

CORDIC in VHDL (1A)

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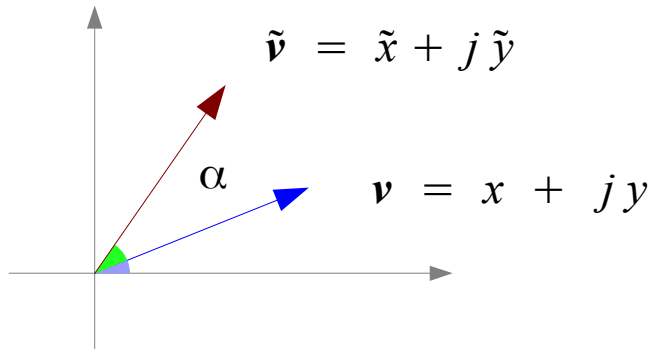
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CORDIC Background

1. G Hampson,
A VHDL Implementation of a CORDIC Arithmetic Processor Chip
Monash University, Technical Report 94-9, 1994

Angle Expansion

$$\tilde{\mathbf{v}} = \mathbf{v} e^{j\alpha}$$



elementary angle

$$\theta_0 = \tan^{-1}(2^0) =$$

$$\theta_1 = \tan^{-1}(2^{-1}) =$$

$$\theta_2 = \tan^{-1}(2^{-2}) =$$

$$\theta_3 = \tan^{-1}(2^{-3}) =$$

α can be expanded by
a set of elementary angles α_i
pseudo-digits q_i

$$\alpha_i \begin{cases} \pi/2 & i = -1 \\ \tan^{-1}(2^{-i}) & i = 0, 1, 2, \dots, n-1 \end{cases}$$

$$q_i \begin{cases} -1 \\ +1 \end{cases}$$

$$\alpha = \sum_{i=-1}^{n-1} q_i \cdot \alpha_i + z_n$$

angle expansion error z_n

$$|z_n| \leq 2^{-(n-1)}$$

Rotating Vector

$$\tilde{\mathbf{v}} = \mathbf{v} e^{j\alpha}$$

$$\alpha = \sum_{i=-1}^{n-1} q_i \cdot \alpha_i + z_n$$

$$= \mathbf{v} \exp\left(j \left(\sum_{i=-1}^{n-1} q_i \cdot \alpha_i + z_n \right)\right)$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$= \mathbf{v} \cdot \left(\prod_{i=-1}^{n-1} e^{jq_i \alpha_i} \right) \cdot e^{jz_n}$$

$$e^{jq_{-1}\alpha_0} = e^{jq_{-1}\frac{\pi}{2}} = e^{\pm j\frac{\pi}{2}} = \pm j = jq_{-1}$$

$$= \mathbf{v} \cdot (jq_{-1}) \left(\prod_{i=0}^{n-1} e^{jq_i \alpha_i} \right) \cdot e^{jz_n}$$

$$\begin{aligned} e^{jq_i \alpha_i} &= \cos(q_i \alpha_i) + j\sin(q_i \alpha_i) \\ &= \cos(q_i \alpha_i) \cdot (1 + j\tan(q_i \alpha_i)) \end{aligned}$$

$$= \mathbf{v} \cdot (jq_{-1}) \left(\prod_{i=0}^{n-1} \cos(\alpha_i) \cdot (1 + jq_i 2^{-i}) \right) \cdot e^{jz_n}$$

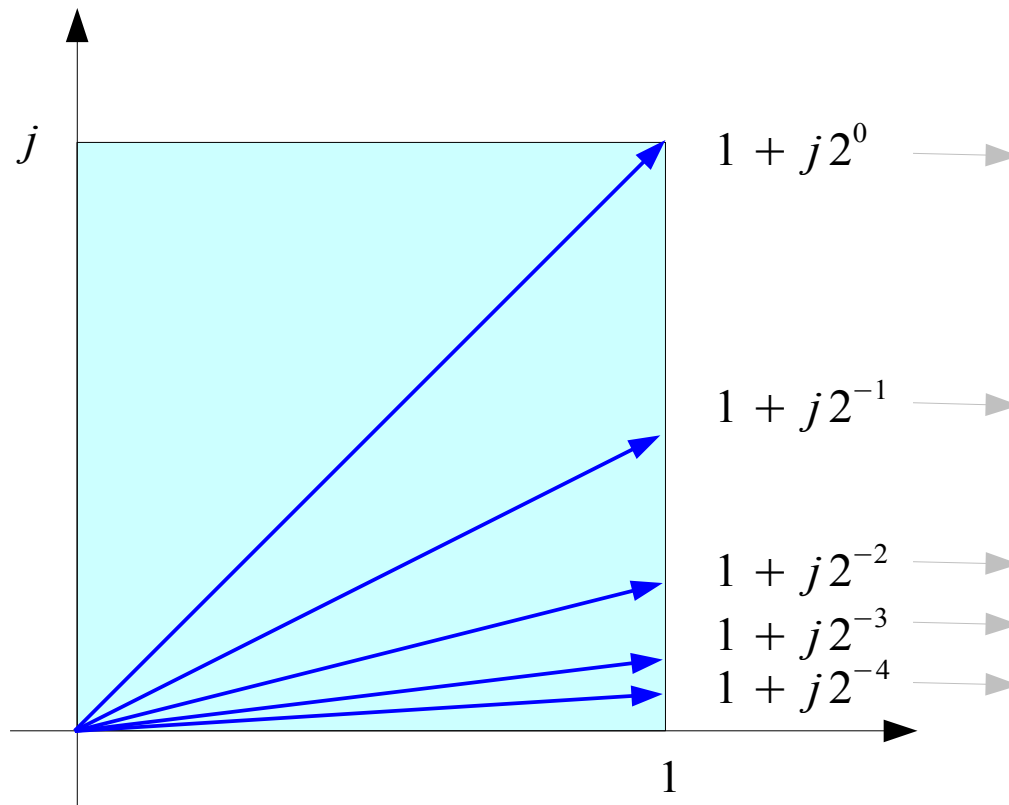
$$= \cos(q_i \alpha_i) \cdot (1 + jq_i 2^{-i})$$

$$= \cos(\alpha_i) \cdot (1 + jq_i 2^{-i})$$

$$= \mathbf{v} \cdot (jq_{-1}) \left(\prod_{i=0}^{n-1} \cos(\alpha_i) \right) \cdot \left(\prod_{i=0}^{n-1} (1 + jq_i 2^{-i}) \right) \cdot e^{jz_n}$$

$$\cos(\pm\alpha_i) = \cos(\alpha_i)$$

Elementary Angle: $\tan^{-1}(K)$



$$\theta_L = \tan^{-1}(2^{-L}) = \tan^{-1}(K)$$

$$\theta_0 = \tan^{-1}(2^0) = 45.00000$$

$$\theta_1 = \tan^{-1}(2^{-1}) = 26.56505$$

$$\theta_2 = \tan^{-1}(2^{-2}) = 14.03624$$

$$\theta_3 = \tan^{-1}(2^{-3}) = 7.12502$$

$$\theta_4 = \tan^{-1}(2^{-4}) = 3.57633$$

Represent arbitrary angle θ

in terms of $\pm\theta_0, \pm\theta_1, \pm\theta_2, \pm\theta_3, \dots, \pm\theta_L, \dots$ $\left(K = \frac{1}{2^L}, L = 0, 1, 2, \dots\right)$

Phase and Magnitude of $1 + jK$ (1)

Cumulative Magnitude

L	$K = \frac{1}{2^L}$	$R = 1 + jK$	Phase of R	Magnitude of R	CORDIC Gain
0	1.0	$1 + j1.0$	45°	1.41421356	1.414213562
1	0.5	$1 + j0.5$	26.56505°	1.11803399	1.581138830
2	0.25	$1 + j0.25$	14.03624°	1.03077641	1.629800601
3	0.125	$1 + j0.125$	7.12502°	1.00778222	1.642484066
4	0.0625	$1 + j0.0625$	3.57633°	1.00195122	1.645688916
5	0.03125	$1 + j0.03125$	1.78991°	1.00048816	1.646492279
6	0.015625	$1 + j0.015625$	0.89517°	1.00012206	1.646693254
7	0.007813	$1 + j0.007813$	0.44761°	1.00003052	1.646743507
...
					1.647 ←

$$R = 1 + jK \xrightarrow[L = 0, 1, 2, \dots]{K = 1/2^L} \sqrt{1^2 + K^2} > 1.0$$

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References

- [1] <http://en.wikipedia.org/>
- [2] G Hampson, A VHDL Implementation of a CORDIC Arithmetic Processor Chip
Monash University, Technical Report 94-9, 1994