Comp. of $\underline{k}^e$ in parent coord. $\{\xi_i\}$: cont'd p.35-3

Recall matrix algebra: $(\underline{A}\,\underline{B})^T = \underline{B}^T \underline{A}^T$ (1)

$$(\underline{A}^{-1})^T = (\underline{A}^T)^{-1} = \underline{A}^{-T} \quad (2)$$

HW 6.7: (HW 2.2) $\underline{A} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 3 & 2 & 6 \end{bmatrix}$ $\underline{B} = \begin{bmatrix} 1 & 3 & 5 \\ 1 & -4 & 1 \\ 2 & 5 & 8 \end{bmatrix}$

Verify (1) and (2).

WA:

{{1,1,1},{2,-1,3},{3,2,6}} * {{1,3,5},{1,-4,1},{2,5,8}}

transpose {{{1,1,1},{2,-1,3},{3,2,6}} * {{1,3,5},{1,-4,1},{2,5,8}}} No

note

transpose [{{1,1,1},{2,-1,3},{3,2,6}} * {{1,3,5},{1,-4,1},{2,5,8}}] Yes

note

{{1,3,5},{1,-4,1},{2,5,8}}^T * {{1,1,1},{2,-1,3},{3,2,6}}^T

invert {transpose {{1,1,1},{2,-1,3},{3,2,6}}}

transpose {invert {{1,1,1},{2,-1,3},{3,2,6}}} No

note

note: transpose [invert {{1,1,1},{2,-1,3},{3,2,6}}] Yes

invert [transpose {{1,1,1},{2,-1,3},{3,2,6}}]

Find and explain the syntax of WA.

(3)-(5) p. 35-2, (4)-(5) p. 35-3:

dot prod

$$k^e_{IJ} = \int_{\bar{\omega} = \square} \underline{B}_I(\underline{\xi}) \cdot \underline{\bar{K}}(\underline{\xi}) \cdot \underline{B}_J(\underline{\xi})\, J(\underline{\xi})\, d\square$$

1×1

Transpose

$$= \int_{\bar{\omega} = \square} \underbrace{\underline{B}_I(\underline{\xi})^T}_{1\times 2}\; \underbrace{\underline{\bar{K}}(\underline{\xi})}_{2\times 2}\; \underbrace{\underline{B}_J(\underline{\xi})}_{2\times 1}\; \underbrace{J(\underline{\xi})}_{1\times 1}\, d\square \qquad (1)$$

$$\underline{B}_I(\underline{\xi}) := \nabla_x N^e_I(\underline{\xi}) = (\underline{J}^e)^{-T}(\underline{\xi})\, \nabla_\xi N^e_I(\underline{\xi}) \qquad (2)$$

(4) p. 35-3

$\underline{B}_J(\underline{\xi})$ similar

(5) p. 35-2

$$\underline{\bar{K}}(\underline{\xi}) := \underline{K}(\underline{x}^e(\underline{\xi})) = \underline{K}(\underbrace{\underline{\varphi}^e(\underline{\xi})}) \qquad (3)$$

$$J(\underline{\xi}) := \det \underline{J}(\underline{\xi}) \qquad (1)$$

Accurate and efficient numerical integration: Gauss-Legendre quadrature (GLQ)

$$I(f) = \int_{-1}^{+1} f(x)\,dx \cong \sum_{i=1}^{\mu} W_i f(x_i) =: I_\mu(f) \qquad (1)$$

$\{x_i, i = 1, \ldots, \mu\}$ = roots of Legendre poly $P_\mu(x)$ of order μ

$\{W_i, i = 1, \ldots, \mu\}$ = Weights associated with $\{x_i\}$

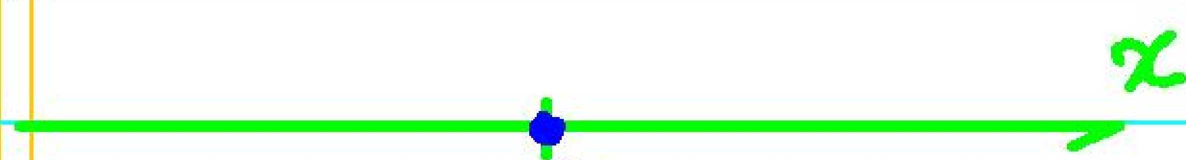
Number of points, n	Points, x_i	Weights, w_i
1	0	2
2	$\pm 1/\sqrt{3}$	1
3	0	$8/9$
	$\pm\sqrt{15}/5$	$5/9$
4	$\pm\sqrt{\left(3-2\sqrt{6/5}\right)/7}$	$\frac{18+\sqrt{30}}{36}$
	$\pm\sqrt{\left(3+2\sqrt{6/5}\right)/7}$	$\frac{18-\sqrt{30}}{36}$
5	0	$128/225$
	$\pm\frac{1}{3}\sqrt{5-2\sqrt{10/7}}$	$\frac{322+13\sqrt{70}}{900}$
	$\pm\frac{1}{3}\sqrt{5+2\sqrt{10/7}}$	$\frac{322-13\sqrt{70}}{900}$

See PEA1 F10
NM1 S11
for more details

http://en.wikipedia.org/wiki/Gaussian_quadrature

Ex:

$\mu = 1$

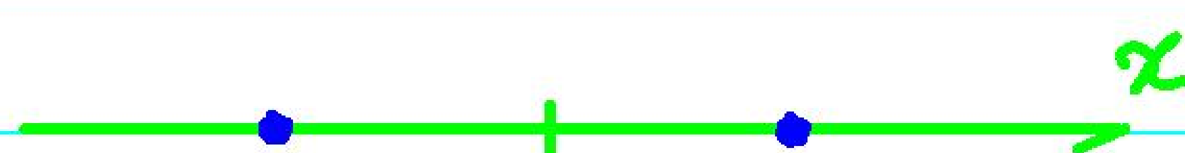


$I_1 = 2 f(0)$

$x_1 = 0$

$W_1 = 2$

$\mu = 2$



$x_1 = \frac{-1}{\sqrt{3}}$ $\qquad$ $x_2 = \frac{1}{\sqrt{3}}$

$W_1 = 1$ $\qquad$ $W_2 = 1$

$$I_2 = 1 \cdot f\left(\frac{-1}{\sqrt{3}}\right) + 1 \cdot f\left(\frac{1}{\sqrt{3}}\right)$$

Thm:

$$I(f) = I_\mu(f) + E_\mu(f) \qquad (1)$$

$$E_\mu(f) = \frac{2^{2\mu+1}(\mu!)^4}{(2\mu+1)\left[(2\mu)!\right]^2} \frac{f^{(2\mu)}(\xi)}{(2\mu)!}, \quad \xi \in [-1,+1] \qquad (2)$$

Let f be a poly of order $2\mu-1$, i.e., $f \in \mathcal{P}_{2\mu-1}$ (set of poly of deg $\leq 2\mu-1$) $\Rightarrow f^{(2\mu)} \equiv 0$

e.g., $f \in \mathcal{P}_1 \Rightarrow f(x) = a_1 x^1 + a_0 \Rightarrow f^{(2)} \equiv 0$

Hence $\mu = 1$ integrates <u>exactly</u> any poly in $\mathcal{P}_1$

$\mu = 2$ " " " $\mathcal{P}_3$

$\mu = 3$ " " " $\mathcal{P}_5$

HW6.8: 1) Verify Table of $\{(w_i, x_i), i = 1, \dots, 5\}$ against NIST Handbook (Lect plan) and FB p. 89.

2) FB, p. 91, pbs. 4.6, 4.7, 4.8

Multivariable int.

$$I = \int_{\square} f(x,y)\, d\square = \int_{\square} f(x_1, x_2)\, d\square$$

$$I_{\mu} = \sum_{i=1}^{\mu} \sum_{j=1}^{\mu} w_i^x\, w_j^y\, f(x_i, y_j)$$

HW6.6 Cont'd p. 35-4: Int. $\underset{\sim}{k}^e$ in parent coord. $\{\xi_i\}$

Find appropriate μ to int. $\underset{\sim}{k}^e$ exactly with GLQ.

Give detailed argument.