

Group Velocity and Phase Velocity (1A)

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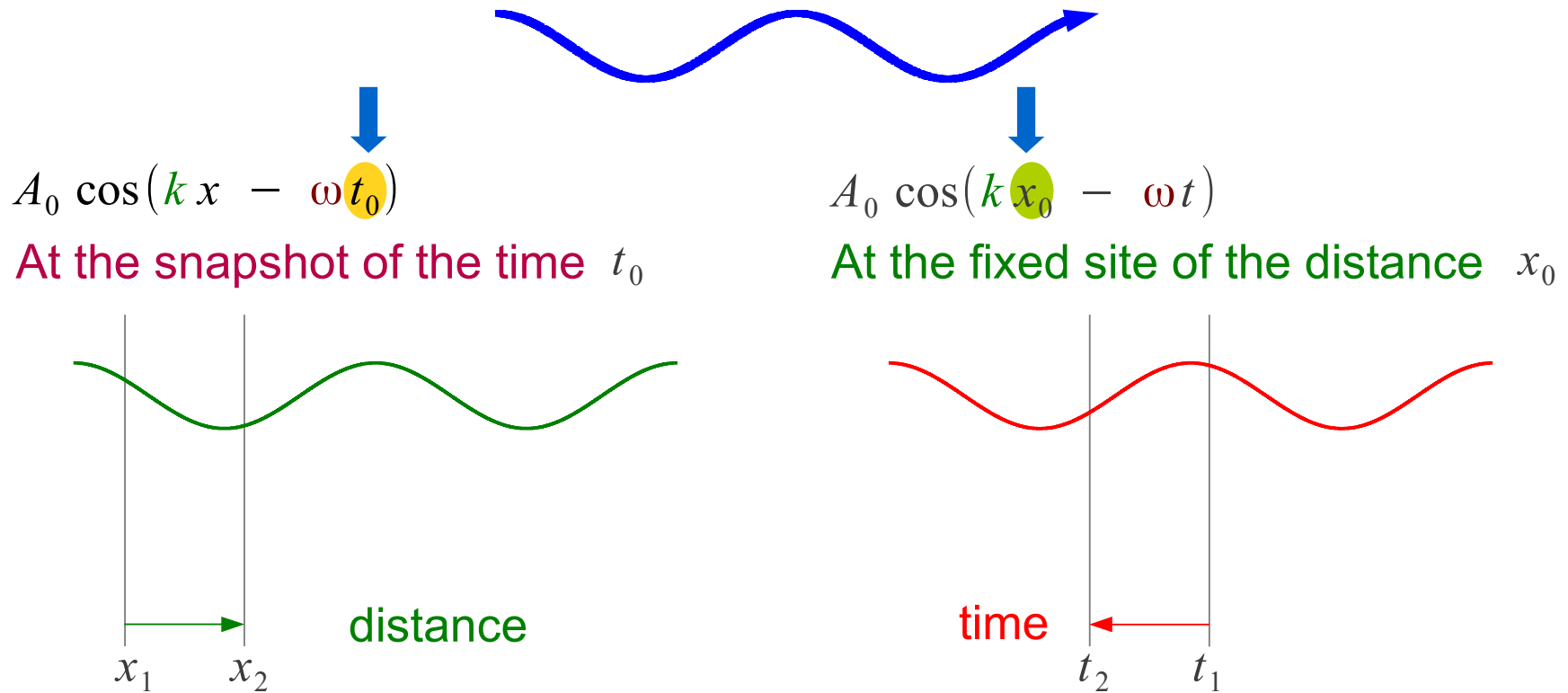
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Wave Equation

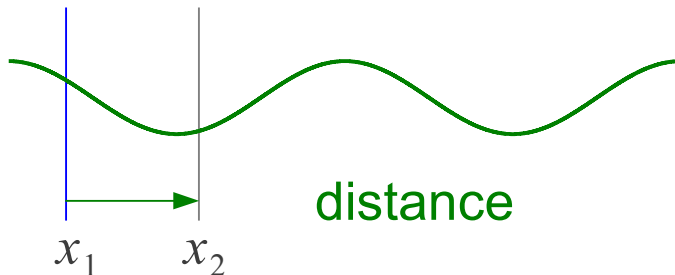
$$A(t, x) = A_0 \cos(kx - \omega t)$$



Wavelength, Frequency

$$A_0 \cos(kx - \omega t_0)$$

At the snapshot of the time t_0

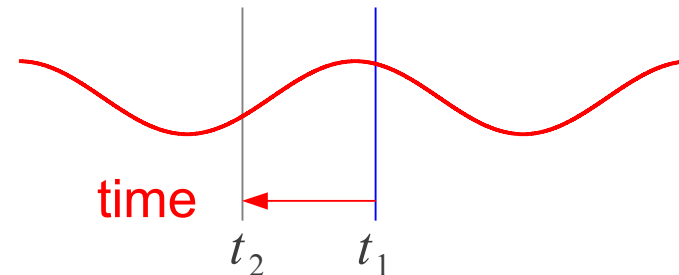


wavelength $\lambda = \frac{2\pi}{k}$

wave number $k = \frac{2\pi}{\lambda}$

$$A_0 \cos(kx_0 - \omega t)$$

At the fixed site of the distance x_0



frequency $f = \frac{\omega}{2\pi}$

period $T = \frac{2\pi}{\omega}$

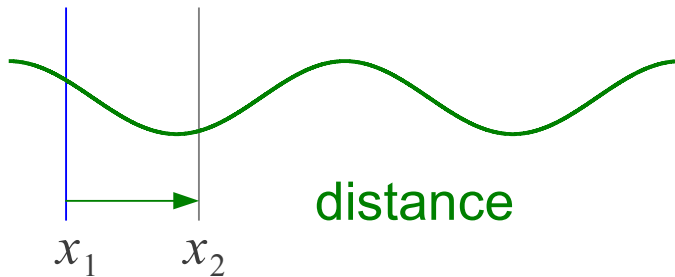
angular frequency $\omega = 2\pi f$

angular frequency $\omega = \frac{2\pi}{T}$

Wave Number, Angular Frequency

$$A_0 \cos(kx - \omega t_0)$$

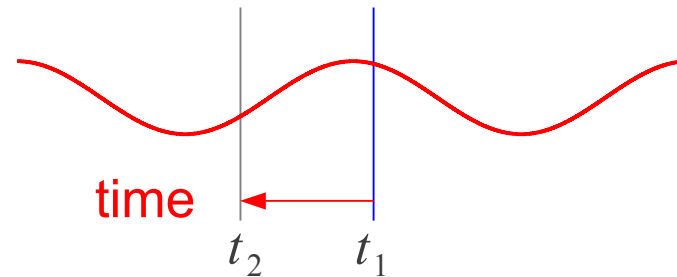
At the snapshot of the time t_0



wave number $k = \frac{2\pi}{\lambda}$
radians per unit distance

$$A_0 \cos(kx_0 - \omega t)$$

At the fixed site of the distance x_0

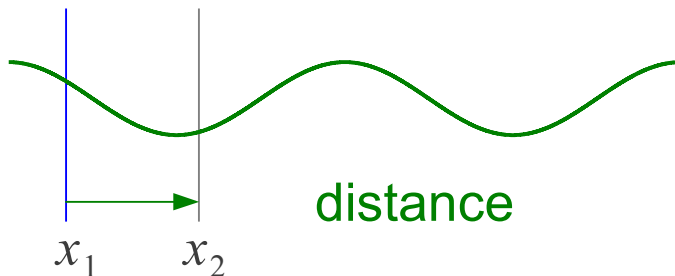


angular frequency $\omega = \frac{2\pi}{T}$
radians per unit time

Phase Velocity (1)

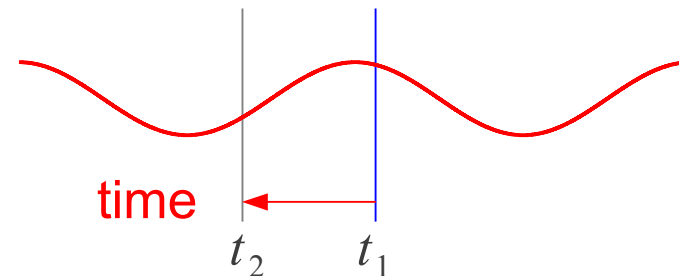
$$A_0 \cos(kx - \omega t_0)$$

At the snapshot of the time t_0



$$A_0 \cos(kx_0 - \omega t)$$

At the fixed site of the distance x_0



wave number $k = \frac{2\pi}{\lambda}$
radians per unit distance

angular frequency $\omega = \frac{2\pi}{T}$
radians per unit time

Phase Velocity $v_p = \frac{\lambda}{T} = \frac{2\pi/k}{2\pi/\omega} = \frac{\omega}{k}$ $v_p = \frac{\omega}{k}$

Phase Velocity (2)

Phase Velocity $v_p = \frac{\omega}{k}$

$$A \cos(kx - \omega t)$$

Given time t ,  ωt oscillations

Corresponding distance x ,  the same oscillations

$$kx = \omega t$$

$$v_p = \frac{x}{t} = \frac{\omega}{k}$$

Phase Velocity, Group Velocity

Phase Velocity $v_p = \frac{\omega}{k}$

Group Velocity $v_g = \frac{\partial \omega}{\partial k}$

Group Delay

References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] <http://www.mathpages.com/>, Phase, Group, and Signal Velocity