

# Derivatives (2A)

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- Partial Derivative
- Directional Derivative
- Tangent and Normal Planes

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Please send corrections (or suggestions) to [youngwlim@hotmail.com](mailto:youngwlim@hotmail.com).

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# Partial Derivatives

Function of one variable  $y = f(x)$

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Function of two variable  $z = f(x, y)$

$$\frac{\partial z}{\partial x} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$$

treating  $y$  as a constant

$$\frac{\partial z}{\partial y} = \lim_{\Delta y \rightarrow 0} \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y}$$

treating  $x$  as a constant

# Partial Derivatives Notations

Function of one variable  $y = f(x)$

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Function of two variable  $z = f(x, y)$

$$\frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} = z_x = f_x$$

$$\frac{\partial z}{\partial x} = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x, y) - f(x, y)}{\Delta x}$$

treating  $y$  as a constant

$$\frac{\partial z}{\partial y} = \frac{\partial f}{\partial y} = z_y = f_y$$

$$\frac{\partial z}{\partial y} = \lim_{\Delta y \rightarrow 0} \frac{f(x, y+\Delta y) - f(x, y)}{\Delta y}$$

treating  $x$  as a constant

# Higher-Order & Mixed Partial Derivatives

## Second-order Partial Derivatives

$$\frac{\partial^2 z}{\partial \mathbf{x}^2} = \frac{\partial}{\partial \mathbf{x}} \left( \frac{\partial z}{\partial \mathbf{x}} \right)$$

$$\frac{\partial^2 z}{\partial \mathbf{y}^2} = \frac{\partial}{\partial \mathbf{y}} \left( \frac{\partial z}{\partial \mathbf{y}} \right)$$

## Third-order Partial Derivatives

$$\frac{\partial^3 z}{\partial \mathbf{x}^3} = \frac{\partial}{\partial \mathbf{x}} \left( \frac{\partial^2 z}{\partial \mathbf{x}^2} \right)$$

$$\frac{\partial^3 z}{\partial \mathbf{y}^3} = \frac{\partial}{\partial \mathbf{y}} \left( \frac{\partial^2 z}{\partial \mathbf{y}^2} \right)$$

## Third-order Partial Derivatives

$$\frac{\partial^2 z}{\partial \mathbf{x} \partial \mathbf{y}} = \frac{\partial}{\partial \mathbf{x}} \left( \frac{\partial z}{\partial \mathbf{y}} \right) = \frac{\partial^2 z}{\partial \mathbf{y} \partial \mathbf{x}} = \frac{\partial}{\partial \mathbf{y}} \left( \frac{\partial z}{\partial \mathbf{x}} \right)$$

# Chain Rule (1)

Function of two variable

$$z = f(u, v)$$

$$u = g(x, y)$$

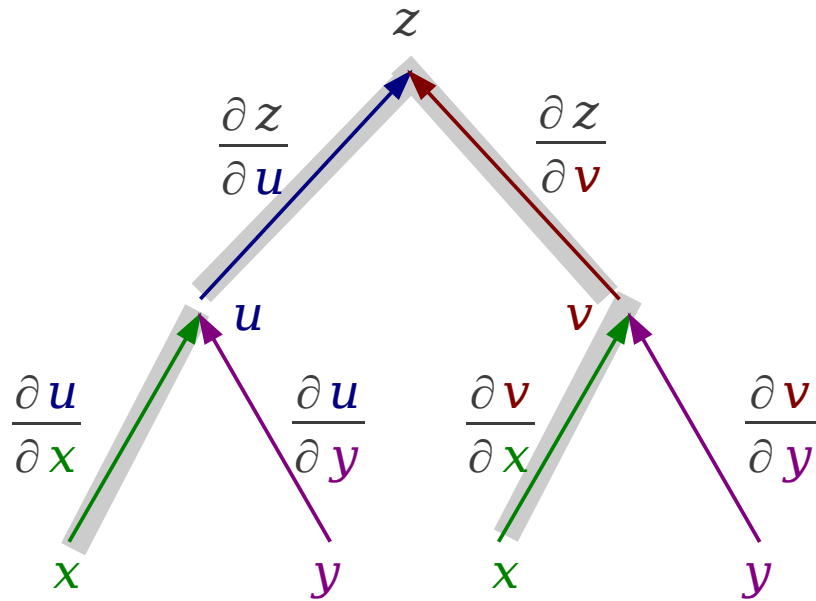
$$v = h(x, y)$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \frac{\partial v}{\partial x}$$

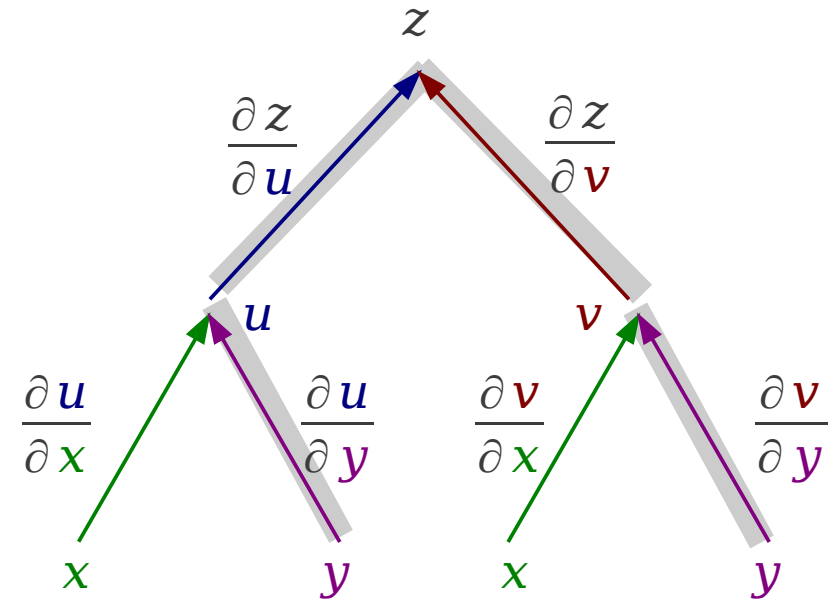
$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \frac{\partial u}{\partial y} + \frac{\partial z}{\partial v} \frac{\partial v}{\partial y}$$

# Chain Rule

Function of two variable  $z = f(u, v)$



$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \frac{\partial v}{\partial x}$$

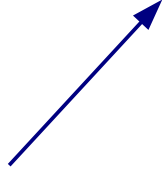


$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \frac{\partial u}{\partial y} + \frac{\partial z}{\partial v} \frac{\partial v}{\partial y}$$

# Chain Rule

Function of two variable

$$y = f(u, v)$$



$$u = g(x, y)$$

$$v = h(x, y)$$



# Line Equations (2)

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# Line Equations (2)

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## References

- [1] <http://en.wikipedia.org/>
- [2] <http://planetmath.org/>
- [3] M.L. Boas, “Mathematical Methods in the Physical Sciences”
- [4] D.G. Zill, “Advanced Engineering Mathematics”