

# Bandpass Sampling (2B)

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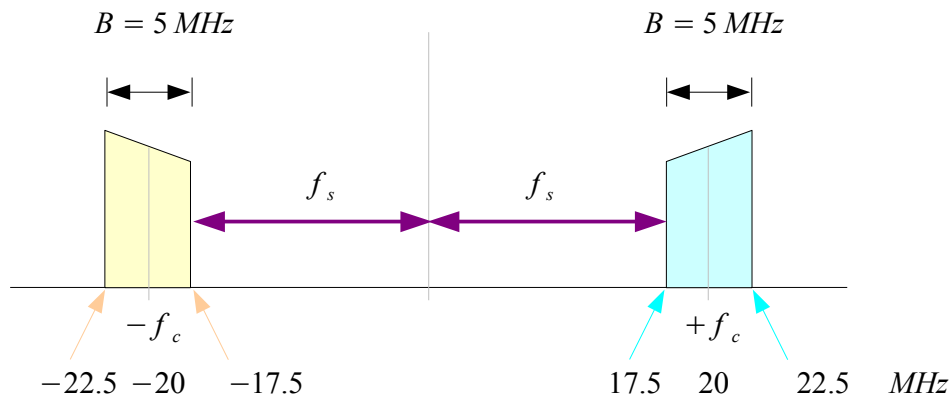
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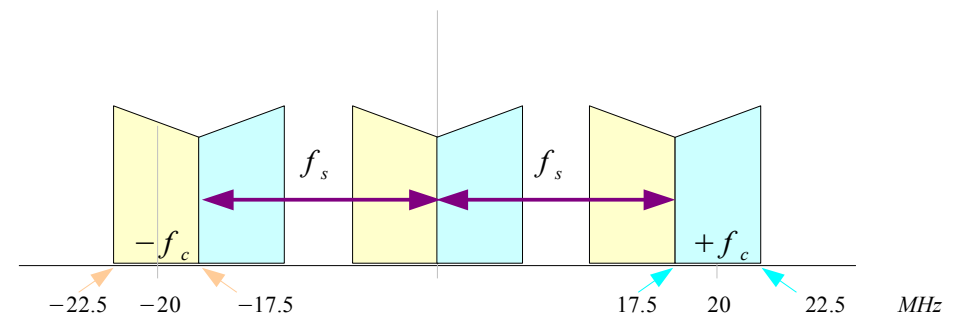
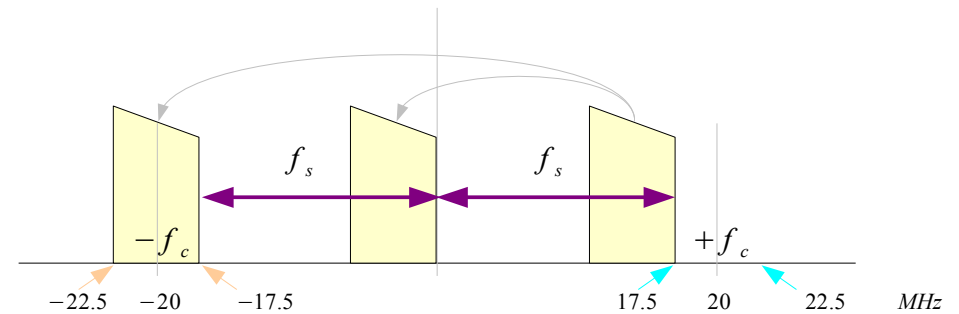
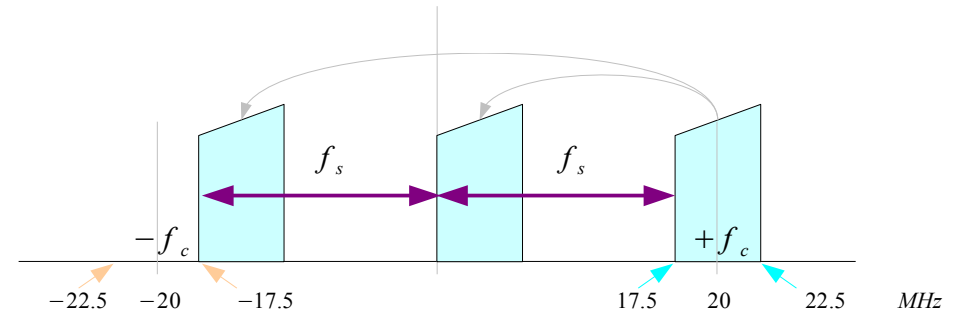
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# Band-limited Signal

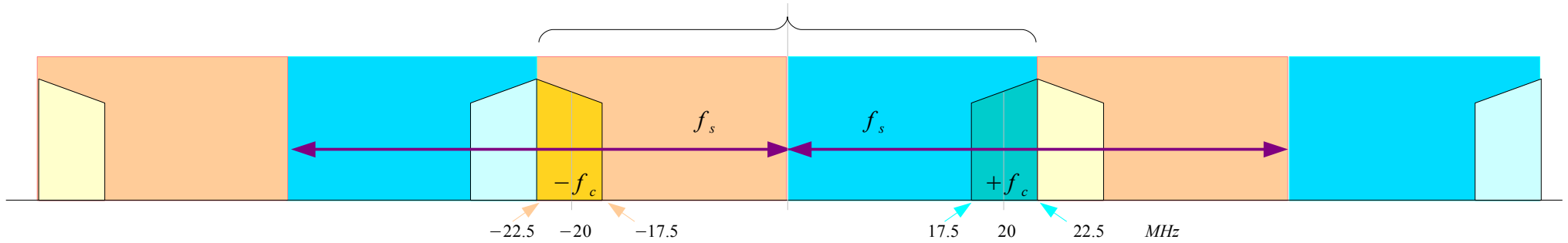


mirror

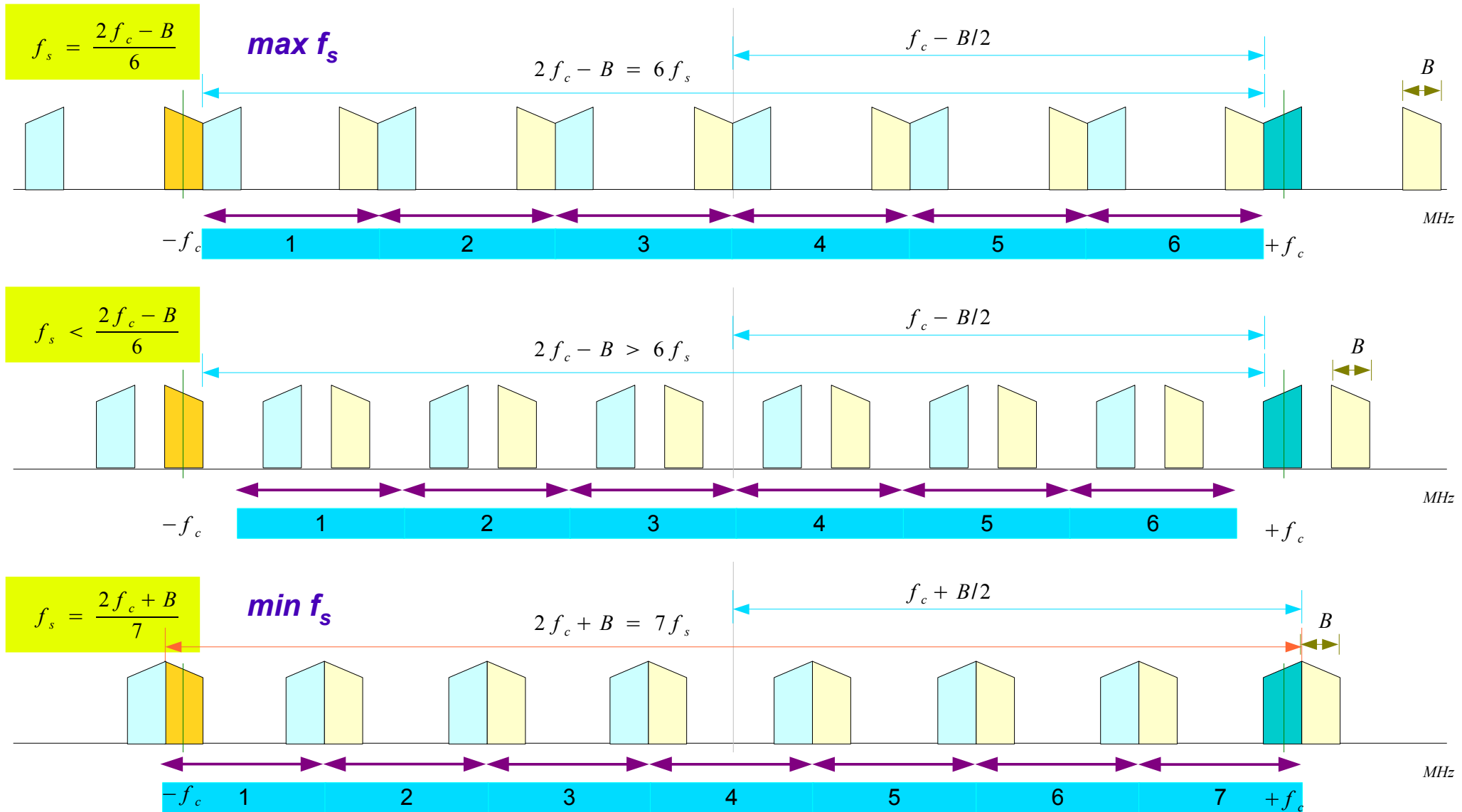
- **Bandpass Sampling**
- **IF filtering**
- **Harmonic Sampling**
- **Sub-Nyquist Sampling**



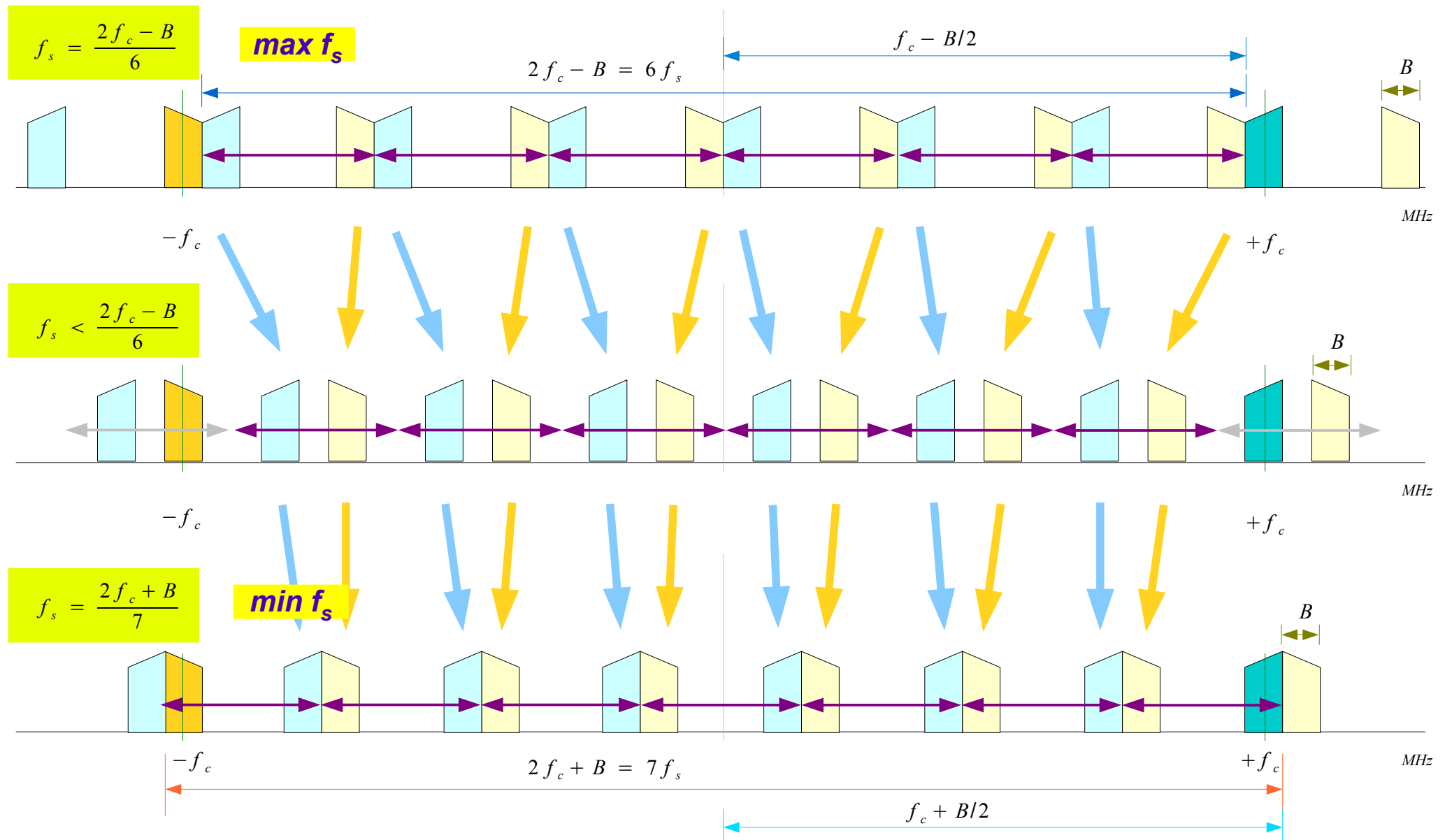
# Low-pass Signal Sampling



# Band-pass Signal Sampling



# Band-pass Signal Sampling



$$\frac{2f_c + B}{m + 1} \leq f_s \leq \frac{2f_c - B}{m}$$

$$f_c = 20 \text{ MHz}$$

$$B = 5 \text{ MHz}$$

$$2B \leq f_s$$

$$\frac{2 \cdot 20 + 5}{1 + 1} = 22.5 \leq f_s \leq \frac{2 \cdot 20 - 5}{1} = 35 \quad \Rightarrow \quad f_s = 22.5 \text{ MHz} \quad (10 \leq f_s)$$

$$\frac{2 \cdot 20 + 5}{2 + 1} = 15 \leq f_s \leq \frac{2 \cdot 20 - 5}{2} = 17.5 \quad \Rightarrow \quad f_s = 17.5 \text{ MHz} \quad (10 \leq f_s)$$

$$\frac{2 \cdot 20 + 5}{3 + 1} = 11.25 \leq f_s \leq \frac{2 \cdot 20 - 5}{3} = 11.67 \quad \Rightarrow \quad f_s = 11.25 \text{ MHz} \quad (10 \leq f_s)$$

$$\frac{2 \cdot 20 + 5}{4 + 1} = 9 \geq \frac{2 \cdot 20 - 5}{4} = 8.75 \quad \Rightarrow \quad \text{X}$$

$$\frac{2 \cdot 20 + 5}{5 + 1} = 7.5 \geq \frac{2 \cdot 20 - 5}{5} = 7.0 \quad \Rightarrow \quad \text{X}$$

$$\frac{2f_c + B}{m + 1} \leq f_s \leq \frac{2f_c - B}{m}$$

$$f_c = 20 \text{ MHz}$$

$$B = 5 \text{ MHz}$$

$$2B \leq f_s$$

$$\frac{f_c + B/2}{B} = R \quad \frac{\text{highest signal frequency}}{\text{bandwidth}}$$

$$\frac{2f_c + B}{(m + 1)B} = f(m, R) \quad \frac{\text{minimum sampling rate}}{\text{bandwidth}}$$

$$\frac{2(f_c + B/2)}{(m + 1)B} = \frac{2R}{m + 1} = f(m, R)$$

$$m = 1 \quad f(1, R) = R$$

$$m = 2 \quad f(2, R) = \frac{2}{3}R$$

$$m = 3 \quad f(3, R) = \frac{1}{2}R$$

$$m = 4 \quad f(4, R) = \frac{2}{5}R$$

$$m = 5 \quad f(5, R) = \frac{1}{3}R$$

$$m = 6 \quad f(6, R) = \frac{2}{7}R$$

$$m = 7 \quad f(7, R) = \frac{1}{4}R$$

$$m = 8 \quad f(8, R) = \frac{2}{9}R$$



$$\frac{2f_c + B}{m + 1} \leq f_s \leq \frac{2f_c - B}{m}$$

$$f_c = 20 \text{ MHz}$$

$$B = 5 \text{ MHz}$$

$$2B \leq f_s$$

$$\frac{f_c + B/2}{B} = R$$

**highest signal frequency**  
**bandwidth**

$$f_H = f_c + B/2$$

$$R = f_H / B$$

$$\frac{2f_c + B}{m + 1} \cdot \frac{1}{B} = f(m, R)$$

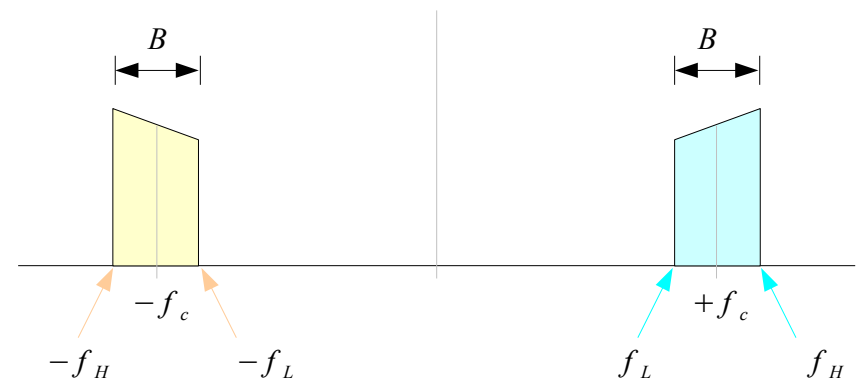
**minimum sampling rate**  
**bandwidth**

$$f_{s, \min} = \frac{2f_c + B}{m + 1} = \frac{2f_H}{k}$$

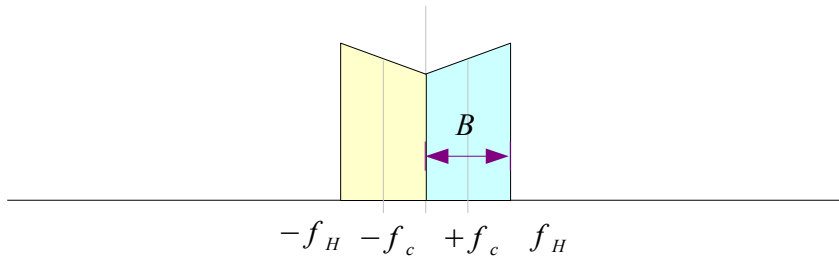
$$\frac{2(f_c + B/2)}{(m + 1)B} = \frac{2R}{m + 1} = f(m, R)$$

$$f(m, R) = \frac{2f_H}{kB} = \frac{2R}{k}$$

$$m + 1 = k$$



$$k = 1 \quad (m = 0)$$

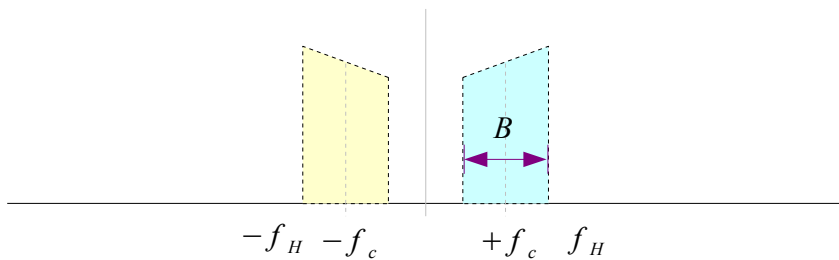


$$f_H = f_c + B/2 = 1B$$

$$f_H / B = R = 1$$

$$f_{s,\min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = 2B$$

$$k = 1 \quad (m = 0)$$

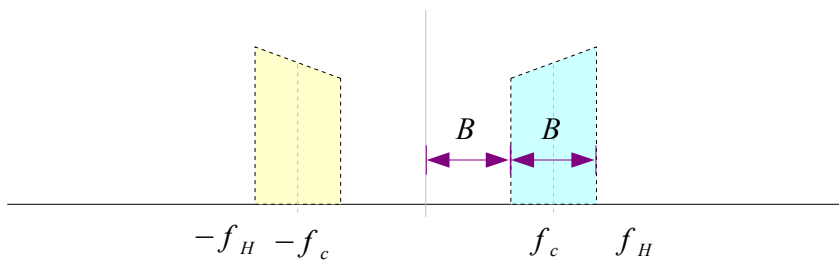


$$f_H = f_c + B/2 = 1.5B$$

$$f_H / B = R = 1.5$$

$$f_{s,\min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = 3B$$

$$k = 1 \quad (m = 0)$$

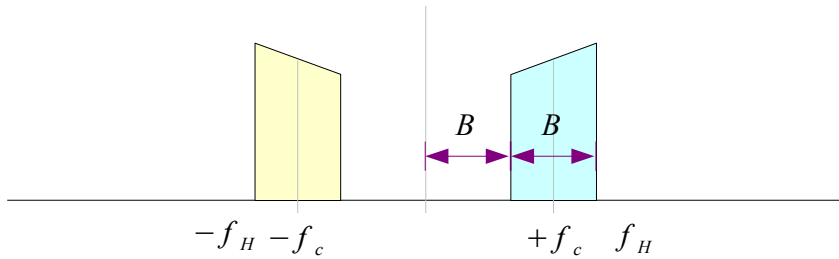


$$f_H = f_c + B/2 = 2B$$

$$f_H / B = R = 2$$

$$f_{s,\min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = 4B$$

$$k = 2 \quad (m = 1)$$

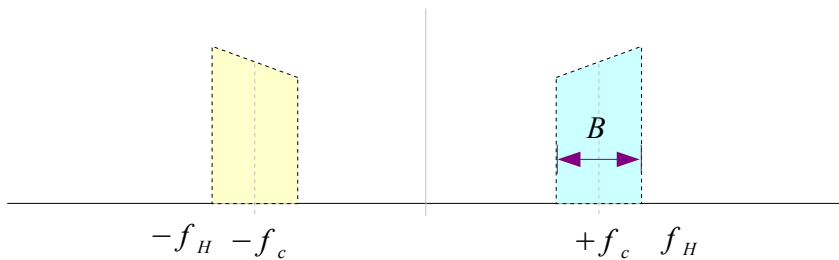


$$f_H = f_c + B/2 = 2B$$

$$f_H / B = R = 2$$

$$f_{s,\min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = 2B$$

$$k = 2 \quad (m = 1)$$

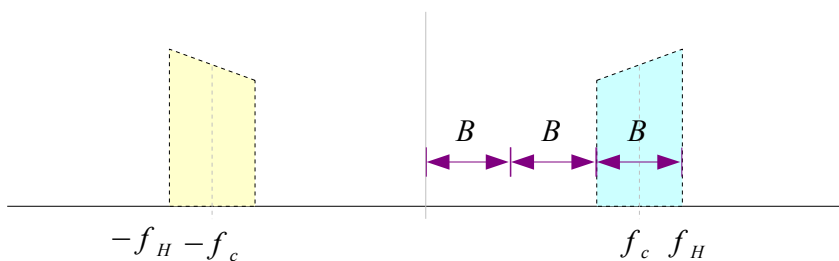


$$f_H = f_c + B/2 = 2.5B$$

$$f_H / B = R = 2.5$$

$$f_{s,\min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = 2.5B$$

$$k = 2 \quad (m = 3)$$

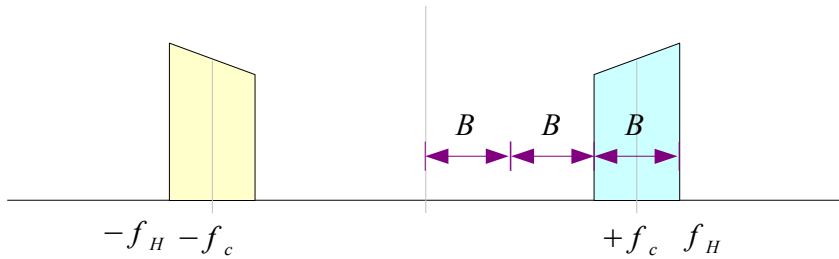


$$f_H = f_c + B/2 = 3B$$

$$f_H / B = R = 3$$

$$f_{s,\min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = 3B$$

$$k = 3 \quad (m = 2)$$

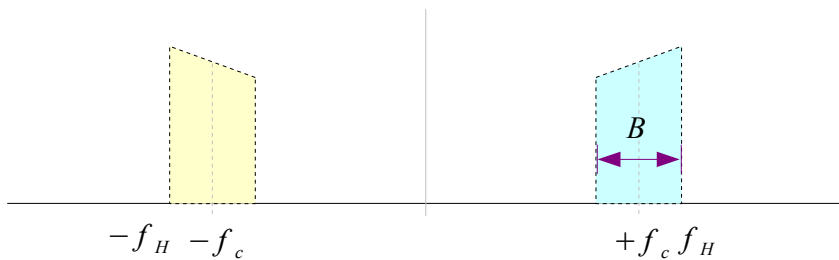


$$f_H = f_c + B/2 = 3B$$

$$f_H / B = R = 3$$

$$f_{s, \min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = 2B$$

$$k = 3 \quad (m = 2)$$

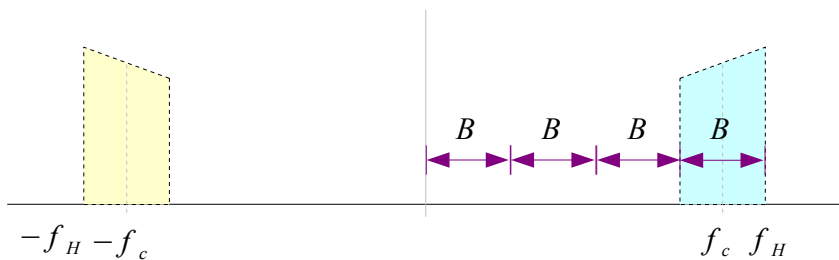


$$f_H = f_c + B/2 = 3.5B$$

$$f_H / B = R = 3.5$$

$$f_{s, \min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = \frac{7}{3}B$$

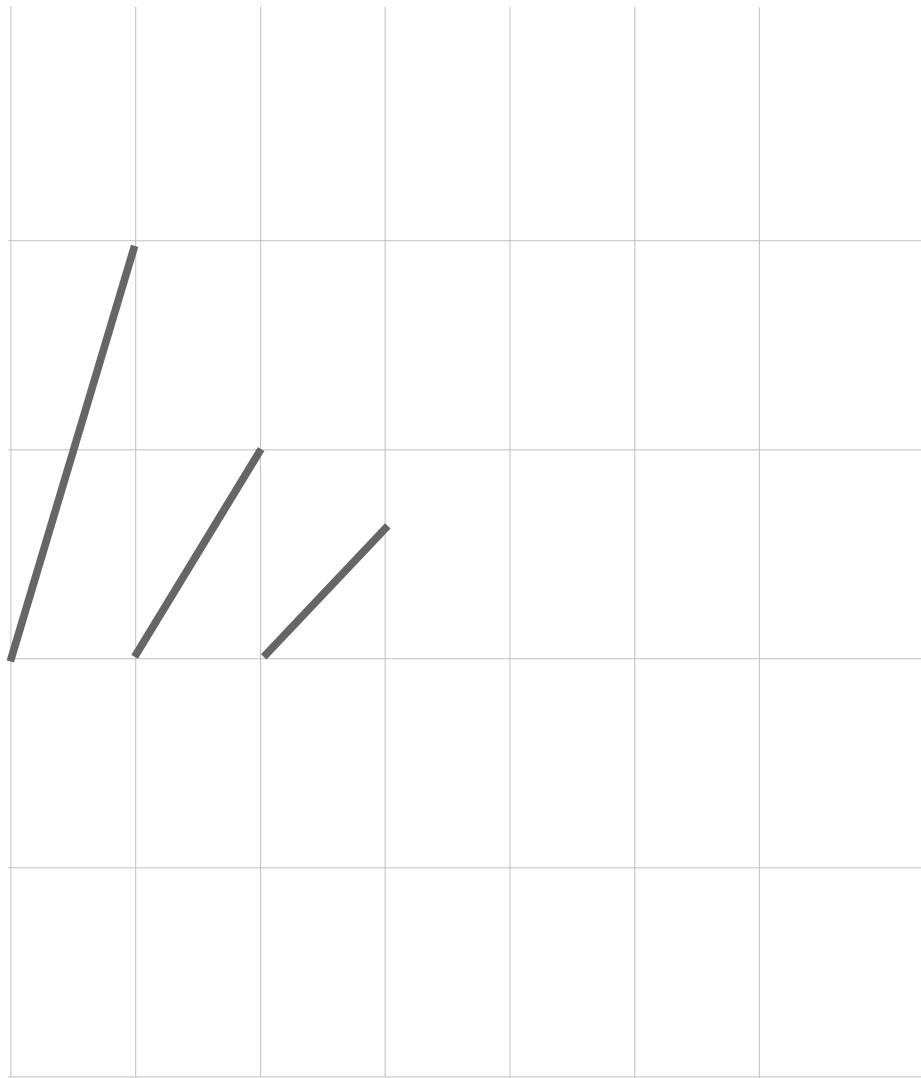
$$k = 3 \quad (m = 2)$$



$$f_H = f_c + B/2 = 4B$$

$$f_H / B = R = 4$$

$$f_{s, \min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = \frac{8}{3}B$$





## References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] A "graphical interpretation" of the DFT and FFT, by Steve Mann
- [4] R. G. Lyons, Understanding Digital Signal Processing, 1997