DFT Frequency (4A)

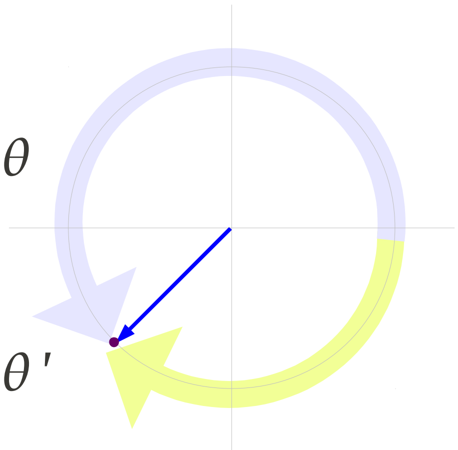
- Negative Frequency
- Angular Frequency
- Fundamental Frequency
- Harmonic Frequency
- Sampling Frequency
- Normalized Frequency

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice and Octave.

Negative Frequency

c.c.w	θ	**c.c.w**	$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$
c.w	θ'	**c.w**	$e^{-j\omega t} = \cos(\omega t) - j\sin(\omega t)$



Angle
- **c.c.w (+)** **Positive Angle** $\theta = \omega t$ $(\theta > 0)$
- **c.w (–)** **Negative Angle** $\theta' = \omega' t$ $(\theta' < 0)$

Angular Speed
- **c.c.w (+)** **Positive Frequency** $\omega = +\frac{2\pi}{T}$ $(\omega > 0)$
- **c.w (–)** **Negative Frequency** $\omega' = -\frac{2\pi}{T}$ $(\omega' < 0)$

Young Won Lim
1/29/11

Angular Frequency

Frequency $f = \frac{1}{T}$ (Hz: cycles per second)

1Hz → event repeats once per second

Angular Frequency $\omega = \frac{2\pi}{T}$ (radians per second)

One revolution = 2π radian

$$\omega = 2\pi f = 2\pi \frac{1}{T}$$ → **Angular Speed**

$$\theta = \omega t = 2\pi f t$$ → **Phase**

DFT Matrix

$$
\begin{bmatrix}
W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\
W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\
W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^0 & W_8^2 & W_8^4 & W_8^6 \\
W_8^0 & W_8^3 & W_8^6 & W_8^1 & W_8^4 & W_8^7 & W_8^2 & W_8^5 \\
W_8^0 & W_8^4 & W_8^0 & W_8^4 & W_8^0 & W_8^4 & W_8^0 & W_8^4 \\
W_8^0 & W_8^{-3} & W_8^{-6} & W_8^{-1} & W_8^{-4} & W_8^{-7} & W_8^{-2} & W_8^{-5} \\
W_8^0 & W_8^{-2} & W_8^{-4} & W_8^{-6} & W_8^0 & W_8^{-2} & W_8^{-4} & W_8^{-6} \\
W_8^0 & W_8^{-1} & W_8^{-2} & W_8^{-3} & W_8^{-4} & W_8^{-5} & W_8^{-6} & W_8^{-7}
\end{bmatrix}
$$

** still symmetric matrix*

c.c.w

c.w

Exponents and Strides

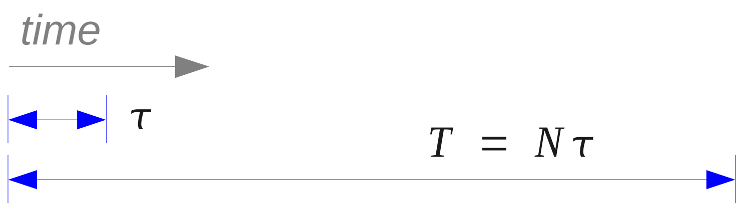
	n=0		n=1		n=2		n=3		n=4		n=5		n=6		n=7
k=0	0	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0
k=1	0	-1	-1	-1	-2	-1	-3	-1	-4	-1	-5	-1	-6	-1	-7
k=2	0	-2	-2	-2	-4	-2	-6	-2	0	-2	-2	-2	-4	-2	-6
k=3	0	-3	-3	-3	-6	-3	-1	-3	-4	-3	-7	-3	-2	-3	-5
k=4	0	-4	-4	-4	0	-4	-4	-4	0	-4	-4	-4	0	-4	-4
k=5	0	+3	+3	+3	+6	+3	+1	+3	+4	+3	+7	+3	+2	+3	+5
k=6	0	+2	+2	+2	+4	+2	+6	+2	0	+2	+2	+2	+4	+2	+6
k=7	0	+1	+1	+1	+2	+1	+3	+1	+4	+1	+5	+1	+6	+1	+7

$$W_N^{nk \pm N} = W_N^{nk}$$

$$W_8^{nk} = e^{-j\left(\frac{2\pi}{8}\right)nk}$$

Fundamental and Harmonic Frequencies

	n=0		n=1		n=2		n=3		n=4		n=5		n=6		n=7
k=0	0	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0
k=1	0	-1	-1	-1	-2	-1	-3	-1	-4	-1	-5	-1	-6	-1	-7
k=2	0	-2	-2	-2	-4	-2	-6	-2	0	-2	-2	-2	-4	-2	-6
k=3	0	-3	-3	-3	-6	-3	-1	-3	-4	-3	-7	-3	-2	-3	-5
k=4	0	-4	-4	-4	0	-4	-4	-4	0	-4	-4	-4	0	-4	-4
k=5	0	+3	+3	+3	+6	+3	+1	+3	+4	+3	+7	+3	+2	+3	+5
k=6	0	+2	+2	+2	+4	+2	+6	+2	0	+2	+2	+2	+4	+2	+6
k=7	0	+1	+1	+1	+2	+1	+3	+1	+4	+1	+5	+1	+6	+1	+7



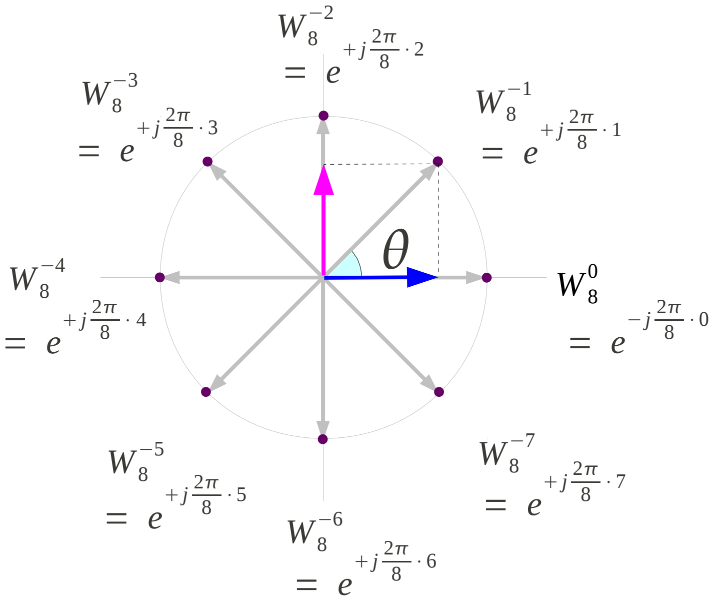
stride	angular speed		Measuring Frequency	Harmonic Frequency
0	cw	0	0	
-1	cw	-1ω	+1f	Fundamental → 1st harmonic
-2	cw	-2ω	+2f	2nd harmonic
-3	cw	-3ω	+3f	3rd harmonic
-4	cw	-4ω	+4f	4th harmonic
+3	ccw	$+3\omega$	-3f	5th harmonic
+2	ccw	$+2\omega$	-2f	6th harmonic
+1	ccw	$+1\omega$	-1f	7th harmonic

Fundamental Frequency f_0

Harmonic Frequency

$$f_k = k \cdot f_0 \qquad k = 1, 2, 3, \cdots$$

Sampling Frequency



Sampling Time τ

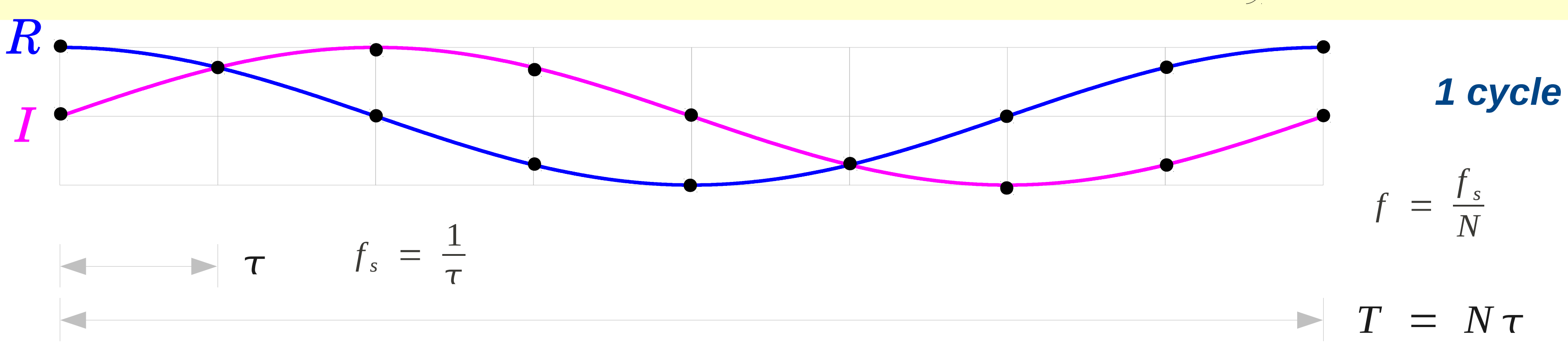
Sampling Frequency $f_s = \frac{1}{\tau}$ *(samples per second)*

Period $T = N\tau$

Fundamental Freq $f = \frac{1}{T}$ *(cycles per second)*

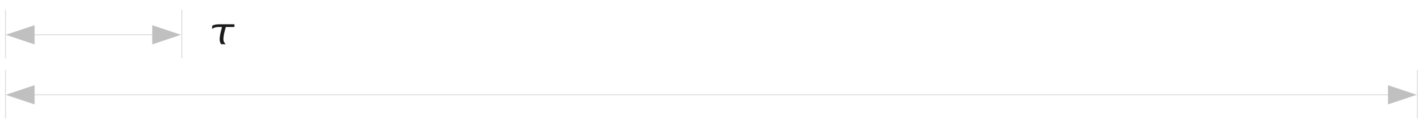
$$f = \frac{f_s}{N} \left(= \frac{1}{N\tau} \right)$$

$$\left(e^{+j\cdot\frac{\pi}{4}\cdot 0} \quad e^{+j\cdot\frac{\pi}{4}\cdot 1} \quad e^{+j\cdot\frac{\pi}{4}\cdot 2} \quad e^{+j\cdot\frac{\pi}{4}\cdot 3} \quad e^{+j\cdot\frac{\pi}{4}\cdot 4} \quad e^{+j\cdot\frac{\pi}{4}\cdot 5} \quad e^{+j\cdot\frac{\pi}{4}\cdot 6} \quad e^{+j\cdot\frac{\pi}{4}\cdot 7} \right)$$



Young Won Lim
1/29/11

Normalized Frequency



Sampling Time τ

Sequence Time Length $T = N\tau$

Sampling Frequency $f_s = \dfrac{1}{\tau}$ *(samples per second)*

Normalized Frequency

$$\frac{f_n}{f_s} = \frac{n}{N}$$ *(cycles per sample)*

Fundamental Frequency $f_0 = \dfrac{1}{T} = \dfrac{1}{N\tau}$

Harmonic Frequencies

$$f_1 = 1 \cdot f_0$$
$$f_2 = 2 \cdot f_0$$
$$f_3 = 3 \cdot f_0$$
$$\cdots$$
$$f_{N-1} = (N-1) \cdot f_0$$

Normalized Frequencies

$$1/N$$
$$2/N$$
$$3/N$$
$$\cdots$$
$$(N-1)/N$$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A "graphical interpretation" of the DFT and FFT, by Steve Mann