

# Apertures (1A)

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Please send corrections (or suggestions) to [youngwlim@hotmail.com](mailto:youngwlim@hotmail.com).

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# Finite Continuous Apertures

Array : a group of sensors

Aperture: the effects of sensors that gather signal energy over finite areas

Space-time signal  $f(\mathbf{x}, t)$

Aperture function  $w(\mathbf{x})$

The sensor's output  $z(\mathbf{x}, t) = w(\mathbf{x}) f(\mathbf{x}, t)$

*multiplication*

Space-time Fourier Transform

$$Z(\mathbf{k}, \omega) = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} W(\mathbf{k} - \mathbf{l}) F(\mathbf{l}, \omega) d\mathbf{l}$$

*convolution*

# Aperture Smoothing Function

Space-time  
signal

$$f(\mathbf{x}, t)$$



$$F(\mathbf{k}, \omega)$$

Aperture  
function

$$w(\mathbf{x})$$



$$W(\mathbf{k})$$

Aperture  
Smoothing  
Function

Sensor  
output

$$z(\mathbf{x}, t)$$



$$Z(\mathbf{k}, \omega)$$

$$Z(\mathbf{k}, \omega) = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} W(\mathbf{k} - \mathbf{l}) F(\mathbf{l}, \omega) d\mathbf{l}$$

# Aperture Smoothing Function

$$f(\mathbf{x}, t) \longleftrightarrow F(\mathbf{k}, \omega)$$

Space-time  
signal

$$F(\mathbf{k}, \omega) = \frac{1}{(2\pi)^4} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(\mathbf{x}, t) e^{j(\mathbf{k}\cdot\mathbf{x} - \omega t)} d\mathbf{x} dt$$

$$w(\mathbf{x}) \longleftrightarrow W(\mathbf{k})$$

Aperture  
function

Aperture  
Smoothing  
Function

$$W(\mathbf{k}) = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} w(\mathbf{x}) e^{j\mathbf{k}\cdot\mathbf{x}} d\mathbf{x}$$

# Co-Array for Continuous Apertures

Co-Array

Autocorrelation of aperture function

$$w(\mathbf{x})$$

Aperture  
function



$$c(\boldsymbol{\chi}) = \int w(\mathbf{x})w(\mathbf{x}+\boldsymbol{\chi})d\mathbf{x}$$

$\boldsymbol{\chi}$  lag

wave's energy content

$$z(\mathbf{x}, t) = w(\mathbf{x}) f(\mathbf{x}, t)$$



$$\int_a (\mathbf{x}_1, \mathbf{x}_1 + \boldsymbol{\chi}; \tau) d\mathbf{x}_1 = c(\boldsymbol{\chi}) R_f(\boldsymbol{\chi}, \tau)$$

# Co-Array for Continuous Apertures

$$w(\mathbf{x})$$



$$c(\boldsymbol{\chi}) = \int w(\mathbf{x})w(\mathbf{x}+\boldsymbol{\chi})d\mathbf{x}$$

$$z(\mathbf{x}, t) = w(\mathbf{x})f(\mathbf{x}, t)$$



$$c(\boldsymbol{\chi})R_f(\boldsymbol{\chi}, \tau) = \int_a R_z(\mathbf{x}_1, \mathbf{x}_1+\boldsymbol{\chi}; \tau) d\mathbf{x}_1$$

$$\boldsymbol{\chi} = \mathbf{x}_2 - \mathbf{x}_1$$

$$R_z(\mathbf{x}_1, \mathbf{x}_2; \tau) = E[z(\mathbf{x}_1, t)z^*(\mathbf{x}_2, t+\tau)]$$

$$= E[w(\mathbf{x}_1)f(\mathbf{x}_1, t)w^*(\mathbf{x}_2)f^*(\mathbf{x}_2, t+\tau)]$$

$$= w(\mathbf{x}_1)w^*(\mathbf{x}_2)E[f(\mathbf{x}_1, t)f^*(\mathbf{x}_2, t+\tau)]$$

$$= w(\mathbf{x}_1)w^*(\mathbf{x}_1+\boldsymbol{\chi})E[f(\mathbf{x}_1, t)f^*(\mathbf{x}_1+\boldsymbol{\chi}, t+\tau)]$$

$$= w(\mathbf{x}_1)w^*(\mathbf{x}_1+\boldsymbol{\chi})R_f(\boldsymbol{\chi}, \tau)$$

$$\int_a R_z(\mathbf{x}_1, \mathbf{x}_1+\boldsymbol{\chi}; \tau) d\mathbf{x}_1 = \int_a w(\mathbf{x}_1)w^*(\mathbf{x}_1+\boldsymbol{\chi})R_f(\boldsymbol{\chi}, \tau)d\mathbf{x}_1$$

$$= \int_a w(\mathbf{x}_1)w^*(\mathbf{x}_1+\boldsymbol{\chi})d\mathbf{x}_1 R_f(\boldsymbol{\chi}, \tau)$$

# Filter-and-Sum Beamforming

$$q_i(t) = w_i(t) * r_i(t)$$

$$b(t) = \sum_{i=0}^{N-1} q_i(t - \tau_i)$$



# Frequency-Domain Beamforming

$$fd(t, \omega) = \sum_{i=0}^{N-1} w_i R_i(t, \omega) e^{+j\omega(t - \tau_i)}$$

$R_i(\omega) \iff r_i(t)$

## References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] D.H. Johnson, et. al., Array Signal Processing: concepts and techniques