

```

::::::::::::
Angles.hpp
::::::::::::
# include <iostream>
# include <iomanip>
# include <fstream>
// # include <cstdlib>
// # include <cmath>
// # include <vector>
// # include <algorithm>

using namespace std;

//-----
// Purpose:
//
// Class Angles Interface Files
//
// Discussion:
//
// Licensing:
//
// This code is distributed under the GNU LGPL license.
//
// Modified:
//
// 2012.05.02
//
// Author:
//
// Young Won Lim
//
// Parameters:
//
//-----

double compute_angle ( int idx, int nIter );
void draw_angle_tree (int nIter, int nAngle);

class Angles
{
public:
    Angles();
    Angles(double *A, int nIter, int nAngle);

    void setA(double *A);
    void setNIter(int nIter);
    void setNAngle(int nAngle);
    int getNIter();
    int getNAngle();

    void plot_unit_circle_angle ();
    void plot_line_angle ();
    void plot_residual_errors ();
    void calc_statistics ();
    void calc_uscale_statistics (double, double);
    void plot_uscale_statistics ();
    void plot_uscale_residual_errors ();
    void plot_uscale_residual_errors (double, double);

private:
    double *A;
    int nIter;
    int nAngle;

```

```

int    Leaf;

double delta_avg;
double delta_std;
double min_angle;
double max_angle;

double ssr;      // sum of the squares of the residuals
double mse;     // mean squared error
double rms;     // root mean square error
double max_err; // maximum of squared errors

};

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Angles.cpp

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:::::::::::::

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```

#include <iostream>
#include <iomanip>
#include <cstdlib>
#include <cmath>
#include <fstream>
#include <vector>
#include <algorithm>

```

```

using namespace std;

```

```

#include "Angles.hpp"
#include "cordic.hpp"

```

```

double pi = 3.141592653589793;
double K = 1.646760258121;

```

```

double threshold =0.001;

```

```

//-----
// Purpose:
//
// Class Angles Implementation Files
//
// Discussion:
//
// Licensing:
//
// This code is distributed under the GNU LGPL license.
//
// Modified:
//
// 2012.05.08
//
// Author:
//
// Young Won Lim
//
// Parameters:
//
//-----

```

```

//-----
// Compute Angles based on the binary tree
// idx - index for leaf nodes of the binary tree
// nIter - no of iteration (corresponds to the level of the tree)
//-----
double compute_angle ( int idx, int nIter )

```

```

{
    double angle = 0.0;
    char    s[32];
    int     i, j;

    // i - bit position starting from lsb
    // j = 2^i
    // (idx & (1 << i)) - i-th bit of idx
    // if each bit is '1', add atan(1/2^i)
    // if each bit is '0', sub atan(1/2^i)
    // s[32] contains the binary representation of idx

    for (i=0; i<nIter; i++) {

        j = 1 << i;
        if (idx & (1 << i)) {
            angle += atan( 1. / j );
            s[nIter-i-1] = '1';
        } else {
            angle -= atan( 1. / j );
            s[nIter-i-1] = '0';
        }

        // cout << "i=" << i << " j=" << j << " 1/j=" << 1./j
        //          << " atan(1/j)=" << atan(1./j)*180/3.1416 << endl;

    }
    s[nIter] = '\0';

    // cout << nIter << " " << idx << " " << s
    //          << " ---> " << angle*180/3.1416 << endl;

    return angle;
}

//-----
// Draw Angle Tree
//-----
void draw_angle_tree (int nIter, int nAngle)
{
    int level = nIter;
    int i, j, k;
    ofstream myout;
    double *A;

    if (nIter > 10) {
        cout << "nIter = " << nIter << " is too large to plot! " << endl;
        return;
    }

    // cout << "nIter = " << nIter << endl;
    // cout << "nAngle = " << nAngle << endl;

    A = (double *) malloc(nAngle * sizeof (double));

    myout.open("angle.dat");

    for (i=0; i<level; ++i) {
        nIter = i;
        nAngle = 1 << nIter;

        for (j=0; j<nAngle; ++j) {
            A[j] = compute_angle(j, nIter);
        }
    }
}

```

```

        // cout << "A[" << j << "]" = " << A[j] << endl;
        myout << A[j]*180/pi << " " << 0.5*i << " 0.0 0.5" << endl;
    }
}

myout.close();

// writing gnuplot commands
myout.open("command.gp");
myout << "plot 'angle.dat' using 1:2:3:4 ";
myout << "with vectors head filled lt 2" << endl;
myout << "pause mouse keypress" << endl;
myout.close();

system("gnuplot command.gp");

//-----
// Accumulated Angle Tree
//-----

myout.open("angle.dat");

for (i=0; i<level; ++i) {
    for (k=0; k<=i; k++) {
        nIter = k;
        nAngle = 1 << nIter;

        for (j=0; j<nAngle; ++j) {
            A[j] = compute_angle(j, nIter);

            //cout << "A[" << j << "]" = " << A[j] << endl;
            myout << A[j]*180/pi << " " << 0.5*i << " 0.0 0.5" << endl;
        }
    }
}

myout.close();

// writing gnuplot commands
myout.open("command.gp");
myout << "plot 'angle.dat' using 1:2:3:4 ";
myout << "with vectors head filled lt 2" << endl;
myout << "pause mouse keypress" << endl;
myout.close();

system("gnuplot command.gp");

free (A);
return;
}

//-----
// Class Angles' Member Functions
//-----
Angles::Angles() : A(NULL), nIter(3), nAngle(8)
{
    Leaf = 1;
}

```

```

cout << "A is not initialized " << endl;
cout << "nIter = " << nIter << endl;
cout << "nAngle = " << nAngle << endl;
}

```

```

Angles::Angles(double *A, int nIter, int nAngle) :
A(A), nIter(nIter), nAngle(nAngle)
{
    if (nAngle == (1 << nIter)) {
        Leaf = 1;
        cout << "A LeafAngles Object is created" << endl;
    } else {
        Leaf = 0;
        cout << "An AllAngles Object is created" << endl;
    }

    cout << "nIter = " << nIter << endl;
    cout << "nAngle = " << nAngle << endl;
}

```

```

void Angles::setNIter(int nIter)
{
    nIter = nIter;
}

```

```

void Angles::setNAngle(int nAngle)
{
    nAngle = nAngle;
}

```

```

int Angles::getNIter()
{
    return nIter;
}

```

```

int Angles::getNAngle()
{
    return nAngle;
}

```

```

//-----
//      Plot angle vectors on the unit circle
//-----

```

```

void Angles::plot_unit_circle_angle ()
{
    int i;
    ofstream myout;

    cout << "* plot_unit_circle_angle ... " ;
    if (Leaf) cout << "(LeafAngles)" << endl;
    else cout << "(AllAngles)" << endl;

    if (nIter > 10) {
        cout << "nIter = " << nIter << " is too large to plot! " << endl;
        return;
    }

    // writing angle data on a unit circle
    myout.open("angle.dat");
    for (i=0; i<nAngle; i++) {
        myout << "0.0 0.0 " << cos(A[i]) << " " << sin(A[i]) << " " << endl;
    }
    myout.close();
}

```

```

// writing gnuplot commands
myout.open("command.gp");
myout << "set size square" << endl;
myout << "set xrange [-1:+1]" << endl;
myout << "set yrange [-1:+1]" << endl;
myout << "set object 1 circle at 0, 0 radius 1" << endl;
myout << "plot 'angle.dat' using 1:2:3:4 ";
myout << "with vectors head filled lt 2" << endl;
myout << "pause mouse keypress" << endl;
myout.close();

system("gnuplot command.gp");

return;
}

//-----
//      Plot angle vectors on the line
//-----
void Angles::plot_line_angle ()
{
    ofstream myout;

    cout << "* plot_line_angle ... ";
    if (Leaf) cout << "(LeafAngles)" << endl;
    else cout << "(AllAngles)" << endl;

    if (nIter > 10) {
        cout << "nIter = " << nIter << " is too large to plot! " << endl;
        return;
    }

    // cout << "nIter = " << nIter << endl;
    // cout << "nAngle = " << nAngle << endl;

    myout.open("angle.dat");

    for (int i=0; i<nAngle; ++i) {
        // cout << "A[" << i << "] = " << A[i] << endl;
        myout << A[i] << " 0.0 0.0 0.5" << endl;
    }

    myout.close();

    // writing gnuplot commands
    myout.open("command.gp");
    myout << "set yrange [0:+2]" << endl;
    myout << "plot 'angle.dat' using 1:2:3:4 ";
    myout << "with vectors head filled lt 2" << endl;
    myout << "pause mouse keypress" << endl;
    myout.close();

    system("gnuplot command.gp");

    return;
}

//-----
//      plot residual errors
//-----

```

```

void Angles::plot_residual_errors ()
{
    int i;
    double x, y, z;
    ofstream myout;

    cout << "* plot_residual_errors ... ";
    if (Leaf) cout << "(LeafAngles)" << endl;
    else cout << "(AllAngles)" << endl;

    if (nIter > 10) {
        cout << "nIter = " << nIter << " is too large to plot! " << endl;
        return;
    }

    // writing residue errors
    myout.open("angle.dat");

    int nBreak =0;
    double mse = 0.0, max_err = 0.0, se = 0.0;
    for (i=0; i<nAngle; i++) {
        x = 1 / K;
        y = 0.0;
        z = A[i];

        cordic(&x, &y, &z, nIter, &nBreak, i, threshold);

        se = z * z;
        mse += se;
        if (se > max_err) max_err = se;

        // cout << "A[" << i << "]=" ;
        // cout << fixed << right << setw(10) << setprecision(7) << A[i];
        // cout << " z=" ;
        // cout << fixed << right << setw(10) << setprecision(7) << z << endl;

        myout << fixed << right << setw(10) << i;
        myout << fixed << right << setw(12) << setprecision(7) << A[i];
        myout << fixed << right << setw(12) << setprecision(7) << z << endl;
    }

    // mse /= nAngle;
    // mse = sqrt(mse);

    cout << "* No of points = " ;
    cout << fixed << right << setw(10) << nAngle ;
    cout << " (nBreak = " << nBreak << " : " ;
    cout << 100. * nBreak / nAngle << " % )" << endl;
    cout << "* Mean Squared Residual Errors = " ;
    cout << fixed << right << setw(12) << setprecision(7) << mse << endl;
    cout << "* Max Squared Residual Error (Uniform Scale) = " ;
    cout << fixed << right << setw(12) << setprecision(7) << max_err << endl;

    myout.close();

    // writing gnuplot commands
    myout.open("command.gp");
    myout << "set autoscale y" << endl;
    myout << "plot 'angle.dat' using 1:3 with linespoints " << endl;
    myout << "pause mouse keypress" << endl;
    myout.close();

    system("gnuplot command.gp");

    return;
}

```

```

}

//-----
// Calculate angle statistics
//-----
void Angles::calc_statistics ()
{
    vector <double> B, D;
    vector <double> ::iterator first, last;
    double mean, std;
    ofstream myout;

    cout << " * calc_statistics... ";
    if (Leaf) cout << "(LeafAngles)" << endl;
    else cout << "(AllAngles)" << endl;

    cout << "nAngle = " << nAngle << endl;

    for (int i=0; i < nAngle; ++i) {
        // cout << "A[" << i << "]= " << setw(12) << setprecision(8) << A[i] << endl;
        // cout << "B[" << i << "]= " << setw(12) << setprecision(8) << B[i] << endl;
    }

    // B : sorted angles array
    for (int i=0; i < nAngle; ++i)
        B.push_back(A[i]);

    sort(B.begin(), B.end());

    // D : difference angle array
    for (int i=0; i < nAngle-1; ++i)
        D.push_back(B[i+1]- B[i]);

    sort(D.begin(), D.end());

    mean = 0.0;
    for (int i=0; i < D.size(); ++i)
        mean += D[i];
    mean /= D.size();

    std = 0.0;
    for (int i=0; i < D.size(); ++i)
        std += ((D[i]-mean) * (D[i]-mean));
    std /= D.size();
    std = sqrt(std);

    min_angle = B[0];
    max_angle = B[B.size()-1];
    delta_avg = mean;
    delta_std = std;

    cout << "max angle      = " << B[0] << endl;
    cout << "min angle      = " << B[B.size()-1] << endl;
    cout << "delta computed = " << (B[B.size()-1] - B[0]) / nAngle ;
    cout << " = (max-min) / nAngle " << endl;
    cout << "delta mean = " << mean << endl;
    cout << "delta std = " << std << endl;

    // write histogram data from delta array
    myout.open("angle.dat");
    double pb ;
    for (int i=0, j, k; i<nAngle-2; i++) {

```



```

    j = i; k = 1;
    while ((D[j+1] - D[j])/D[j] < 0.01) {
        k++;
        j++;
    }
    pb = (double) k / D.size();
    myout << fixed << right << setw(12) << setprecision(7) << D[i] ;
    myout << " " << pb << endl;
    i = j;
}
myout.close();

// writing gnuplot commands
myout.open("command.gp");
myout << "set boxwidth 0.9 relative" << endl;
myout << "set style fill solid 1.0 border lt -1" << endl;
myout << "plot 'angle.dat' with linespoints" << endl;
myout << "pause mouse keypress" << endl;
myout.close();

system("gnuplot command.gp");

// write angle-delta data
myout.open("angle.dat");
for (int i=0; i<B.size()-1; i++) {
    myout << B[i] << " " << B[i+1] - B[i] << endl;
}
myout.close();

// writing gnuplot commands
myout.open("command.gp");
myout << "set boxwidth 0.9 relative" << endl;
myout << "set style fill solid 1.0 border lt -1" << endl;
myout << "plot 'angle.dat' with linespoints" << endl;
myout << "pause mouse keypress" << endl;
myout.close();

system("gnuplot command.gp");

return;
}

//-----
// Calculate statistics on the uniform scale
//-----
// ssr      : sum of the squares of the residuals
// mse      : mean squared error
// rms      : root mean square error
// max_err  : maximum of squared errors
//-----
// ssr      : sum of the squares of the residuals
// ang = min_angle + delta_avg * offFactor ;
// ang += (delta_avg / resFactor);
//-----
void Angles::calc_uscale_statistics (double resFactor, double offFactor)
{
    int n;
    double x, y, z;
    ofstream myout;

    cout << "* calc_uscale_statistics ... ";

```

```

if (Leaf) cout << "(LeafAngles Resolution)" << endl;
else cout << "(AllAngles Resolution)" << endl;

// sr      : square error of a data point

double ang = min_angle + delta_avg * offFactor ;
double se = 0.0 ;
int nBreak =0;
n = 0;

ssr = mse = rms = max_err = 0.0;

while (ang < max_angle) {
    x = 1 / K;
    y = 0.0;
    z = ang;

    cordic(&x, &y, &z, nIter, &nBreak, n, threshold);

    se = (z * z);
    ssr += se;
    if (se > max_err) max_err = se;

    // cout << fixed << right << setw(10) << setprecision(7) << A[i];
    // cout << fixed << right << setw(10) << setprecision(7) << z << endl;

    ang += (delta_avg / resFactor);
    n++;
}

mse = ssr / n;
rms = sqrt(mse);

max_err = sqrt(max_err);

cout << "  Angles = (" ;
cout << min_angle << " : " << delta_avg << " : " << max_angle << ")" ;
cout << endl << "  --> total " << n << " points" ;
cout << " (nBreak = " << nBreak << " : " ;
cout << 100. * nBreak / n << " % )" << endl;

cout << "  SSR: Sum of Squared Residuals      = " ;
cout << fixed << right << setw(12) << setprecision(7) << ssr << endl;
cout << "  MSR: Mean Squared Residuals          = " ;
cout << fixed << right << setw(12) << setprecision(7) << mse << endl;
cout << "  RMS: Root Mean Squared Residuals      = " ;
cout << fixed << right << setw(12) << setprecision(7) << rms << endl;
cout << "  Max Residual Error                    = " ;
cout << fixed << right << setw(12) << setprecision(7) << max_err << endl;

return;
}

//-----
//  Plot uniform scale statistics
//-----
//  ang = min_angle + delta_avg * offFactor ;
//  ang += (delta_avg / resFactor);
//-----
void Angles::plot_uscale_statistics ( )
{

```

```

int    i, j;
double resFactor, offFactor;
ofstream myout;

cout << "* plot_uscale_statistics ... ";
if (Leaf) cout << "(LeafAngles Resolution)" << endl;
else cout << "(AllAngles Resolution)" << endl;

if (nIter > 10) {
    cout << "nIter = " << nIter << " is too large to plot! " << endl;
    return;
}

// ssr : sum of the squares of the residuals
// mse : mean squared error
// rms : root mean square error
// sr  : square error of a data point
// max_err : maximum of squared errors

// writing residue errors
myout.open("angle.dat");

for (i=0; i<4; i++) {
    resFactor = i + 1.0;

    myout << fixed << right << setw(10) << resFactor;

    for (j=0; j<4; j++) {
        offFactor = (j+1) / 4.;

        cout << " ===== " << i << " === " << j ;
        cout << " ===== " << endl;

        calc_uscale_statistics(resFactor, offFactor);

        myout << fixed << right << setw(22) << setprecision(7) << ssr;
    }

    myout << endl;
}

myout.close();

// writing gnuplot commands
myout.open("command.gp");
myout << "set autoscale y" << endl;
myout << "plot 'angle.dat' using 1:2 with linespoints, " ;
myout << " 'angle.dat' using 1:3 with linespoints, " ;
myout << " 'angle.dat' using 1:4 with linespoints, " ;
myout << " 'angle.dat' using 1:5 with linespoints " << endl;
myout << "pause mouse keypress" << endl;
myout.close();

system("gnuplot command.gp");

return;
}

//-----
// Plot residual errors on the uniform scale
//-----
// ang = min_angle + delta_avg * offFactor ;

```

```

//  ang += (delta_avg / resFactor);
//-----
void Angles::plot_uscale_residual_errors
(double resFactor, double offFactor )
{
    int n;
    double x, y, z;
    ofstream myout;

    cout << " * plot_uscale_residual_errors ... ";
    if (Leaf) cout << "(LeafAngles Resolution)" << endl;
    else cout << "(AllAngles Resolution)" << endl;

    if (nIter > 10) {
        cout << "nIter = " << nIter << " is too large to plot! " << endl;
        return;
    }

    double ang = min_angle + delta_avg * offFactor ;
    double se = 0.0 ;
    int nBreak =0;
    n = 0;

    // writing residue errors
    myout.open("angle.dat");

    while (ang < max_angle) {
        x = 1 / K;
        y = 0.0;
        z = ang;

        cordic(&x, &y, &z, nIter, &nBreak, n, threshold);

        se = (z * z);

        // cout << fixed << right << setw(10) << setprecision(7) << A[i];
        // cout << fixed << right << setw(10) << setprecision(7) << z << endl;

        myout << fixed << right << setw(10) << n;
        myout << fixed << right << setw(22) << setprecision(7) << ang;
        myout << fixed << right << setw(22) << setprecision(7) << z;
        myout << fixed << right << setw(22) << setprecision(7) << se << endl;

        ang += (delta_avg / resFactor);
        n++;
    }

    cout << " Angles = (" ;
    cout << min_angle << " : " << delta_avg << " : " << max_angle << ")" ;
    cout << endl << " --> total " << n << " points" ;
    cout << " (nBreak = " << nBreak << " : " ;
    cout << 100. * nBreak / n << " % )" << endl;

    myout.close();

    // writing gnuplot commands
    myout.open("command.gp");
    myout << "set autoscale y" << endl;
    myout << "plot 'angle.dat' using 1:3 with linespoints " << endl;
    myout << "pause mouse keypress" << endl;
    myout.close();

    system("gnuplot command.gp");
}

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    return;
}

void Angles::plot_uscale_residual_errors ()
{
    plot_uscale_residual_errors (1.0, 1.0);
}

/*****

for (i=0; i<20; i+=4) {
    for (j=0; j<4; ++j) {
        r = atan( 1. / (1 << (i+j)) ) / atan( 1. / (1 << i) ) * 100;
        cout << "index = " << i+j << " --> r = " << r << endl;
    }
}

return 0;
}
*****/

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*****/

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Angles_tb.cpp

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```

```

#include <cstdlib>
#include <cmath>
#include <iostream>
#include <iomanip>
#include <fstream>

```

```

using namespace std;

```

```

#include "cordic.hpp"
#include "Angles.hpp"

```

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//-----

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// Purpose:

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// Explore Angles Space using Class Angles

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// Discussion:

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// Licensing:

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// This code is distributed under the GNU LGPL license.

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// Modified:

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// 2012.05.08

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// Author:

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// Young Won Lim

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// Parameters:

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//
//-----

int main (int argc, char * argv[])
{
    int    nIter = 3;
    int    nAngle = 1 << nIter;
    int    i, j, k;
    int    level, leaves;
    double *A, *All;

    if (argc > 1 ) {
        nIter = atoi(argv[1]);
        nAngle = 1 << nIter;
    }

    // cout << "nIter = " << nIter << endl;
    // cout << "nAngle = " << nAngle << endl;

    A = (double *) malloc ((1<<nIter) * sizeof (double));
    All = (double *) malloc (2* (1<<nIter) * sizeof (double));

    for (j=0; j<nAngle; ++j) {
        A[j] = compute_angle(j, nIter);
        // cout << "A[" << j << "]=" << setw(12) << setprecision(8) << A[j] << endl;
    }

    for (i=0, k=0; i<=nIter; ++i) {
        level = i;
        leaves = 1 << level;

        // cout << "level = " << level << "leaves = " << leaves << endl;

        for (j=0; j<leaves; ++j) {
            All[j+k] = compute_angle(j, level);
            // cout << "All[" << j+k << "] = " << All[j+k] << endl;
        }

        k += leaves;
    }

    Angles LeafAngles(A, nIter, nAngle);
    Angles AllAngles(All, nIter, 2*nAngle-1);

    // -----
    //   Plot angle vectors on the unit circle
    // -----
    // LeafAngles.plot_unit_circle_angle();
    // AllAngles.plot_unit_circle_angle();

    // -----
    //   Plot angle on the line axis
    // -----
    // LeafAngles.plot_line_angle();
    // AllAngles.plot_line_angle();

    // -----
    //   Plot Angle Tree
    // -----

```

```

// draw_angle_tree (nIter, nAngle);

// -----
// Find Angles Statistics --> member data
// -----
LeafAngles.calc_statistics();
AllAngles.calc_statistics();

// -----
// Plot residue errors at the leaf node angles
// -----
// LeafAngles.plot_residual_errors();
// AllAngles.plot_residual_errors();

// -----
// Calculate Uniform Scale Statistics --> member data
// -----
LeafAngles.calc_uscale_statistics(1.0, 1.0);
AllAngles.calc_uscale_statistics(1.0, 1.0);

// -----
// Plot Uniform Scale Statistics
// -----
LeafAngles.plot_uscale_statistics();
AllAngles.plot_uscale_statistics();

// -----
// Plot residue errors at the leaf node angles
// -----
LeafAngles.plot_uscale_residual_errors();
AllAngles.plot_uscale_residual_errors();

return 0;
}

:::::::::::::
cordic.hpp
:::::::::::::
void cordic ( double *x, double *y, double *z, int n, int *, int, double =0.0 );
:::::::::::::
cordic.cpp
:::::::::::::
# include <cstdlib>
# include <iostream>
# include <iomanip>
# include <cmath>
# include <ctime>

using namespace std;

# include "cordic.hpp"

//*****80

```

```

void cordic ( double *x, double *y, double *z, int n,
             int *nBreak, int nBreakInit, double threshold)

/*****80
// CORDIC returns the sine and cosine using the CORDIC method.
//
// Licensing:
//
//   This code is distributed under the GNU LGPL license.
//
// Modified:
//
//   2012.04.17
//
// Author:
//
//   Based on MATLAB code in a Wikipedia article.
//
//   Modifications by John Burkardt
//
//   Further modified by Young W. Lim
//
// Parameters:
//
//   Input:
//     *x: x coord of an init vector
//     *y: y coord of an init vector
//     *z: angle (-90 <= angle <= +90)
//     n: number of iteration
//       A value of 10 is low. Good accuracy is achieved
//       with 20 or more iterations.
//
//   Output:
//     *xo: x coord of a final vector
//     *yo: y coord of a final vector
//     *zo: angle residue
//
// Local Parameters:
//
//   Local, real ANGLES(60) = arctan ( (1/2)^(0:59) );
//
//   Local, real KPROD(33), KPROD(j) = product ( 0 <= i <= j ) K(i),
//   K(i) = 1 / sqrt ( 1 + (1/2)^(2i) ).
//
//
//
{
# define ANGLES_LENGTH 60
# define KPROD_LENGTH 33

double angle;
double angles[ANGLES_LENGTH] = {
  7.8539816339744830962E-01,
  4.6364760900080611621E-01,
  2.4497866312686415417E-01,
  1.2435499454676143503E-01,
  6.2418809995957348474E-02,
  3.1239833430268276254E-02,
  1.5623728620476830803E-02,
  7.8123410601011112965E-03,
  3.9062301319669718276E-03,
  1.9531225164788186851E-03,
  9.7656218955931943040E-04,
  4.8828121119489827547E-04,
  2.4414062014936176402E-04,
  1.2207031189367020424E-04,
  6.1035156174208775022E-05,
  3.0517578115526096862E-05,
  1.5258789061315762107E-05,
  7.6293945311019702634E-06,
  3.8146972656064962829E-06,

```



```
1.9073486328101870354E-06,  
9.5367431640596087942E-07,  
4.7683715820308885993E-07,  
2.3841857910155798249E-07,  
1.1920928955078068531E-07,  
5.9604644775390554414E-08,  
2.9802322387695303677E-08,  
1.4901161193847655147E-08,  
7.4505805969238279871E-09,  
3.7252902984619140453E-09,  
1.8626451492309570291E-09,  
9.3132257461547851536E-10,  
4.6566128730773925778E-10,  
2.3283064365386962890E-10,  
1.1641532182693481445E-10,  
5.8207660913467407226E-11,  
2.9103830456733703613E-11,  
1.4551915228366851807E-11,  
7.2759576141834259033E-12,  
3.6379788070917129517E-12,  
1.8189894035458564758E-12,  
9.0949470177292823792E-13,  
4.5474735088646411896E-13,  
2.2737367544323205948E-13,  
1.1368683772161602974E-13,  
5.6843418860808014870E-14,  
2.8421709430404007435E-14,  
1.4210854715202003717E-14,  
7.1054273576010018587E-15,  
3.5527136788005009294E-15,  
1.7763568394002504647E-15,  
8.8817841970012523234E-16,  
4.4408920985006261617E-16,  
2.2204460492503130808E-16,  
1.1102230246251565404E-16,  
5.5511151231257827021E-17,  
2.7755575615628913511E-17,  
1.3877787807814456755E-17,  
6.9388939039072283776E-18,  
3.4694469519536141888E-18,  
1.7347234759768070944E-18 };
```

```
double c2;  
double factor;  
int j;  
double kprod[KPROD_LENGTH] = {  
0.70710678118654752440,  
0.63245553203367586640,  
0.61357199107789634961,  
0.60883391251775242102,  
0.60764825625616820093,  
0.60735177014129595905,  
0.60727764409352599905,  
0.60725911229889273006,  
0.60725447933256232972,  
0.60725332108987516334,  
0.60725303152913433540,  
0.60725295913894481363,  
0.60725294104139716351,  
0.60725293651701023413,  
0.60725293538591350073,  
0.60725293510313931731,  
0.60725293503244577146,  
0.60725293501477238499,  
0.60725293501035403837,  
0.60725293500924945172,  
0.60725293500897330506,  
0.60725293500890426839,  
0.60725293500888700922,  
0.60725293500888269443,  
0.60725293500888161574,
```

```

0.60725293500888134606,
0.60725293500888127864,
0.60725293500888126179,
0.60725293500888125757,
0.60725293500888125652,
0.60725293500888125626,
0.60725293500888125619,
0.60725293500888125617 };
double pi = 3.141592653589793;
double poweroftwo;
double s2;
double sigma;
double sign_factor;
double theta;

double xn, yn;

//
// Initialize loop variables:
//
theta = *z;

xn = *x;
yn = *y;

poweroftwo = 1.0;
angle = angles[0];

angle = atan( 1. );

//
// Iterations
//
for ( j = 1; j <= n; j++ )
{
    if ( theta < 0.0 )
    {
        sigma = -1.0;
    }
    else
    {
        sigma = 1.0;
    }

    //
    // scaling factor is not yet corrected
    //
    static int cntBreak = 0;
    if ( nBreakInit == 0 ) cntBreak = 0;
    if ( abs(*z) < threshold ) {
        *nBreak = ++cntBreak;

        // cout << "cntBreak= " << cntBreak;
        // cout << " z= " << *z;
        // cout << " < " << threshold << endl;
        break;
    }

    factor = sigma * poweroftwo;

    *x =          xn - factor * yn;
    *y = factor * xn +          yn;

    xn = *x;
    yn = *y;

//
// Update the remaining angle.

```

```

//
theta = theta - sigma * angle;

poweroftwo = poweroftwo / 2.0;
//
// Update the angle from table, or eventually by just dividing by two.
//
if ( ANGLES_LENGTH < j + 1 )
{
angle = angle / 2.0;
}
else
{
angle = angles[j];
}

angle = atan( 1. / (1 << j));

*z = theta;
}
//
// Adjust length of output vector to be [cos(beta), sin(beta)]
//
// KPROD is essentially constant after a certain point, so if N is
// large, just take the last available value.
//
// if ( 0 < n )
// {
//   *c = *c * kprod [ i4_min ( n, KPROD_LENGTH ) - 1 ];
//   *s = *s * kprod [ i4_min ( n, KPROD_LENGTH ) - 1 ];
// }
//
// Adjust for possible sign change because angle was originally
// not in quadrant 1 or 4.
//
// *c = sign_factor * *c;
// *s = sign_factor * *s;

return;
# undef ANGLES_LENGTH
# undef KPROD_LENGTH
}

```