$n=2$

Team 6, Cook

$$\frac{dV_y}{dt} = \frac{-k}{m(t)} V_y^2 - g$$

Let $y = V_y$, and y_1 be a particular solution

$$w = y - y_1$$

$$\frac{dw}{dt} = \frac{dy}{dt} - \frac{dy_1}{dt} = \frac{-k}{m(t)}(y^2 - y_1^2)$$

$$= \frac{-k}{m(t)}(y-y_1)(y+y_1)$$

$$= \frac{-k}{m(t)}(y-y_1)(y-y_1+2y_1)$$

$$= \frac{-k}{m(t)} w(w+2y_1)$$

↳ "known" function

$$\frac{dw}{dt} = \frac{-k}{m(t)} w^2 + \frac{-2y_1 k}{m(t)} w$$

$$a(t) = -k/m(t) \qquad b(t) = -2y_1(t)k/m(t)$$

$$\frac{dw}{dt} = a(t)w^2 + b(t)w$$

$$u = w^{-1}$$

$$\frac{du}{dt} = -w^{-2}\frac{dw}{dt}$$

$$= -w^{-2}\left[a(t)w^2 + b(t)w\right]$$

$$= a(t) + b(t)w^{-1}$$

$$\frac{du}{dt} = a(t) + b(t)u$$

$$-a(t) - b(t)u + \frac{du}{dt} = 0$$

$$-(a(t) + b(t)u)\,dt + du = 0$$

$$\phi_t = \frac{\partial \phi}{\partial t} \qquad \phi_u = \frac{\partial \phi}{\partial u}$$

To be exact $\phi_{tu} = \phi_{ut}$

$\phi_{tu} \neq 0, \quad \phi_{ut} = 0$

$$h(u,t)\left[-(a(t)+b(t)u)\,dt + du = 0\right]$$

$$\phi_t = -h(a(t)+b(t)u) \qquad \phi_u = h$$

$$\phi_{tu} = -\frac{\partial h}{\partial u}(a(t)+b(t)u) - h(b(t)) = \phi_{ut} = \frac{\partial h}{\partial t}$$

assume $\partial h/\partial u = 0$, or $h = f(t)$

$$-h\,b(t) = \frac{\partial h}{\partial t}$$

$$h = e^{-\int b(t)\,dt}$$

$$\phi_t = -e^{-\int b(t)dt}(a(t)+b(t)u)$$

$$\phi = \int \phi_t\,dt = \int\left(-a(t)e^{-\int b(t)dt} - b(t)ue^{-\int b(t)dt}\right)dt$$

$$= -\int a(t)e^{-\int b(t)dt} - u\int b(t)e^{-\int b(t)dt} + f(u) = k_2 \text{ - constant}$$

$$\phi = \int \phi_u\,du = \int e^{-\int b(t)dt}\,du = ue^{-\int b(t)dt} + f(t) = k_2$$

$$\phi = -\int a(t)e^{-\int b(t)dt} - u\int b(t)e^{-\int b(t)dt} + ue^{-\int b(t)dt} = k_2$$

$$u = \frac{k_2 + \int a(t)e^{-\int b(t)dt}\,dt}{e^{-\int b(t)dt} - \int b(t)e^{-\int b(t)dt}\,dt}$$

$$u = w^{-1} = \frac{1}{y-y_1}$$

$$\therefore \quad y = \left[\frac{k_2 + \int a(t)e^{-\int b(t)dt}\,dt}{e^{-\int b(t)dt} - \int b(t)e^{-\int b(t)dt}\,dt}\right]^{-1} + y_1(t)$$

$$\int y\,dy = \log\left[\frac{k_2 + \int a(t)e^{-\int b(t)dt}\,dt}{e^{-\int b(t)dt} - \int b(t)e^{-\int b(t)dt}\,dt}\right] \frac{d}{dt}\left[\frac{\int a(t)e^{-\int b(t)dt}\,dt}{e^{-\int b(t)dt} - \int b(t)e^{-\int b(t)dt}\,dt}\right]$$

$$+ \int y_1(t)\,dt$$

$$\frac{d}{dt}\left[e^{-\int b(t)dt}\right] = -b(t)e^{-\int b(t)dt}$$

$$\rightarrow \int -b(t)e^{-\int b(t)dt} = e^{-\int b(t)dt}$$

$$\frac{d}{dt}\left[\frac{\int a(t)e^{-\int b(t)dt}}{\int -b(t)e^{-\int b(t)dt} - \int b(t)e^{-\int b(t)dt}}\right] = \frac{d}{dt}\int \frac{a(t)}{-2b(t)}dt = \frac{a(t)}{-2b(t)}$$

$$\int y\,dy = \log\left[\frac{k_2 + \int a(t)e^{-\int b(t)dt}dt}{-2\int b(t)e^{-\int b(t)dt}dt}\right]\frac{a(t)}{-2b(t)} + \int y_1(t)dt$$

$$V_y = \left[\frac{k_2 + \int a(t)e^{-\int b(t)dt}}{-2\int b(t)e^{-\int b(t)dt}dt}\right]^{-1} + y_1(t) \quad \text{Vertical Velocity}$$

$$y = -\log\left[\frac{k_2 + \int a(t)e^{-\int b(t)dt}dt}{-2\int b(t)e^{-\int b(t)dt}dt}\right]\frac{a(t)}{2b(t)} + \int y_1(t)dt \quad \text{Vertical Position}$$

$$a(t) = \frac{-k}{m(t)} \qquad b(t) = \frac{-2y_1(t)k}{m(t)}$$