} X_c\left(j\left(\omega - \frac{2\pi k}{T_s}\right)\right)$$

CTFT of a sampled signal

DFT and DTFT

DTFT of a sampled signal

$$X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j\hat{\omega} n}$$

$$\hat{\omega} = \omega T_s$$

$$\hat{\omega} \rightarrow \hat{\omega}_k \qquad 0 \le \hat{\omega}_k < 2\pi \qquad 0 \le k < N \qquad 0 \le n < L$$

$$X(e^{j\hat{\omega}_k}) = \sum_{n=0}^{L-1} x[n]\, e^{-j\hat{\omega}_k n}$$

$$\hat{\omega}_k = \left(\frac{2\pi}{N}\right) k$$

DFT of a sampled signal

$$X[k] = \qquad X(e^{j(2\pi/N)k}) = \sum_{n=0}^{L-1} x[n]\, e^{-j(2\pi/N)kn}$$

DTFT *sampled in frequency*

DFT and CTFT

DFT of a sampled signal

$$X[k] = X(e^{j(2\pi/N)k}) = \sum_{n=0}^{L-1} x[n]\, e^{-j(2\pi/N)kn}$$

DTFT *sampled in frequency*

$$X(e^{j\omega T_s})\Big|_{\omega = \frac{2\pi k}{N T_s}}$$

CTFT evaluated at $\omega = \frac{2\pi k}{N T_s}$

$$= \frac{1}{T_s} \sum_{l=-\infty}^{+\infty} X_c(j(\omega - l\omega_s))\Big|_{\omega = \frac{2\pi k}{N T_s}}$$

$$= \frac{1}{T_s} \sum_{l=-\infty}^{+\infty} X_c(j(\omega - l\frac{2\pi}{T_s}))\Big|_{\omega = \frac{2\pi k}{N T_s}}$$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003