Group Velocity and Phase Velocity (1A)

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5/14/12

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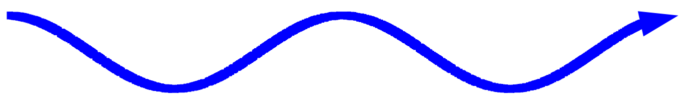
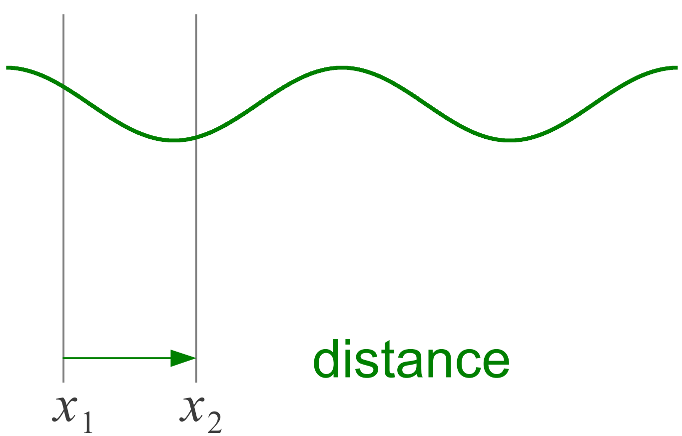
Wave Equation

$$A(t, t) = A_0 \cos(kx - \omega t)$$



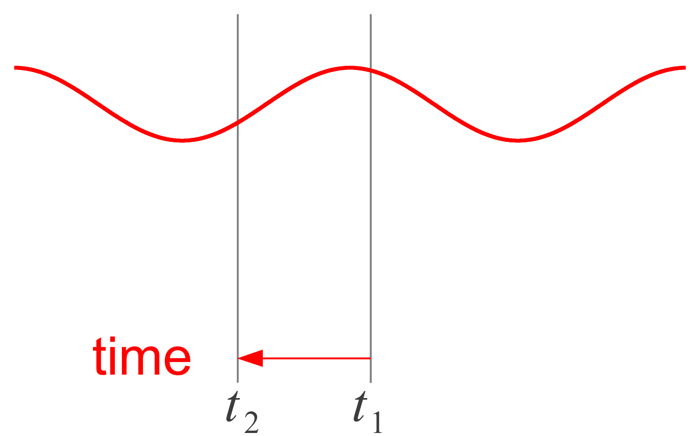
$A_0 \cos(kx - \omega t_0)$

At the snapshot of the time t_0



$A_0 \cos(kx_0 - \omega t)$

At the fixed site of the distance x_0



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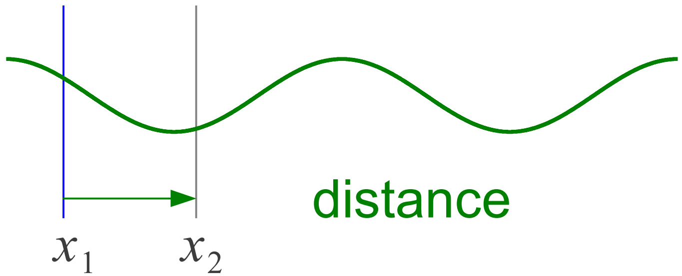
Wavelength, Frequency

$A_0 \cos(k\,x - \omega\, t_0)$

At the snapshot of the time t_0

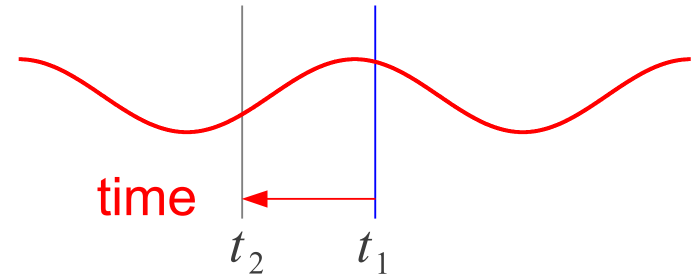


wavelength $\lambda = \dfrac{2\pi}{k}$

wave number $k = \dfrac{2\pi}{\lambda}$

$A_0 \cos(k\,x_0 - \omega\, t)$

At the fixed site of the distance x_0



frequency $f = \dfrac{\omega}{2\pi}$

period $T = \dfrac{2\pi}{\omega}$

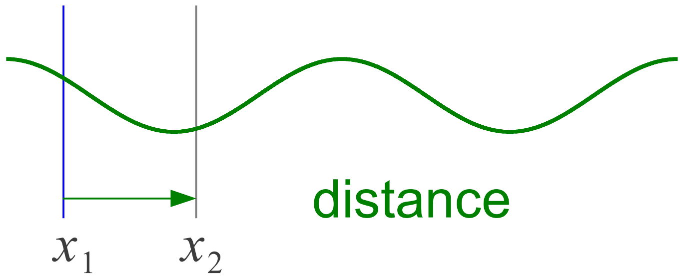
angular frequency $\omega = 2\pi f$

angular frequency $\omega = \dfrac{2\pi}{T}$

Wave Number, Angular Frequency

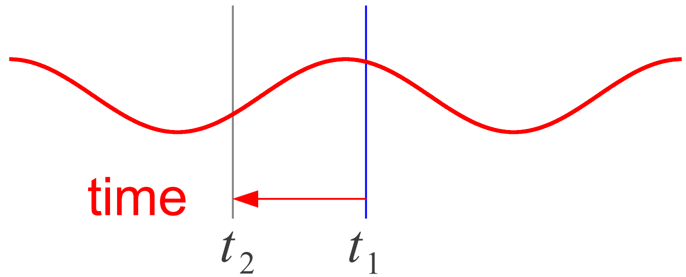
$A_0 \cos(k x - \omega t_0)$

At the snapshot of the time t_0



$A_0 \cos(k x_0 - \omega t)$

At the fixed site of the distance x_0



wave number $k = \dfrac{2\pi}{\lambda}$

radians per unit distance

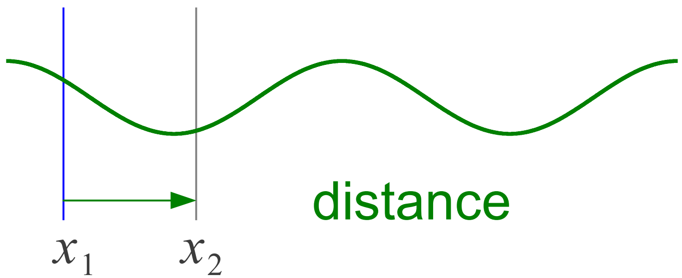
angular frequency $\omega = \dfrac{2\pi}{T}$

radians per unit time

Phase Velocity (1)

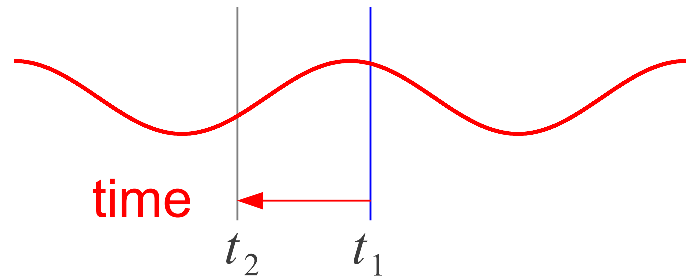
$A_0 \cos(kx - \omega t_0)$

At the snapshot of the time t_0



$A_0 \cos(kx_0 - \omega t)$

At the fixed site of the distance x_0



wave number $k = \dfrac{2\pi}{\lambda}$

radians per unit distance

angular frequency $\omega = \dfrac{2\pi}{T}$

radians per unit time

Phase Velocity $v_p = \dfrac{\lambda}{T} = \dfrac{2\pi/k}{2\pi/\omega} = \dfrac{\omega}{k}$

$$v_p = \frac{\omega}{k}$$

Phase Velocity (2)

Phase Velocity $v_p = \frac{\omega}{k}$

$$A\cos(kx - \omega t)$$

*Given time **t**,* → ωt *oscillations*

*Corresponding distance **x**,* → *the same oscillations*

$$kx = \omega t$$

$$v_p = \frac{x}{t} = \frac{\omega}{k}$$

Phase Velocity, Group Velocity

Phase Velocity $$v_p = \frac{\omega}{k}$$

Group Velocity $$v_g = \frac{\partial \omega}{\partial k}$$

Group Delay

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] http://www.mathpages.com/, Phase, Group, and Signal Velocity