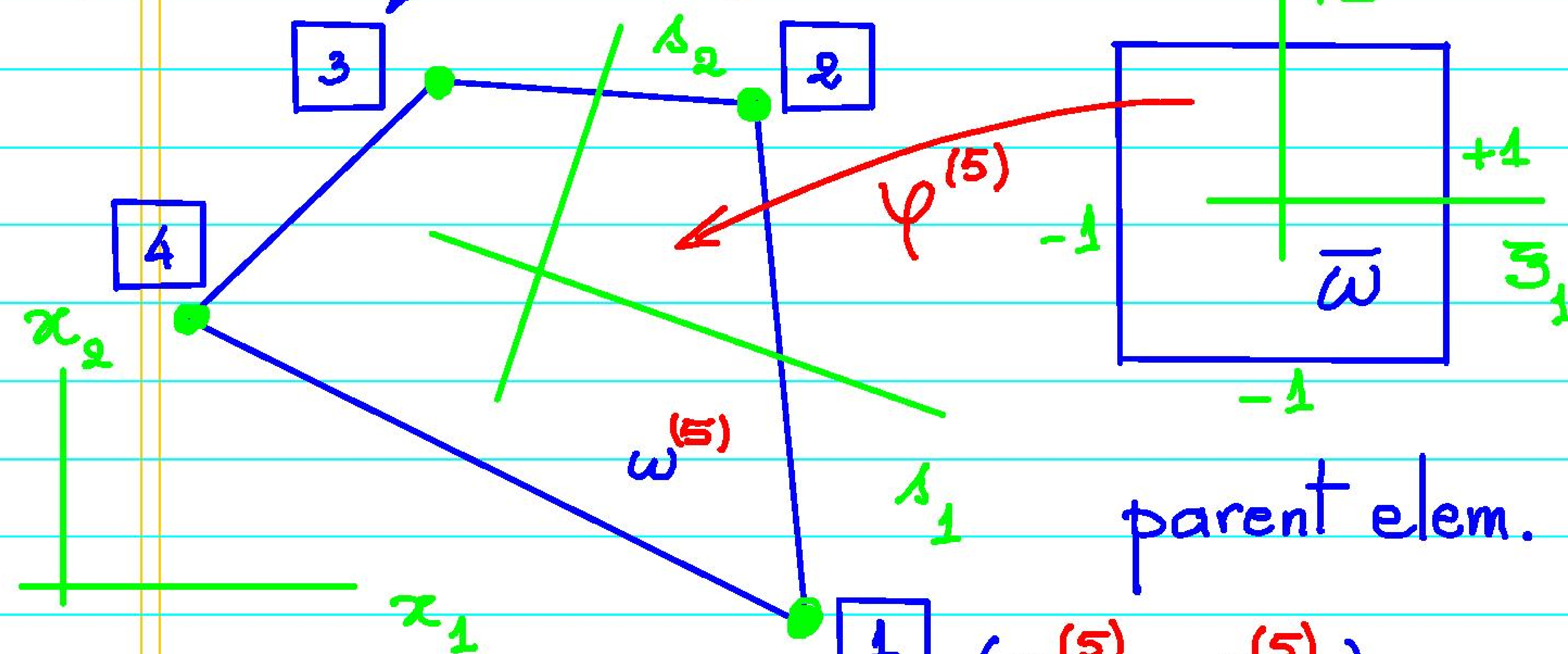
Ex for (1) p. 34-5: Consider elem $e=5$ fig p. 30-1

$$\left(x^{(5)}_{1,3}, x^{(5)}_{2,3}\right) = \left(x^{(5)}_{3}, y^{(5)}_{3}\right)$$



$$\left(x^{(5)}_{1,1}, x^{(5)}_{2,1}\right) = \left(x^{(5)}_{1}, y^{(5)}_{1}\right)$$

$$x^{(5)}_i = \varphi^{(5)}(\underline{\xi}) = \sum_{I=1}^{n_e} N_I(\underline{\xi})\, x^{(5)}_{i,I}, \quad i=1,2$$

$$\begin{Bmatrix} x^{(5)}_1 \\ x^{(5)}_2 \end{Bmatrix} = \begin{Bmatrix} x^{(5)} \\ y^{(5)} \end{Bmatrix} = \sum_{I=1}^{n_e} N_I(\underline{\xi}) \begin{Bmatrix} x^{(5)}_I \\ y^{(5)}_I \end{Bmatrix}$$

HW6.4: FB, pp. 148-149, pbs. 6.1, 6.2, 6.3.

Comp. of $\underline{k}^e$ in parent coord. $\{\xi_i\}$: cont'd

(5) p.34-3: $$\underline{k}^e = \left[k^e_{IJ} \right] \quad (1)$$

$$k^e_{IJ} = \int_{\omega^e} (\nabla N^e_I) \cdot \underline{\kappa} \cdot (\nabla N^e_J)\, d\omega^e \quad (2)$$

$$k^e_{IJ} = \int_{\omega^e} \left[\nabla_x N^e_I(\underline{\xi})\right] \cdot \underline{\kappa}(x) \cdot \left[\nabla_x N^e_J(\underline{\xi})\right] d\omega^e_x \quad (3)$$

Note! Convert to $\{\xi_i\}$ coord.

(1) p.34-4: $$d\omega^e_x = \underbrace{\det \underline{J}(\underline{\xi})}_{J(\underline{\xi})}\, d\omega^e_\xi \quad (4)$$

$$d\Omega = dx_1\, dx_2 \qquad\qquad d\xi_1\, d\xi_2 = d\bar{\omega}$$

$$\underline{\kappa}(\underline{x}) = \underline{\kappa}(\underline{x}^e(\underline{\xi})) = \underline{\kappa}(\underbrace{\varphi^e(\underline{\xi})}_{(1)\ p.34\text{-}5}) \quad (5)$$

$$\nabla_x N^e_I(\underline{\xi}) = \begin{bmatrix} \dfrac{\partial}{\partial x_1} \\ \dfrac{\partial}{\partial x_2} \end{bmatrix} N^e_I(\underline{\xi}) \quad (6)$$

$$\frac{\partial}{\partial x_i} = \frac{\partial}{\partial \xi_j} \frac{\partial \xi_j}{\partial x_i} \Rightarrow \underset{\text{col}}{\left\lfloor \frac{\partial}{\partial x_i} \right\rfloor} = \underset{\text{col}}{\left\lfloor \frac{\partial}{\partial \xi_j} \right\rfloor} \left[\frac{\partial \xi_j}{\partial x_i} \right] \quad \begin{matrix} \text{row} \\ \text{col} \end{matrix} \qquad (1)$$

$$\underbrace{\left\lfloor \frac{\partial}{\partial x_1} \quad \frac{\partial}{\partial x_2} \right\rfloor}_{\nabla_x^T} = \underbrace{\left\lfloor \frac{\partial}{\partial \xi_1} \quad \frac{\partial}{\partial \xi_2} \right\rfloor}_{\nabla_\xi^T} \underbrace{\begin{bmatrix} \frac{\partial \xi_1}{\partial x_1} & \frac{\partial \xi_1}{\partial x_2} \\ \frac{\partial \xi_2}{\partial x_1} & \frac{\partial \xi_2}{\partial x_2} \end{bmatrix}}_{\underline{J}^{-1}(\underline{\xi})} \qquad (2)$$

$$\Rightarrow \boxed{\nabla_x = \underline{J}^{-T}(\underline{\xi}) \; \nabla_\xi} \qquad (3)$$

(6) p. 35-2:
$$\boxed{\nabla_x N_I^e(\underline{\xi}) = \left(\underline{J}^e\right)^{-T}(\underline{\xi}) \; \nabla_\xi N_I^e(\underline{\xi})} \qquad (4)$$

(1) p. 34-4:
$$\underline{J}^e(\underline{\xi}) = \left[\frac{\partial x_i^e(\underline{\xi})}{\partial \xi_j} \right] \qquad (5)$$

p.34-1 Data set 1

G2DM1.0/D1: $\Omega = \bar{\omega} = \square$ biunit square

PDE: (4) p.33-2: $\underline{K} = \underline{\underline{I}}$ (identity matrix)

$f = 0 \quad , \quad \dfrac{\partial u}{\partial t} = 0$

Ess. bc: $q = 2$ on $\partial\Omega$

Nat. bc: none

HW6.5: Use 2D LIBF p.29-2 with $n = m = 2, 4, 6, 8, \ldots$ to solve G2DM1.0/D1 for u^h until accuracy 10^{-6} at center $(x, y) = (0, 0)$.

HW6.6: solve G2DM1.0/D1 using 2D LLEBF until accuracy 10^{-6} at center $(x, y) = (0, 0)$.

Note: Verify your results with any FE code, with detailed documentation.