Introduction to Systems of Linear Equations

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Linear Equations

2-dim → Line equation

$$a\,x + b\,y = c$$ (a, b not both 0)

3-dim → Plane equation

$$a\,x + b\,y + c\,z = d$$ (a, b, c not both 0)

n-dim → Hyper-Plane equation

$$a_1\,x_1 + a_2\,x_2 + \cdots + a_n\,x_n = b$$ (a_i not both 0)

(n=2) $a_1\,x_1 + a_2\,x_2 = b$ → $a\,x + b\,y = c$

(n=3) $a_1\,x_1 + a_2\,x_2 + a_3\,x_3 = b$ → $a\,x + b\,y + c\,z = d$

Homogeneous Linear Equations

2-dim ➡ Line equation

$$a\,x + b\,y = 0$$

(a, b not both 0)

3-dim ➡ Plane equation

$$a\,x + b\,y + c\,z = 0$$

(a, b, c not both 0)

n-dim ➡ Hyper-Plane equation

$$a_1\,x_1 + a_2\,x_2 + \cdots + a_n\,x_n = 0$$

(a_i not both 0)

(n=2) $a_1\,x_1 + a_2\,x_2 = 0$ ➡ $a\,x + b\,y = 0$

(n=3) $a_1\,x_1 + a_2\,x_2 + a_3\,x_3 = 0$ ➡ $a\,x + b\,y + c\,z = 0$

System of Linear Equation

(Eq 1) $\quad a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n = b_1$

(Eq 2) $\quad a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n = b_2$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

(Eq m) $\quad a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n = b_m$

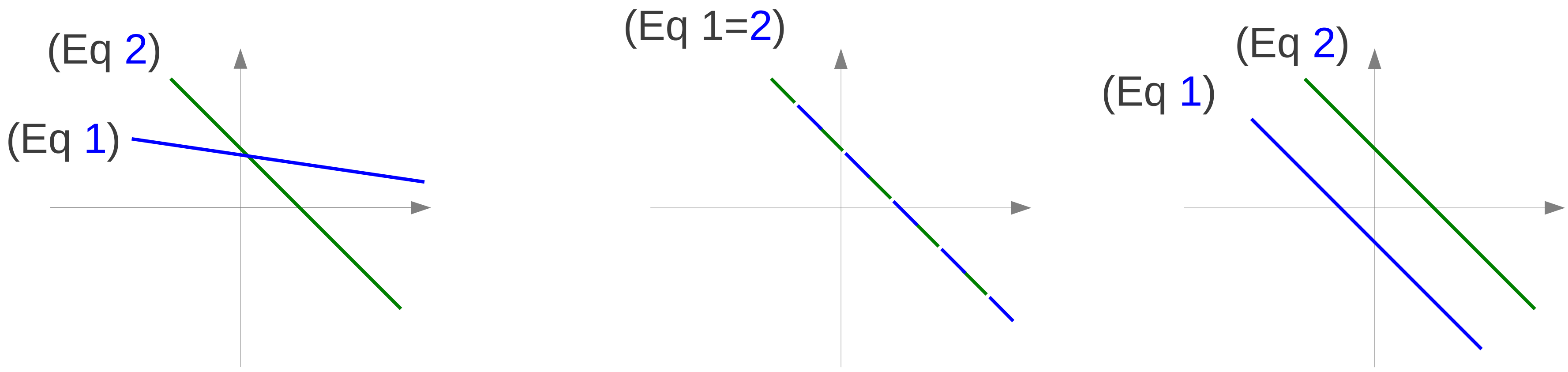
m equations

n unknowns

Linear Systems of 2 Unknowns

(Eq 1) $a_{11} x_1 + a_{12} x_2 = b_1$

(Eq 2) $a_{21} x_1 + a_{22} x_2 = b_2$



One solution

Too many solutions

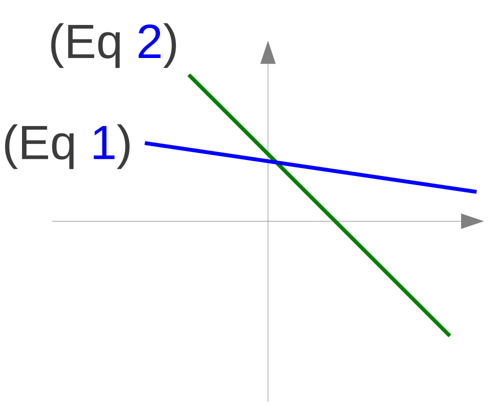
No solution

Linear Systems of 2 Unknowns

$$\begin{cases} x - y = 1 \\ 2x + y = 6 \end{cases}$$

$$2x - 2y = 2$$
$$2x + y = 6$$
$$-3y = -4$$
$$\left(\frac{7}{3}, \frac{4}{3}\right)$$

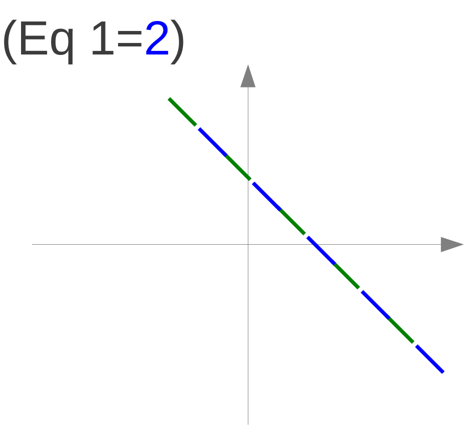
One solution



$$\begin{cases} 4x - 2y = 1 \\ 16x - 8y = 4 \end{cases}$$

$$4x - 2y = 1$$
$$4x - 2y = 1$$
$$0 = 0$$

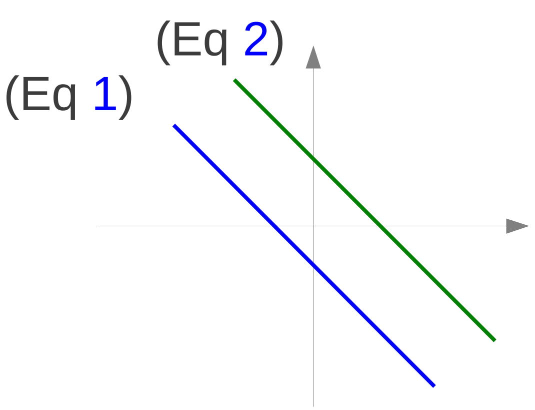
Too many solutions



$$\begin{cases} x + y = 4 \\ 3x + 3y = 6 \end{cases}$$

$$x + y = 4$$
$$x + y = 2$$
$$4 = 2$$

No solution

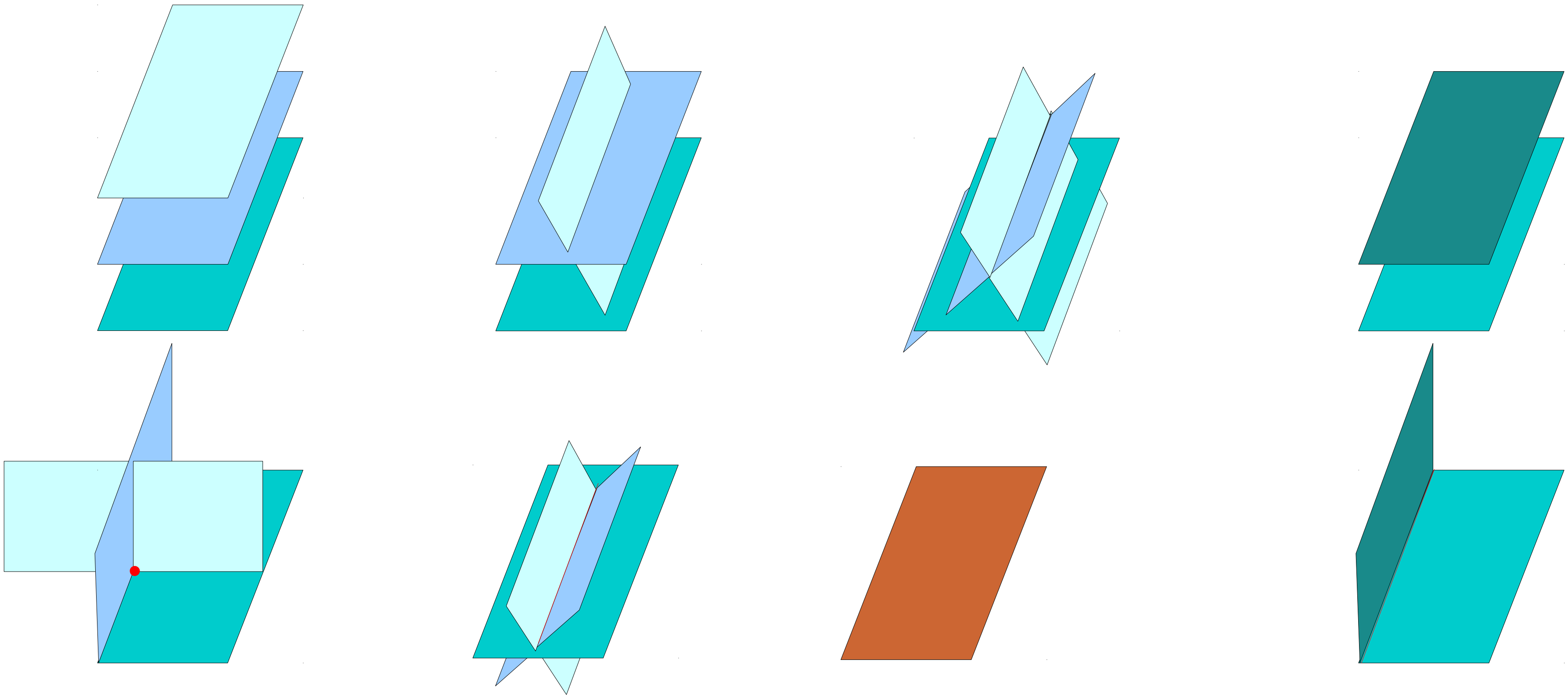


Linear Systems of 3 Unknowns

(Eq 1) $a_{11} x_1 + a_{12} x_2 + a_{13} x_3 = b_1$

(Eq 2) $a_{21} x_1 + a_{22} x_2 + a_{23} x_3 = b_2$

(Eq 3) $a_{31} x_1 + a_{32} x_2 + a_{33} x_3 = b_3$



Linear Equations

(Eq 1) $a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n = b_1$

(Eq 2) $a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n = b_2$

$\vdots$

(Eq 3) $a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n = b_m$

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

Linear Equations

$$a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n = b_1$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{bmatrix} b_1 \end{bmatrix}$$

$$\sum_{j=1}^{n} a_{1j} \cdot x_j = b_1$$

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mxn Matrix

row index

nx1 Vector

Linear Equations

$$a_{21} \, x_1 + a_{22} \, x_2 + \cdots + a_{2n} \, x_n = b_2$$

$$\begin{bmatrix} a_{21} & a_{22} & \cdots & a_{2n} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{bmatrix} b_2 \end{bmatrix}$$

$$\sum_{j=1}^{n} a_{2j} \cdot x_j = b_2$$

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Linear Equations

$$a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n = b_m$$

$$\begin{bmatrix} a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = b_m$$

$$\sum_{j=1}^{n} a_{mj} \cdot x_j = b_m$$

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Storing Magnetic Energy

References

[1] http://en.wikipedia.org/
[2] Anton, et al., Elementary Linear Algebra, 10th ed, Wiley, 2011
[3] Anton, et al., Contemporary Linear Algebra,