

Time Domain Techniques (3B)

for Noisy Signals

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Signal Detrending / Spike Removal

Remove low frequency noise

a constant
a line
a long period sinusoid

least square fitting a low frequency function $tr[i]$

subtracting the trend from the measurements $y[i] = x[i] - tr[i]$

example)

removing the dc component

$$y[i] = x[i] - tr[i] = x[i] - \frac{1}{N} \sum_{i=0}^{N-1} x[i-1]$$

removing spikes

$$y[i] = x[i] \quad \text{if } |x[i] - y[i-1]| < \text{threshold}$$

$$y[i] = \frac{x[i-1] + x[i+1]}{2} \quad \text{else}$$

Stacking

Remove background noise

Assume noise has a zero mean

Measuring multiple times (ensemble)

Averaging across the ensemble

Reduces the noise level in the averaged signal
Increases the SNR of the correlated component (signal)

$$E[X[i]] \rightarrow x[i]$$

$$E[N[i]] \rightarrow 0$$

$$E[N^2[i]] \rightarrow \frac{\sigma_n}{\sqrt{M}} \quad SNR \propto \sqrt{M}$$

Moving Average Filter

L-point Running Averager

$$y[n] = \frac{1}{L} \sum_{k=0}^{L-1} x[n-k] = \frac{1}{L} \{x[n] + x[n-1] + \dots + x[n-L+1]\}$$

$$Y(e^{j\hat{\omega}}) = \frac{1}{L} \sum_{k=0}^{L-1} e^{-j\hat{\omega}k} X(e^{j\hat{\omega}}) = \frac{1}{L} \{X(e^{j\hat{\omega}}) + X(e^{j\hat{\omega}})e^{-j\hat{\omega}1} + \dots + X(e^{j\hat{\omega}})e^{-j\hat{\omega}(L-1)}\}$$

$$H(e^{j\hat{\omega}}) = \frac{1}{L} \sum_{k=0}^{L-1} e^{j\hat{\omega}k}$$

$$\begin{aligned} &= \frac{1}{L} \{1 + e^{-j\hat{\omega}1} + \dots + e^{-j\hat{\omega}(L-1)}\} = \frac{1}{L} \left(\frac{1 - e^{-j\hat{\omega}L}}{1 - e^{-j\hat{\omega}}} \right) \\ &= \frac{1}{L} \left(\frac{e^{-j\hat{\omega}L/2}}{e^{-j\hat{\omega}/2}} \right) \left(\frac{e^{+j\hat{\omega}L/2} - e^{-j\hat{\omega}L/2}}{e^{+j\hat{\omega}/2} - e^{-j\hat{\omega}/2}} \right) = \frac{1}{L} \left(\frac{e^{+j\hat{\omega}L/2} - e^{-j\hat{\omega}L/2}}{e^{+j\hat{\omega}/2} - e^{-j\hat{\omega}/2}} \right) e^{-j\hat{\omega}(L-1)/2} \\ &= \left(\frac{\sin(\hat{\omega}L/2)}{L \sin(\hat{\omega}/2)} \right) e^{-j\hat{\omega}(L-1)/2} \end{aligned}$$

Dirichlet Function

$$H(e^{j\hat{\omega}}) = D_L(e^{j\hat{\omega}}) e^{-j\hat{\omega}(L-1)/2}$$

$$D_L(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}L/2)}{L \sin(\hat{\omega}/2)}$$

Moving Average Filter

Frequency Response

$$H(e^{j\hat{\omega}}) = D_L(e^{j\hat{\omega}}) e^{-j\hat{\omega}(L-1)/2}$$

Dirichlet Function

$$D_L(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega} L/2)}{L \sin(\hat{\omega}/2)}$$

$$\begin{aligned} D_L(e^{j(\hat{\omega} + 2\pi)}) &= \frac{\sin((\hat{\omega} + 2\pi)L/2)}{L \sin((\hat{\omega} + 2\pi)/2)} \\ &= \frac{\sin(\hat{\omega} L/2 + L\pi)}{L \sin(\hat{\omega}/2 + \pi)} \end{aligned}$$

$$\begin{cases} +D_L(e^{j\hat{\omega}}) & \text{for an odd } L \quad (\text{period: } 2\pi) \\ -D_L(e^{j\hat{\omega}}) & \text{for an even } L \end{cases}$$

$$0 \leq \hat{\omega} \leq +\pi$$

$$0 \leq \hat{\omega}/2 \leq +\frac{\pi}{2}$$

$$0 \leq \sin(\hat{\omega}/2) \leq +1$$

a quarter period

Envelope: $\frac{1}{\sin(\hat{\omega}/2)}$

$$0 \leq \hat{\omega} L/2 \leq +L \frac{\pi}{2}$$

$$-1 \leq \sin(\hat{\omega} L/2) \leq +1$$

L quarter periods

Zeros: $\hat{\omega} = \frac{2\pi}{L} k$

$$\sin(\hat{\omega} L/2) = 0$$

$$D_L(e^{-j\hat{\omega}}) = \frac{\sin(-\hat{\omega} L/2)}{L \sin(-\hat{\omega}/2)} = D_L(e^{j\hat{\omega}})$$

an even function

$$\lim_{\hat{\omega} \rightarrow 0} D_L(e^{j\hat{\omega}}) = \lim_{\hat{\omega} \rightarrow 0} \frac{L/2 \cos(\hat{\omega} L/2)}{L/2 \cos(\hat{\omega}/2)} = 1$$

max value $D_L(e^{j\hat{\omega}}) = 1$ when $\hat{\omega} = 0$

Correlation: Identifying Similarities

References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] J.C. Santamarina, D Fratta, "Discrete Signals and Inverse Problems: An Introduction for Engineers and Scientists", 2005