Report 3 Problem Set (from Section 7 Notes)

1) Consider the ODE y'' - 10y' + 25y = r(x) with the initial conditions y(0) = 4, y'(0) = -5. Let it have the following excitation $r(x) = 7e^{5x} - 2x^2$. Find the solution. Plot this solution and the solution from the example on page 7-3 of the notes.

2) Develop the 2nd homogeneous solution for the case of the double real root as a limiting case of distinct roots. Consider two distinct real roots in the form of:

   $\lambda_1 = \lambda$ and $\lambda_2 = \lambda + \epsilon$ perturbation ϵ

   A. Find the homogeneous L2-ODE-CC having the above distinct roots.
   B. Show that the following is a homogeneous solution (See Sec. 7 notes for elaboration):

   $$\frac{e^{(\lambda+\epsilon)x} - e^{\lambda x}}{\epsilon}$$

   C. Find the limit of the homogeneous solution from part 2 as $\epsilon \rightarrow 0$ (think l'Hopital's rule)
   D. Take the derivative of $e^{\lambda x}$ with respect to λ
   E. Compare the results of parts 3 and 4, and relate to the result by variation of parameters.
   F. Compute the homogeneous solution in 2 with the values of λ=5 and ϵ=.001, and compare to the value obtained from the exact 2nd homogeneous solution.

3) Find the complete solution for the equation y'' - 3y' + 2y = r(x) with an excitation of $r(x) = 4x^2$ and the initial conditions y(0) = 1, y'(0) = 0. Plot the solution y(x).

4) Use the Basic Rule 1 and the Sum Rule 3 to show that the appropriate particular solution for the equation y'' - 3y' + 2y = 4x^2 - 6x^5 is of the form below with n=5.

$$y_p(x) = \sum_{j=0}^{n} c_j x^j$$

5) Complete the solution of y" - 3y' + 2y = $4x^2 - 6x^5$ as follows:
   A. Obtain the coeffictions of the variables x, x^2, x^3, and x^5 as given in the notes on page 7-14.
   B. Verify all equations by long-hand expansion of the following series:

$$\sum_{j=2}^{5} c_j \cdot j \cdot (j-1) \cdot x^{j-2} - 3\sum_{j=1}^{5} c_j \cdot j \cdot x^{j-1} + 2\sum_{j=0}^{5} c_j x^j = 4x^2 - 6x^5$$

   C. Put the system of equations for $\{c_0,\ldots,c_5\}$ in matrix form
   D. Solve for the coefficients $\{c_0,\ldots,c_5\}$ by back substitution
   E. Consider the initial conditions y(0)=1, y'(0)=0. Find the solution y(x) and plot it.

6) Solve the equation y" - 3y' + 2y = $4x^2 - 6x^5$ with the initial conditions y(0)=1, y'(0)=0 as follows. Considering the following equations.

$$y''_{p,1} - 3y'_{p,1} + 2y_{p,1} = r_1(x) := 4x^2$$
$$y''_{p,2} - 3y'_{p,2} + 2y_{p,2} = r_2(x) := -6x^5$$

   The particular solution for $y_{p,1}$ was found in R3.3. Find the particular solution $y_{p,2}$ and then obtain the solution y for the equation and initial conditions given.

7) Expand the series on both sides of the following summations to verify these equalities. (Correction: in the first equation, n = 5)

$$\sum_{j=2}^{n} c_j j(j-1)x^{j-2} = \sum_{k=0}^{3} c_{k+2}(k+2)(k+1)x^k = \sum_{j=0}^{3} c_{j+2}(j+2)(j+1)x^j$$

$$\sum_{j=1}^{5} c_j \cdot j \cdot x^{j-1} = \sum_{k=0}^{4} c_{k+1}(k+1)x^k = \sum_{j=0}^{4} c_{j+1}(j+1)x^j$$

8) K2011 p.84 pbs 5, 6.
   Find a (real) general solution. State which rule you are using. Show each step of your work.

$$y'' + 3y' + 2y = e^{-x}cos(x)$$
$$y'' + y' + (\pi^2 + \frac{1}{4})y = e^{-x/2}sin(\pi x)$$

9) K2011 p.85 pbs 13, 14.
Solve the initial value problem. State which rule you are using. Show each step of your calculation in detail.

$$8y'' - 6y' + y = \cosh(x)) \quad y(0) = 0.2 \;\; y'(0) = 0.05$$

$$y'' + 4y' + 4y = e^{-2x} \sin(2x) \quad y(0) = 1 \;\; y'(0) = -1.5$$

10) Obtain equations (2), (3), (n-2), (n-1), (n), and set up the matrix **A** as in the notes on p.7-16 for the general case, with the matrix coefficients for rows 1, 2, 3, (n-2), (n-1), n filled in, as obtained from equations (1), (2), (3), (n-2), (n-1), (n).