Time Domain Techniques (3B)

for Noisy Signals

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Signal Detrending / Spike Removal

Remove low frequency noise

a constant
a line
a long period sinusoid

least square fitting a low frequency function $tr[i]$

subtracting the trend from the measurements $y[i] = x[i] - tr[i]$

example)

removing the dc component

$$y[i] = x[i] - tr[i] = x[i] - \frac{1}{N}\sum_{i=0}^{N-1} x[i-1]$$

removing spikes

$$y[i] = x[i] \qquad if \ \left| x[i] - y[i-1] \right| < threshold$$

$$y[i] = \frac{x[i-1] + x[i+1]}{2} \qquad else$$

Stacking

Remove background noise

Assume noise has a zero mean

Measuring multiple times (ensemble)

Averaging across the ensemble

Reduces the noise level in the averaged signal
Increases the SNR of the correlated component (signal)

$$\boldsymbol{E}[X[i]] \rightarrow x[i]$$

$$\boldsymbol{E}[N[i]] \rightarrow 0$$

$$\boldsymbol{E}[N^2[i]] \rightarrow \frac{\sigma_n}{\sqrt{M}}$$

$$SNR \propto \sqrt{M}$$

Moving Average Filter

L-point Running Averager

$$y[n] = \frac{1}{L}\sum_{k=0}^{L-1} x[n-k] \qquad = \frac{1}{L}\{x[n] + x[n-1] + \cdots + x[n-L+1]\}$$

$$Y(e^{j\hat{\omega}}) = \frac{1}{L}\sum_{k=0}^{L-1} e^{-j\hat{\omega}k}\, X(e^{j\hat{\omega}}) \qquad = \frac{1}{L}\{X(e^{j\hat{\omega}}) + X(e^{j\hat{\omega}})e^{-j\hat{\omega}1} + \cdots + X(e^{j\hat{\omega}})e^{-j\hat{\omega}(L-1)}\}$$

$$H(e^{j\hat{\omega}}) = \frac{1}{L}\sum_{k=0}^{L-1} e^{j\hat{\omega}k} \qquad = \frac{1}{L}\{1 + e^{-j\hat{\omega}1} + \cdots + e^{-j\hat{\omega}(L-1)}\} \qquad = \frac{1}{L}\left(\frac{1-e^{-j\hat{\omega}L}}{1-e^{-j\hat{\omega}}}\right)$$

$$= \frac{1}{L}\left(\frac{e^{-j\hat{\omega}L/2}}{e^{-j\hat{\omega}/2}}\right)\left(\frac{e^{+j\hat{\omega}L/2} - e^{-j\hat{\omega}L/2}}{e^{+j\hat{\omega}/2} - e^{-j\hat{\omega}/2}}\right) \qquad = \frac{1}{L}\left(\frac{e^{+j\hat{\omega}L/2} - e^{-j\hat{\omega}L/2}}{e^{+j\hat{\omega}/2} - e^{-j\hat{\omega}/2}}\right) e^{-j\hat{\omega}(L-1)/2}$$

$$= \left(\frac{\sin(\hat{\omega}L/2)}{L\sin(\hat{\omega}/2)}\right) e^{-j\hat{\omega}(L-1)/2}$$

Dirichlet Function

$$H(e^{j\hat{\omega}}) = D_L(e^{j\hat{\omega}})\, e^{-j\hat{\omega}(L-1)/2}$$

$$D_L(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}L/2)}{L\sin(\hat{\omega}/2)}$$

Moving Average Filter

Frequency Response

$$H(e^{j\hat{\omega}}) = D_L(e^{j\hat{\omega}})\, e^{-j\hat{\omega}(L-1)/2}$$

Dirichlet Function

$$D_L(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}L/2)}{L\sin(\hat{\omega}/2)}$$

$$D_L(e^{j(\hat{\omega}+2\pi)}) = \frac{\sin((\hat{\omega}+2\pi)L/2)}{L\sin((\hat{\omega}+2\pi)/2)} = \frac{\sin(\hat{\omega}L/2 + L\pi)}{L\sin(\hat{\omega}/2+\pi)}$$

$$\begin{cases} +D_L(e^{j\hat{\omega}}) & \text{for an odd } L \\ -D_L(e^{j\hat{\omega}}) & \text{for an even } L \end{cases}$$

(period: 2π)

$0 \le \hat{\omega} \le +\pi$

$0 \le \hat{\omega}/2 \le +\frac{\pi}{2}$ $\quad$ $0 \le \hat{\omega}L/2 \le +L\frac{\pi}{2}$

$0 \le \sin(\hat{\omega}/2) \le +1$ $\quad$ $-1 \le \sin(\hat{\omega}L/2) \le +1$

a quarter period $\quad$ L quarter periods

Envelope: $\frac{1}{\sin(\hat{\omega}/2)}$ $\quad$ Zeros: $\hat{\omega} = \frac{2\pi}{L}k$

$\sin(\hat{\omega}L/2) = 0$

$$D_L(e^{-j\hat{\omega}}) = \frac{\sin(-\hat{\omega}L/2)}{L\sin(-\hat{\omega}/2)} = D_L(e^{j\hat{\omega}})$$

an even function

$$\lim_{\hat{\omega}\to 0} D_L(e^{j\hat{\omega}}) = \lim_{\hat{\omega}\to 0} \frac{L/2\cos(\hat{\omega}L/2)}{L/2\cos(\hat{\omega}/2)} = 1$$

max value $D_L(e^{j\hat{\omega}}) = 1$ *when* $\hat{\omega} = 0$

Correlation: Identifying Similarities

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] J.C. Santamarina, D Fratta, “Discrete Signals and Inverse Problems: An Introduction for Engineers and Scientists”, 2005