# Sampling Basics(1A)

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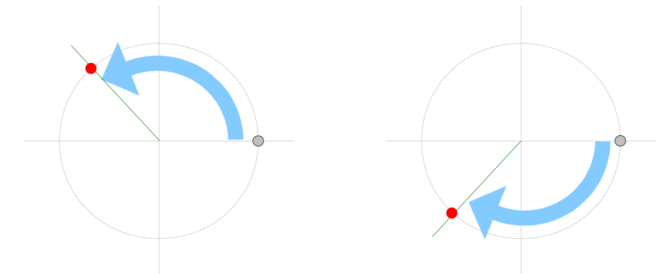
# Measuring Rotation Rate

## Angular Speed (Frequency)

$$\omega = \frac{2\pi}{T} \qquad \frac{\textit{angular displacement}}{\textit{time}}$$

$$\omega = 2\pi f \qquad 2\pi \times \textit{linear frequency}$$

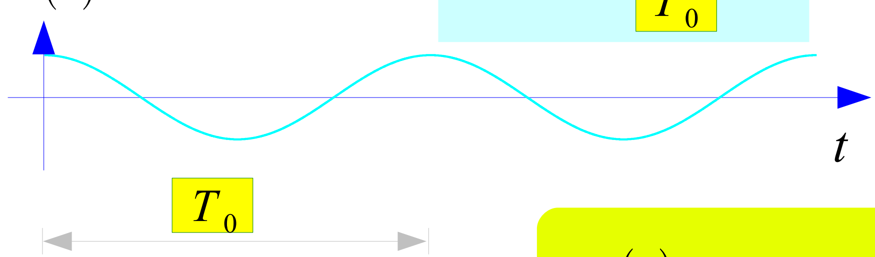
*For 1 second* *For 1 second*



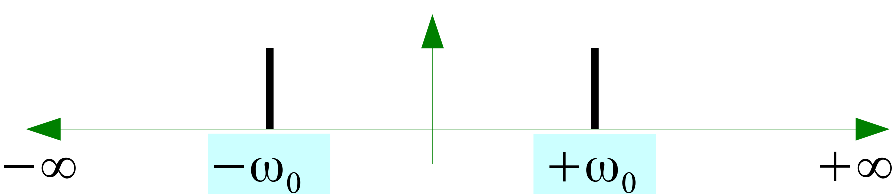
$+\omega_0 \ (rad/sec)$ $-\omega_0 \ (rad/sec)$

### Time Domain

$$\omega_0 = \frac{2\pi}{T_0}$$




### Frequency Domain




$$x(t) = A\cos(\omega_0 t) = \frac{A}{2}e^{j\omega_0 t} + \frac{A}{2}e^{-j\omega_0 t}$$



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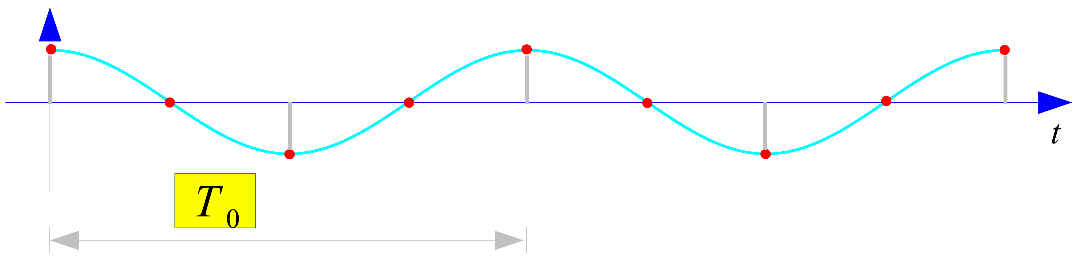
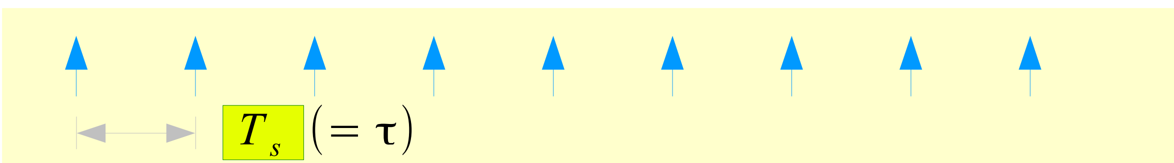
# Sampling

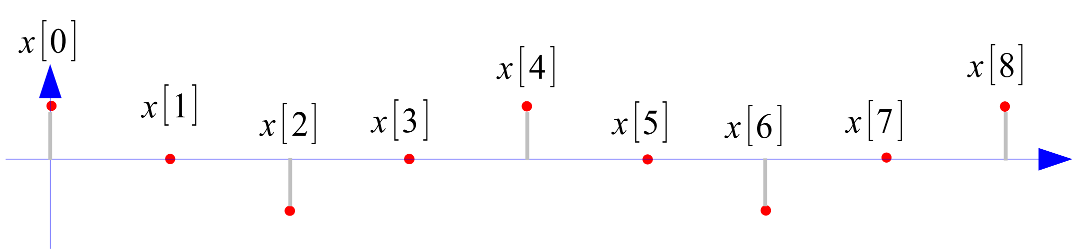
**continuous-time signals**

$$x(t) = A\cos(\omega_0 t)$$




**discrete-time sequence**




**Sampling Time**

$$T_s \quad (= \tau)$$

**Sequence Time Length**

$$T = N \cdot T_s$$

**Sampling Frequency**

$$\boxed{f_s = \frac{1}{T_s} \quad \text{(samples / sec)}}$$

**Signal's Frequency**

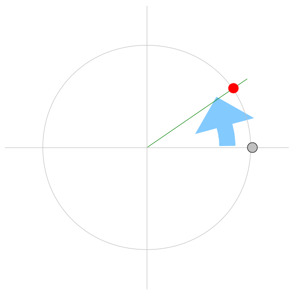
$$\boxed{f_0 = \frac{1}{T_0} \quad \text{(cycles / sec)}}$$

# Angular Frequencies in Sampling

## *continuous-time signals*

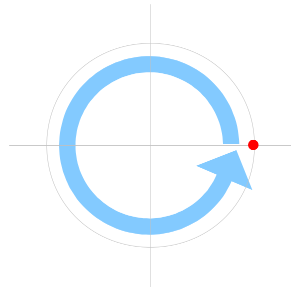
*For 1 second*

$\omega_0 = 2\pi f_0 \ (rad/sec)$

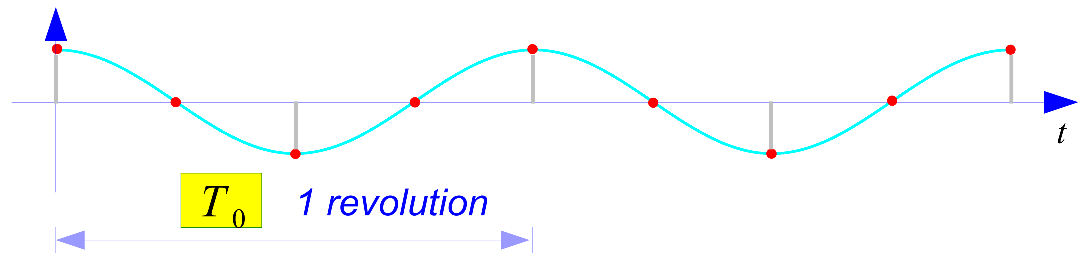


*For 1 revolution*

$2\pi \ (rad) \ / \ T_0 \ (sec)$



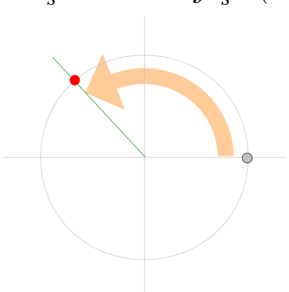
$x(t) = A \cos(\omega_0 t)$




## *sampling sequence*

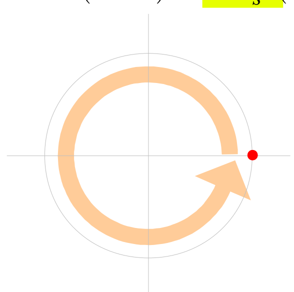
*For 1 second*

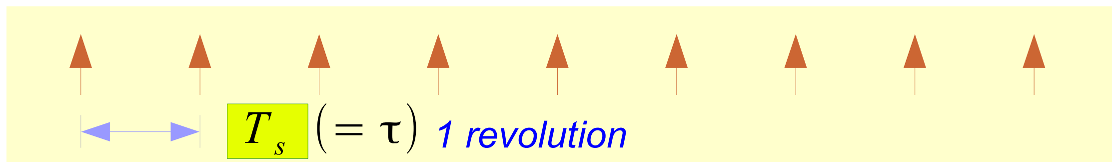
$\omega_s = 2\pi f_s \ (rad/sec)$



*For 1 revolution*

$2\pi \ (rad) \ / \ T_s \ (sec)$





# Sampling of Sinusoid Functions

$$x(t) = A\cos(\omega t + \phi)$$

$$t \rightarrow n\,T_s$$

$$x[n] = x(n\,T_s)$$

$$= A\cos(\omega \cdot n\,T_s + \phi)$$

$$= A\cos(\boxed{\omega \cdot T_s}\, n + \phi)$$

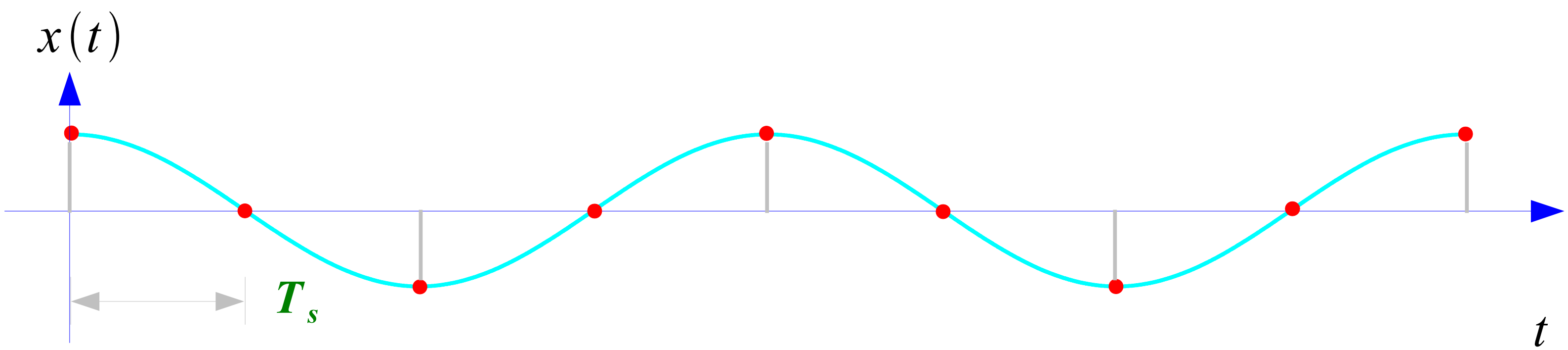
$$= A\cos(\boxed{\hat{\omega}} \cdot n + \phi)$$

$$\hat{\omega} = \omega \cdot T_s = \frac{\omega}{1/T_s}$$

$$\hat{\omega} = \frac{\omega}{\boxed{f_s}} = 2\pi \frac{f}{\boxed{f_s}}$$

Normalized to $f_s$

**Normalized Radian Frequency**



# Normalized Radian Frequency (1)

**continuous-time signals**

$x(t)$

**Sampling** $\longrightarrow$

$t \rightarrow n\,T_s$

**discrete-time sequence**

$x[n] = x(nT_s)$

**Angular Frequency**

$\omega \quad (rad\,/\,sec)$

**Sampling** $\longrightarrow$

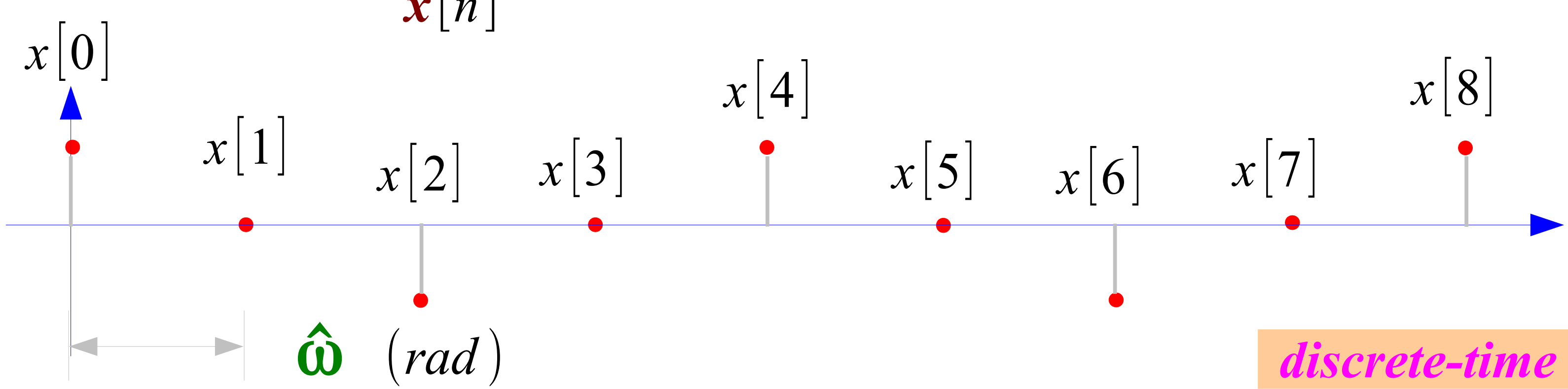
$\times\, T_s$

**Normalized Radian Frequency**

$\hat{\omega} = \omega \cdot T_s \quad (rad\,/\,sample)$

Angular Speed X Sampling Time

$\boldsymbol{x}[n]$




*discrete-time sequence*

# Normalized Radian Frequency (2)

## *Normalized Frequency*

$$\frac{f_0}{f_s} \quad \frac{(cycle\ /\ sec)}{(sample\ /\ sec)} \Rightarrow \frac{f_0}{f_s} \quad (cycle\ /\ sample)$$

## *Normalized Radian Frequency*

$$2\pi \frac{(rad)}{(cycle)} \cdot \frac{f_0}{f_s} \frac{(cycle)}{(sample)} \Rightarrow \frac{\omega_0}{f_s} \quad (rad\ /\ sample)$$

*Signal's relative angle position after each of $T_s$ second*

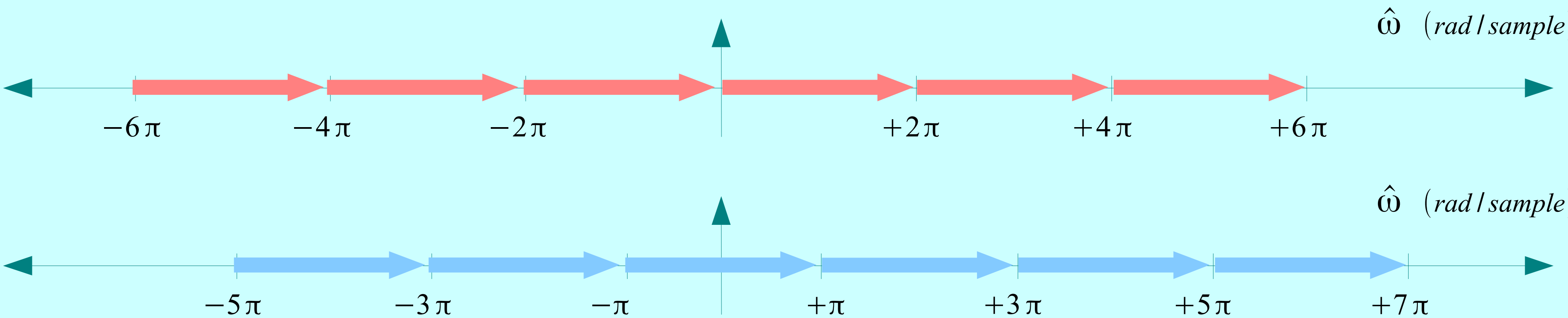
$$\hat{\omega} = \omega T_s$$

***Normalized Radian Frequency***
*can be viewed as*
*"the angular displacement of a signal during the period of its sample time $T_s$"*

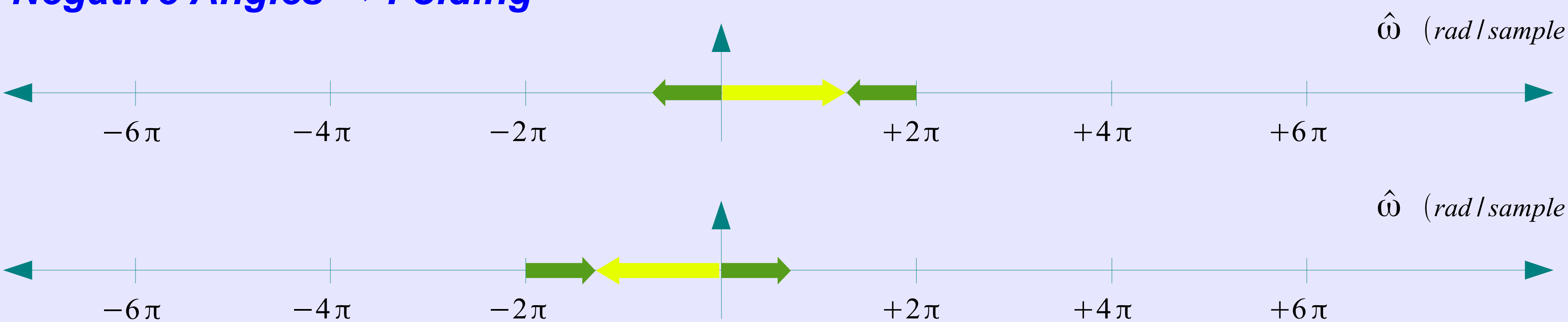
- Negative Angles
  → folding
- Co-terminal Angles
  → periodic

# Periodic and Folding

## *Co-terminal Angles → Periodic*




## *Negative Angles → Folding*



## References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A “graphical interpretation” of the DFT and FFT, by Steve Mann