Apertures (1A)

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Finite Continuous Apertures

Array : a group of sensors

Aperture: the effects of sensors that gather signal energy <u>over finite areas</u>

Space-time signal $f(\boldsymbol{x}, t)$

Aperture function $w(\boldsymbol{x})$

The sensor's output $z(\boldsymbol{x}, t) = w(\boldsymbol{x})\, f(\boldsymbol{x}, t)$ ← *multiplication*

Space-time Fourier Transform

$$Z(\boldsymbol{k}, \omega) = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} W(\boldsymbol{k} - \boldsymbol{l})\, F(\boldsymbol{l}, \omega)\, d\boldsymbol{l}$$

convolution

Aperture Smoothing Function

Space-time signal	$f(\boldsymbol{x}, t)$	⟺	$F(\boldsymbol{k}, \omega)$
Aperture function	$w(\boldsymbol{x})$	⟺	$W(\boldsymbol{k})$ — Aperture Smoothing Function
Sensor output	$z(\boldsymbol{x}, t)$	⟺	$Z(\boldsymbol{k}, \omega)$

$$Z(\boldsymbol{k}, \omega) = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} W(\underline{\boldsymbol{k} - \boldsymbol{l}})\, F(\underline{\boldsymbol{l}}, \omega)\, d\boldsymbol{l}$$

Aperture Smoothing Function

$f(\boldsymbol{x}, t) \Leftrightarrow F(\boldsymbol{k}, \omega)$

Space-time signal

$$F(\boldsymbol{k}, \omega) = \frac{1}{(2\pi)^4} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(\boldsymbol{x}, t)\, e^{j(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t)}\, d\boldsymbol{x}\, dt$$

$w(\boldsymbol{x}) \Leftrightarrow W(\boldsymbol{k})$

Aperture function

Aperture Smoothing Function

$$W(\boldsymbol{k}) = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} w(\boldsymbol{x})\, e^{j\boldsymbol{k}\cdot\boldsymbol{x}}\, d\boldsymbol{x}$$

Co-Array for Continuous Apertures

Co-Array Autocorrelation of aperture function

$w(\boldsymbol{x})$

Aperture function

$$c(\chi) = \int w(\boldsymbol{x}) w(\boldsymbol{x}+\chi) d\,\boldsymbol{x}$$

χ lag

wave's energy content

$$z(\boldsymbol{x},t) = w(\boldsymbol{x})\, f(\boldsymbol{x},t)$$

$$\int_a (\boldsymbol{x}_1, \boldsymbol{x}_1+\chi;\, \tau) d\,\boldsymbol{x}_1 = c(\chi)\, R_f(\chi, \tau)$$

Co-Array for Continuous Apertures

$$w(\boldsymbol{x}) \quad \Rightarrow \quad c(\chi) = \int w(\boldsymbol{x})w(\boldsymbol{x}+\chi)\,d\,\boldsymbol{x}$$

$$z(\boldsymbol{x},t) = w(\boldsymbol{x})\,f(\boldsymbol{x},t) \quad \Rightarrow \quad c(\chi)\,R_f(\chi,\tau) = \int_a R_z(\boldsymbol{x}_1,\boldsymbol{x}_1+\chi;\,\tau)\,d\,\boldsymbol{x}_1$$

$$\chi = \boldsymbol{x}_2 - \boldsymbol{x}_1$$

$$\begin{aligned}
R_z(\boldsymbol{x}_1,\boldsymbol{x}_2;\,\tau) &= E[z(\boldsymbol{x}_1,t)\,z^*(\boldsymbol{x}_2,\,t+\tau)] \\
&= E[\,w(\boldsymbol{x}_1)f(\boldsymbol{x}_1,t)\,w^*(\boldsymbol{x}_2)f^*(\boldsymbol{x}_2,\,t+\tau)\,] \\
&= w(\boldsymbol{x}_1)w^*(\boldsymbol{x}_2)\,E[\,f(\boldsymbol{x}_1,t)f^*(\boldsymbol{x}_2,\,t+\tau)\,] \\
&= w(\boldsymbol{x}_1)w^*(\boldsymbol{x}_1+\chi)\,E[\,f(\boldsymbol{x}_1,t)f^*(\boldsymbol{x}_1+\chi,\,t+\tau)\,] \\
&= w(\boldsymbol{x}_1)w^*(\boldsymbol{x}_1+\chi)\,R_f(\chi,\tau)
\end{aligned}$$

$$\begin{aligned}
\int_a R_z(\boldsymbol{x}_1,\boldsymbol{x}_1+\chi;\,\tau)\,d\,\boldsymbol{x}_1 &= \int_a w(\boldsymbol{x}_1)w^*(\boldsymbol{x}_1+\chi)\,R_f(\chi,\tau)\,d\,\boldsymbol{x}_1 \\
&= \int_a w(\boldsymbol{x}_1)w^*(\boldsymbol{x}_1+\chi)\,d\,\boldsymbol{x}_1\;R_f(\chi,\tau)
\end{aligned}$$

Filter-and-Sum Beamforming

$$q_i(t) = w_i(t) * r_i(t)$$

$$b(t) = \sum_{i=0}^{N-1} q_i(t - \tau_i)$$

Frequency-Domain Beamforming

$$fd(t, \boldsymbol{\omega}) = \sum_{i=0}^{N-1} w_i R_i(t, \boldsymbol{\omega}) e^{+j\boldsymbol{\omega}(t - \boldsymbol{\tau}_i)}$$

$R_i(\omega) \Longleftrightarrow r_i(t)$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] D.H. Johnson, et. al., Arrary Signal Processing: concepts and techniques