General Vector Space (2A)

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10/31/12

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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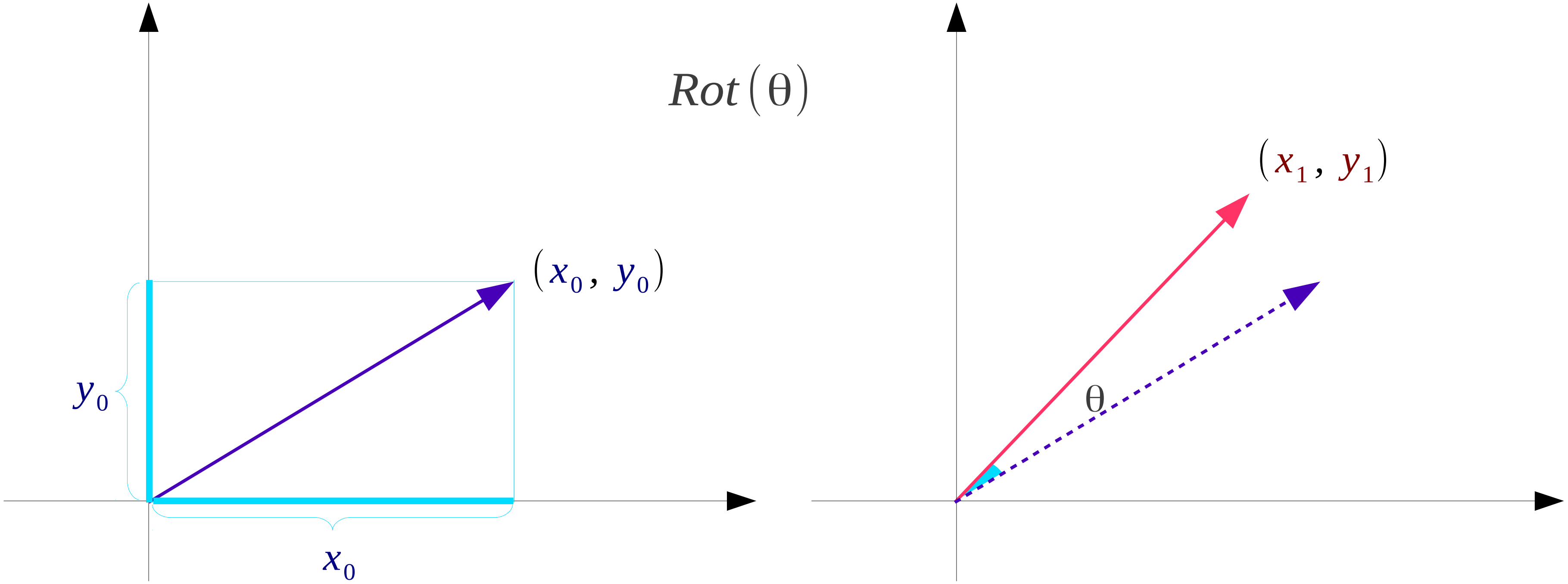
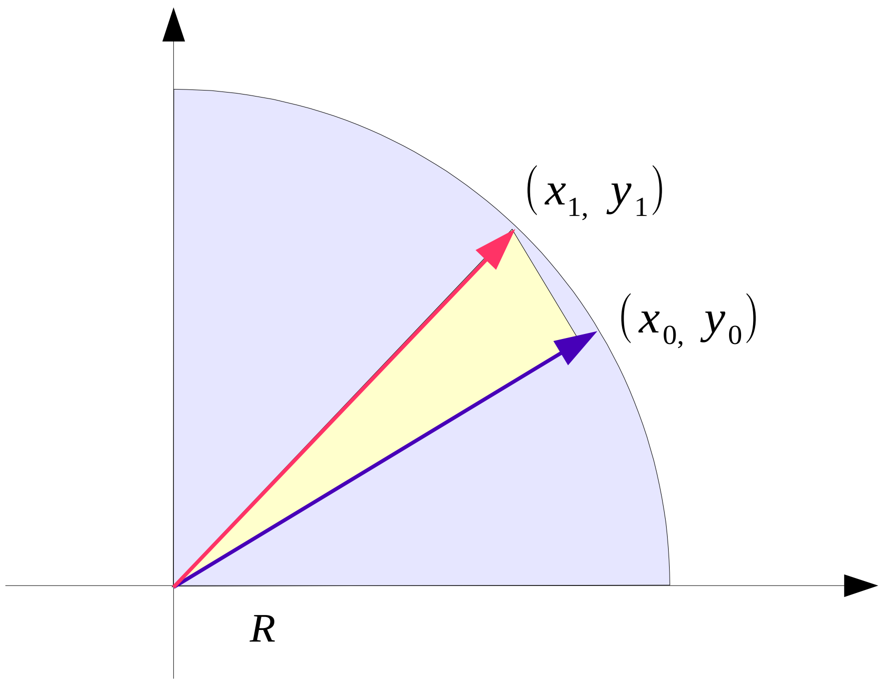
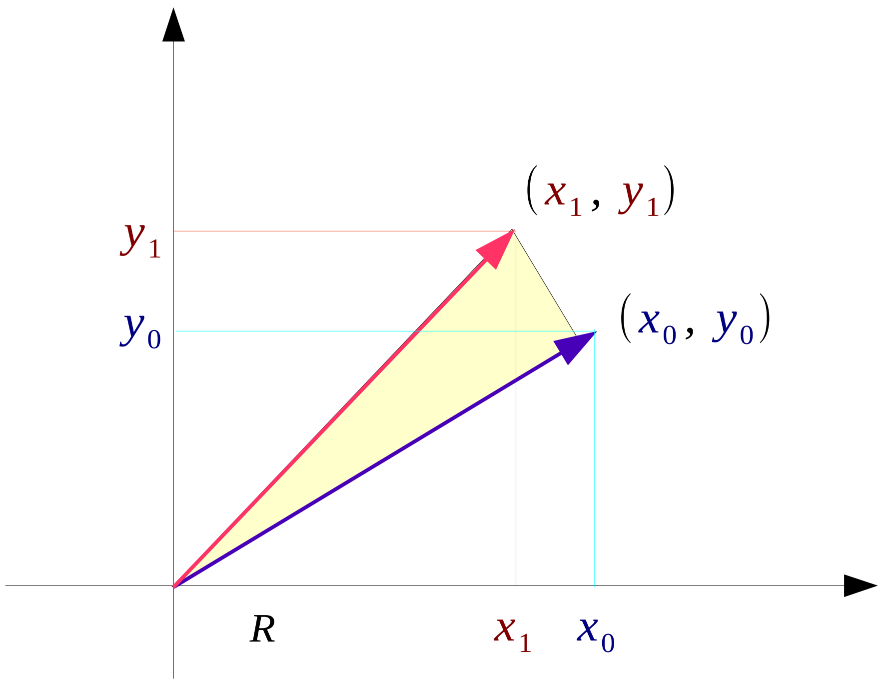
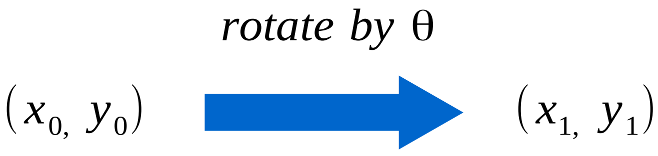
Vector Rotation (1)



Vector Rotation (2)



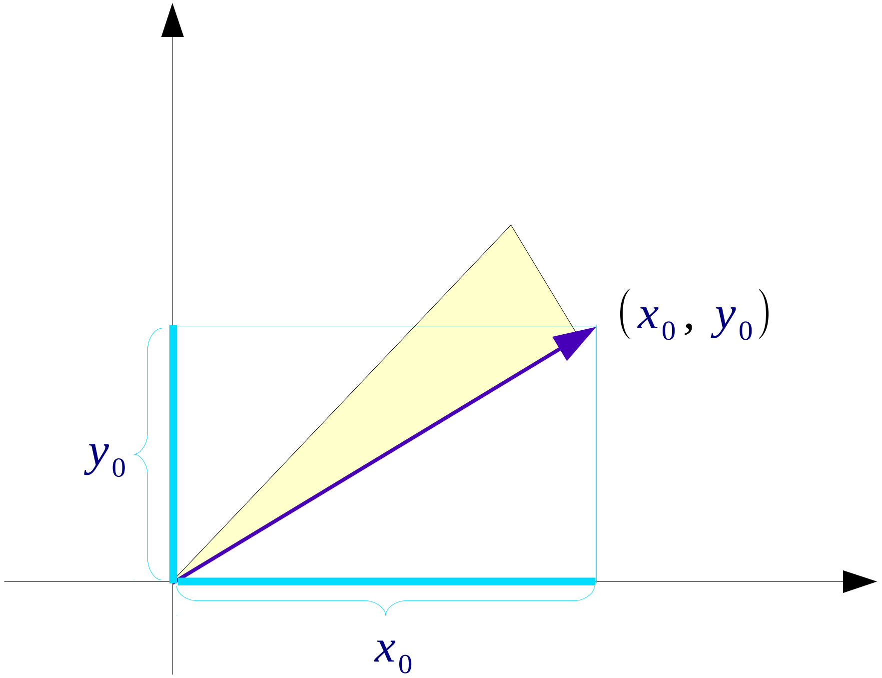


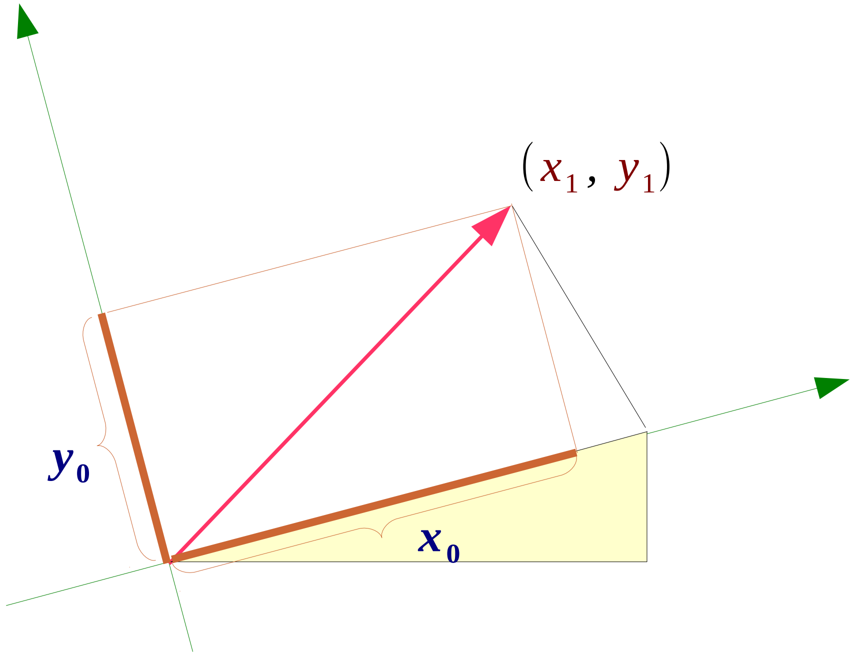


$$x_1 = x_0 \cos\theta - y_0 \sin\theta$$

$$y_1 = x_0 \sin\theta + y_0 \cos\theta$$

Vector Rotation (3)





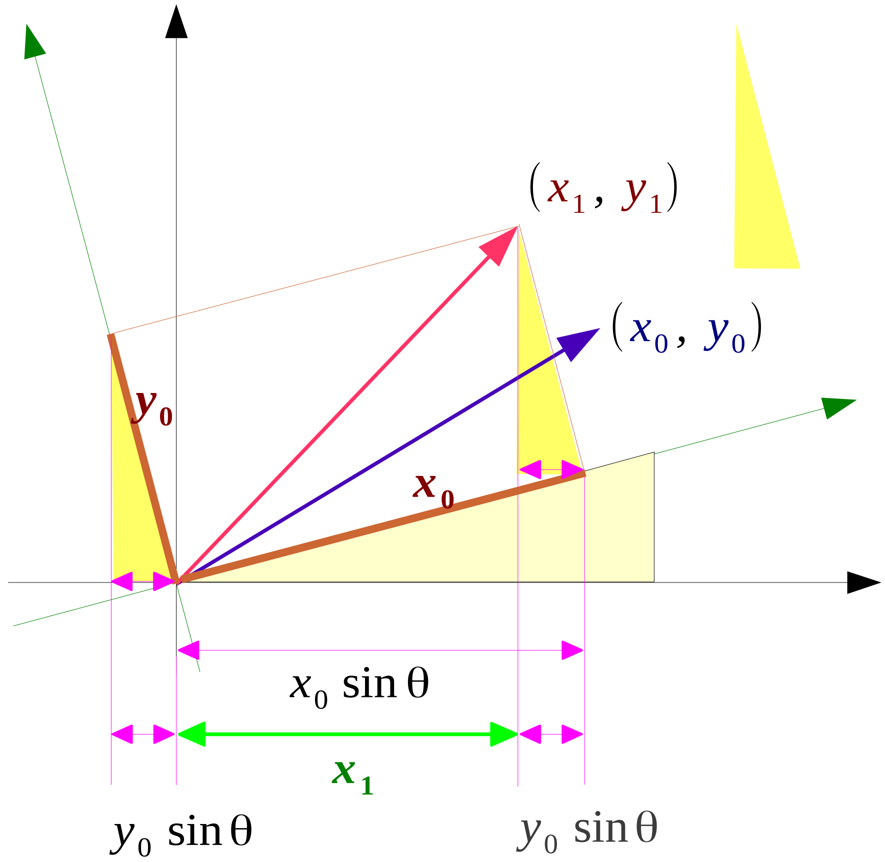
In the rotated coordinate

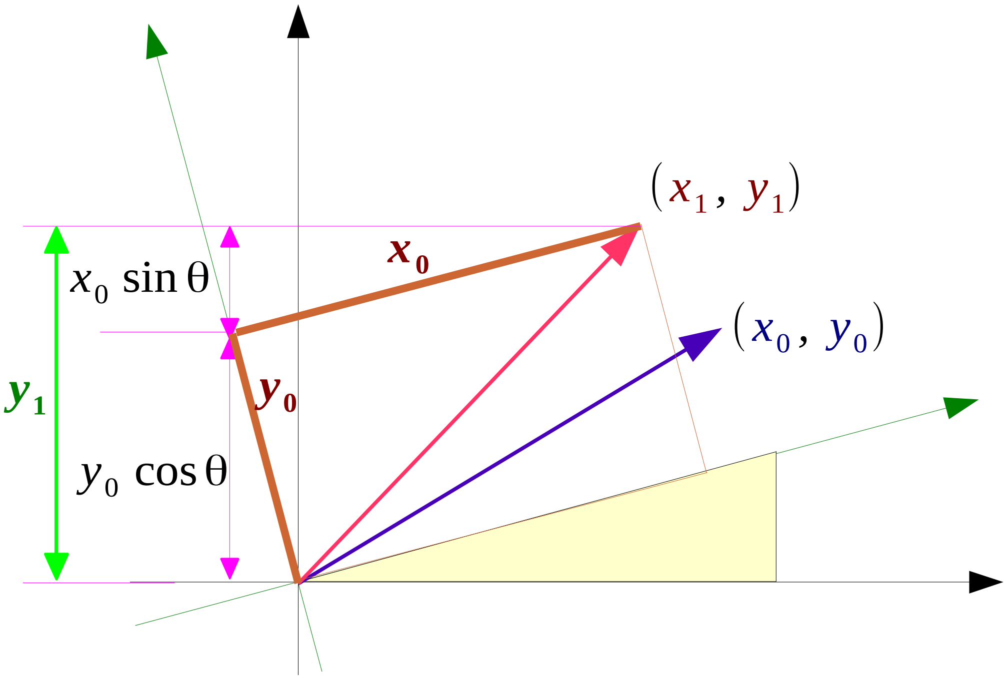
invariant length x_0, y_0

Vector Rotation (4)

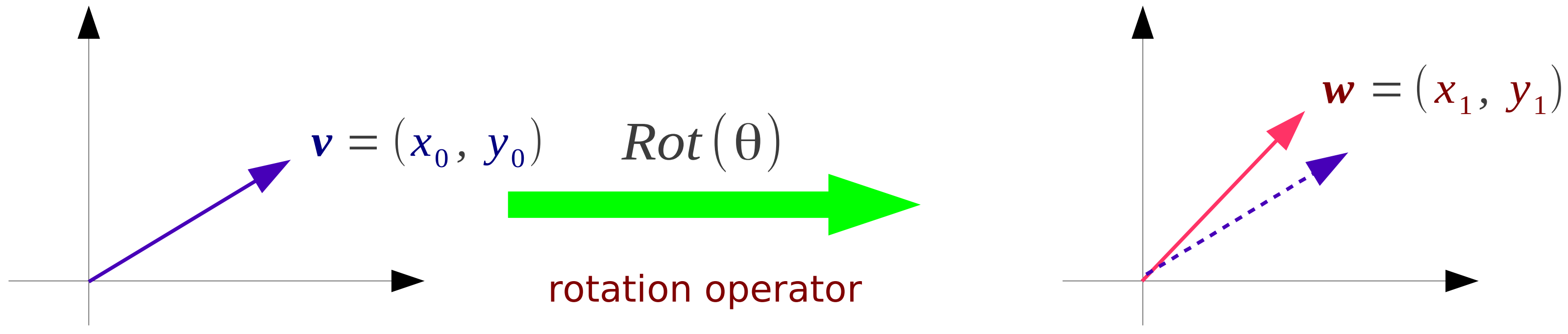
$$x_1 = x_0 \cos\theta - y_0 \sin\theta$$

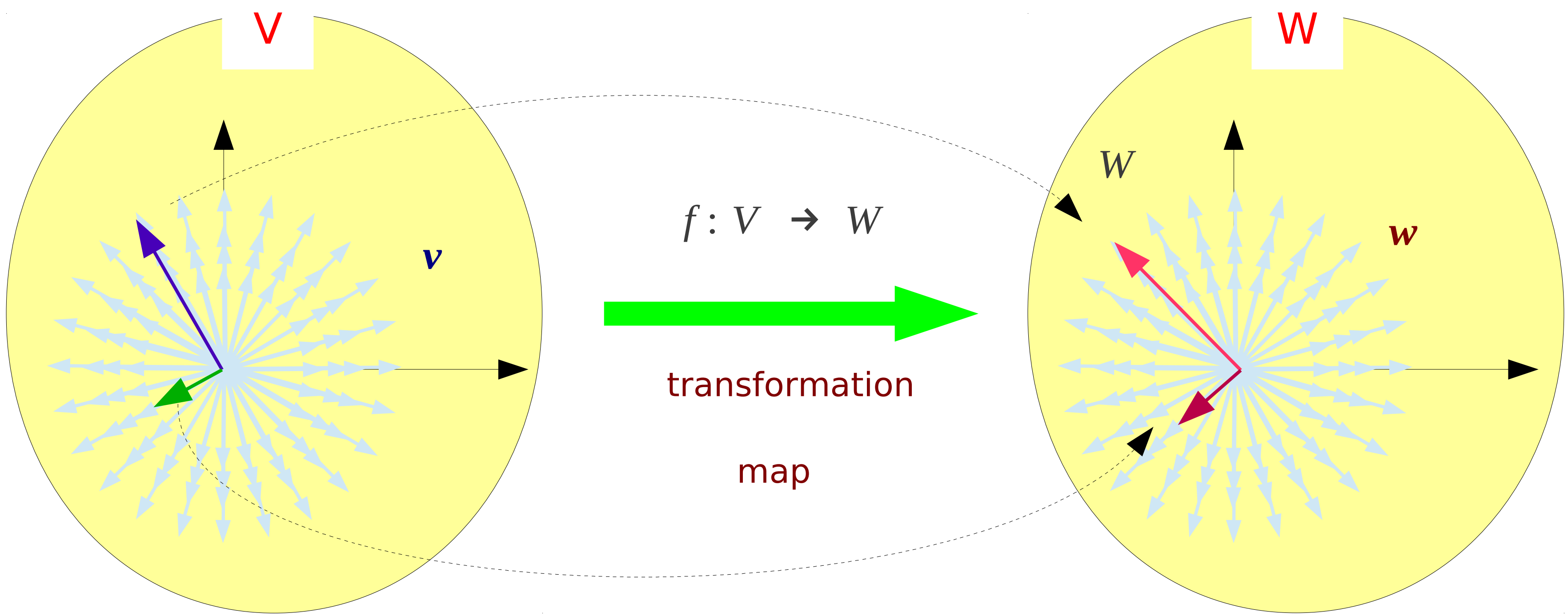
$$y_1 = x_0 \sin\theta + y_0 \cos\theta$$



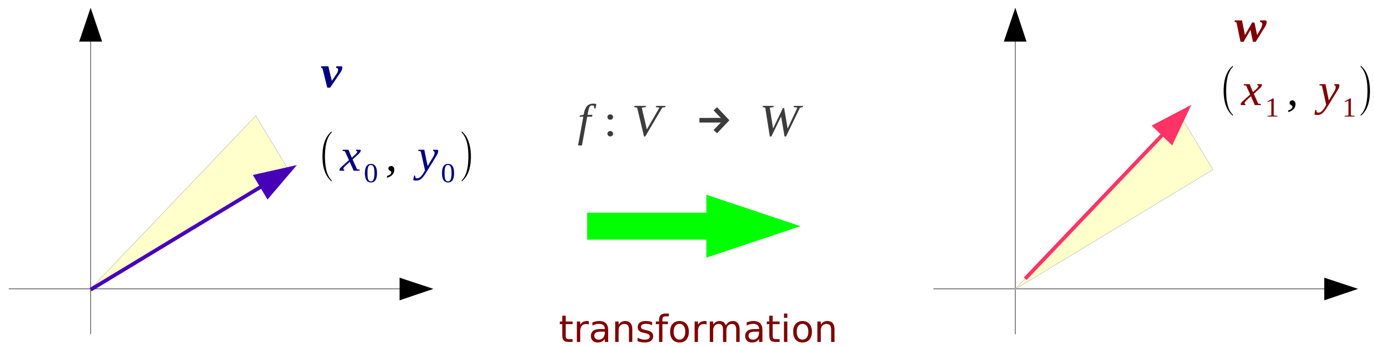


Transformation





Matrix Transformation



$$x_1 = x_0 \cos\theta - y_0 \sin\theta$$
$$y_1 = x_0 \sin\theta + y_0 \cos\theta$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_0 \\ y_1 \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

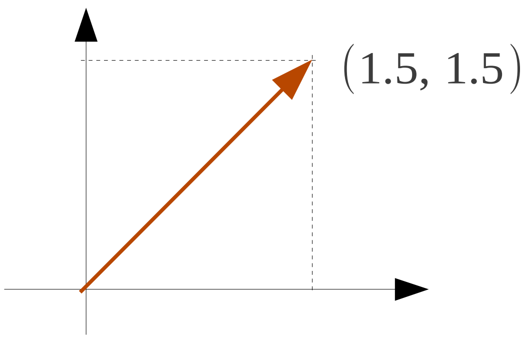
$$\boldsymbol{w} = \boldsymbol{A} \qquad \boldsymbol{x}$$

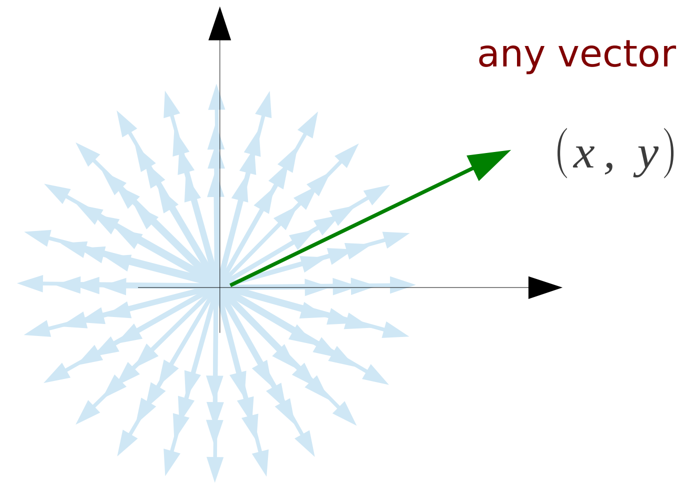
$$\boldsymbol{w} = T_{\boldsymbol{A}}(\boldsymbol{x})$$

$$\boldsymbol{x} \xrightarrow{\;T_{\boldsymbol{A}}\;} \boldsymbol{w}$$

Coordinates and Coordinates Systems

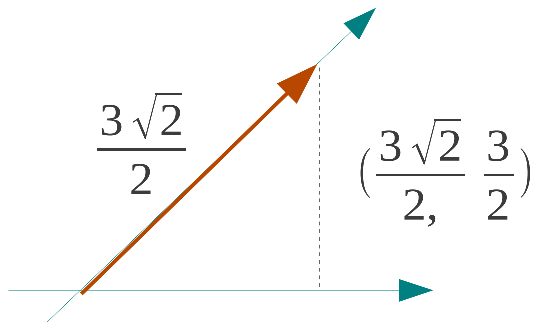
Rectangular Coordinate System

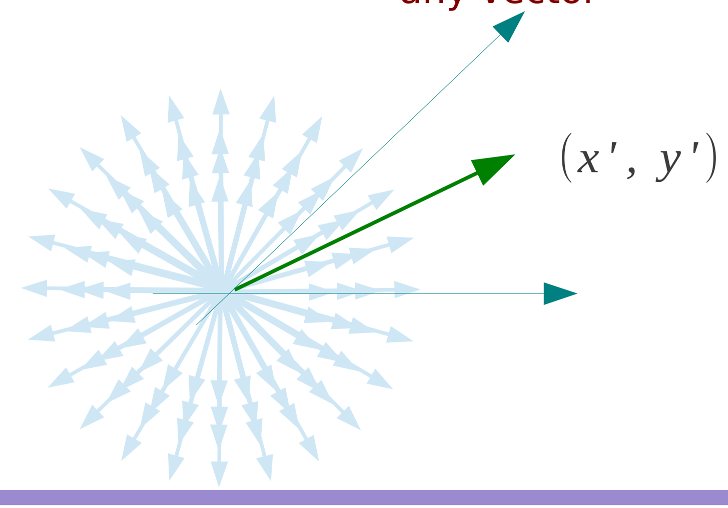




coordinates $(1.5\,,\;1.5)$

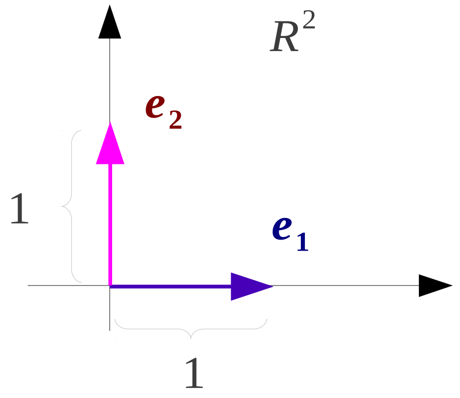
Non-Rectangular Coordinate System





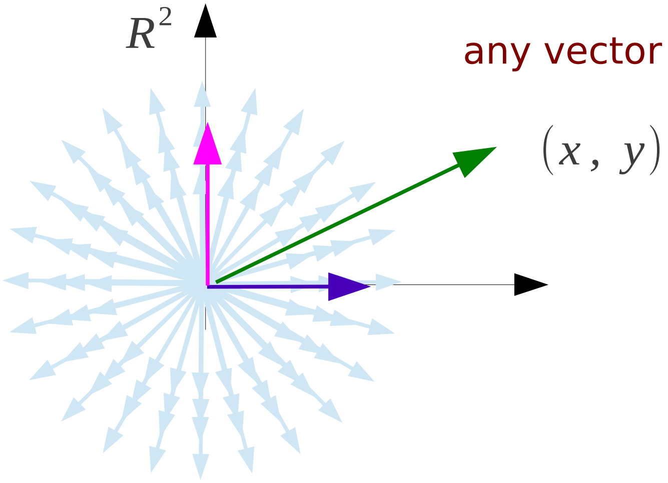
coordinates $(\frac{3\sqrt{2}}{2}, \frac{3}{2})$

Standard Basis

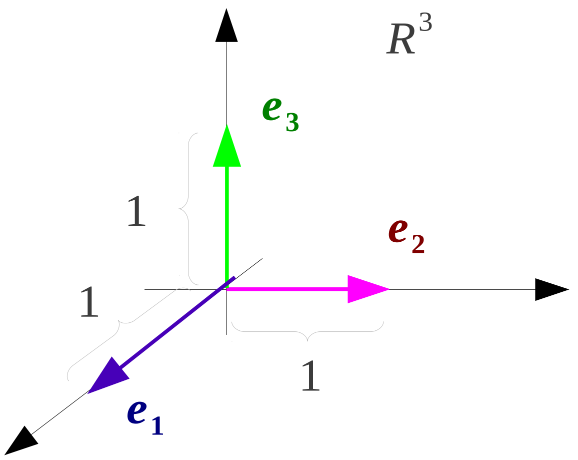


$e_1 = (1, 0)$

$e_2 = (0, 1)$



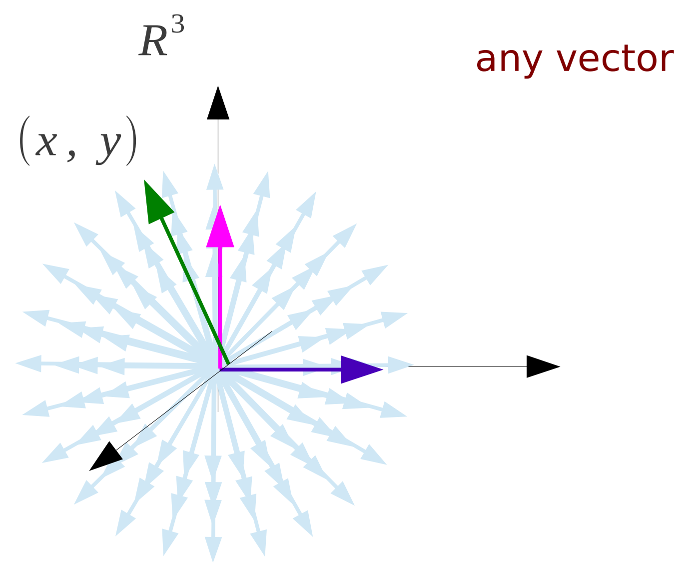
standard basis $\{e_1, e_2\}$ **spans** R^2



$e_1 = (1, 0, 0)$

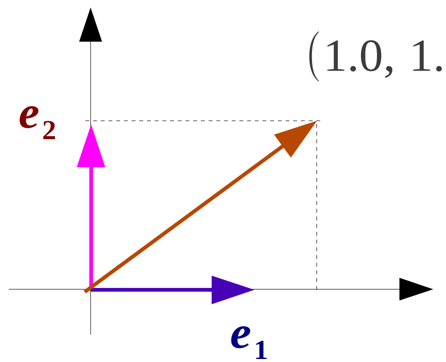
$e_2 = (0, 1, 0)$

$e_3 = (0, 0, 1)$



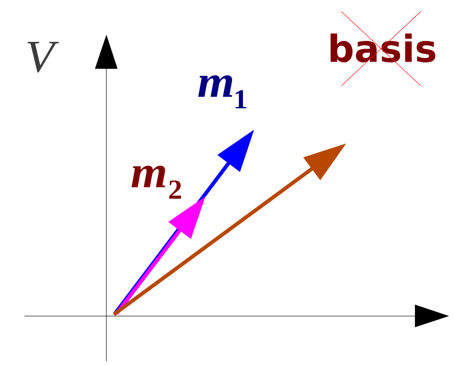
standard basis $\{e_1, e_2, e_3\}$ **spans** R^3

Basis and Coordinates



$(1.0,\ 1.5) = 1.0\,\boldsymbol{e_1} + 1.5\,\boldsymbol{e_2}$

$= 1.5\begin{bmatrix}1\\0\end{bmatrix} + 1.5\begin{bmatrix}0\\1\end{bmatrix}$



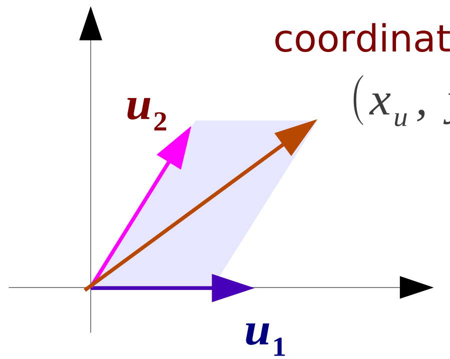
collinear vectors → linearly dependent vectors

basis $\{\boldsymbol{e_1},\ \boldsymbol{e_2}\}$

coordinates $(1.5,\ 1.5)$

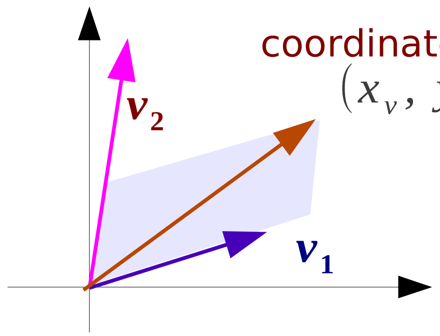
many bases but the same number of basis vectors

basis $\{\boldsymbol{u_1},\ \boldsymbol{u_2}\}$



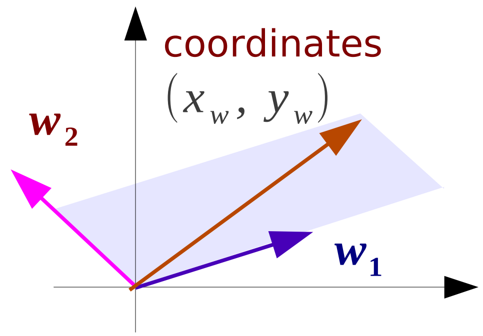
coordinates $(x_u,\ y_u)$

basis $\{\boldsymbol{v_1},\ \boldsymbol{v_2}\}$



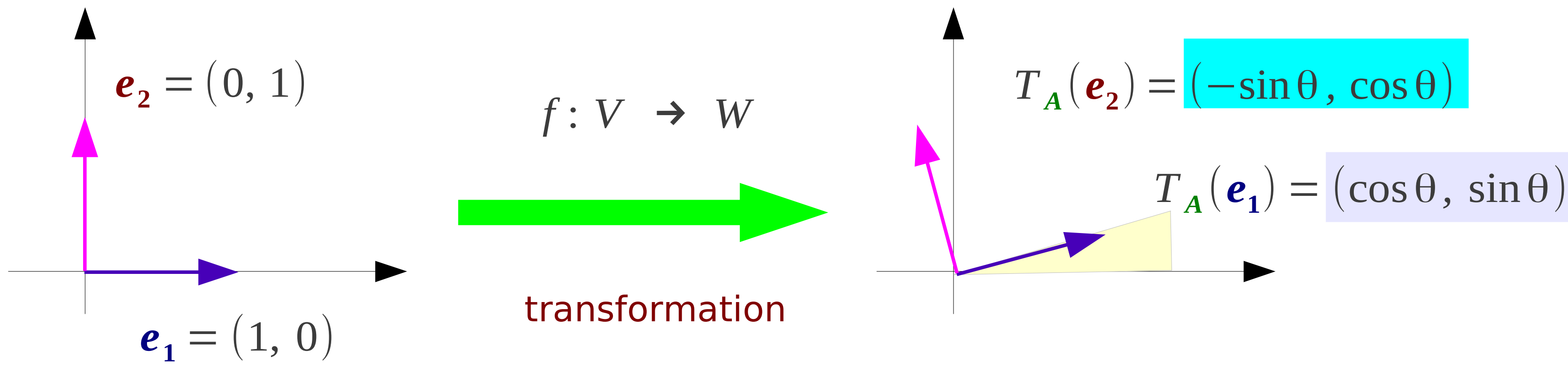
coordinates $(x_v,\ y_v)$

basis $\{\boldsymbol{w_1},\ \boldsymbol{w_2}\}$



coordinates $(x_w,\ y_w)$

Standard Basis & Standard Matrix



standard basis

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_0 \\ y_1 \end{bmatrix}$$

$$\boldsymbol{w} = A\,\boldsymbol{x}$$

$$\boldsymbol{w} = T_A(\boldsymbol{x})$$

$$\boldsymbol{x} \xrightarrow{T_A} \boldsymbol{w}$$

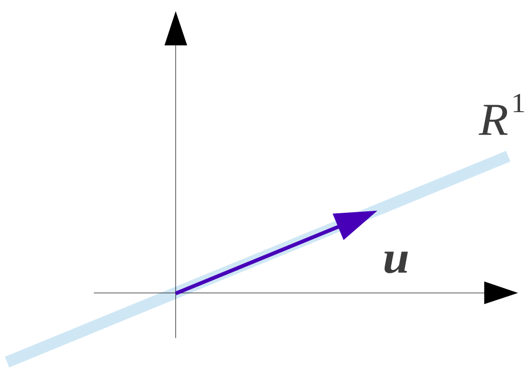
standard matrix

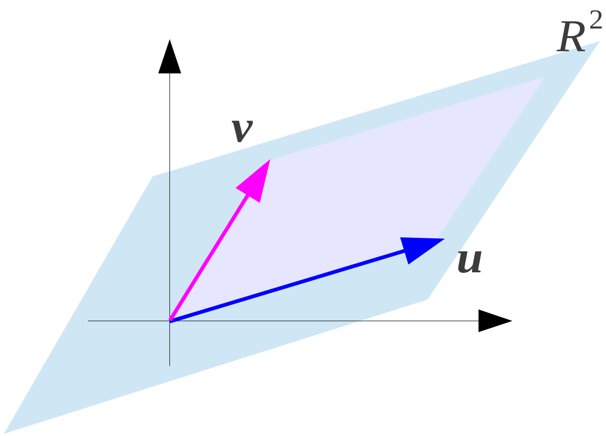
$$A = \begin{pmatrix} T_A(\boldsymbol{e}_1) & T_A(\boldsymbol{e}_2) & & T_A(\boldsymbol{e}_n) \end{pmatrix}$$

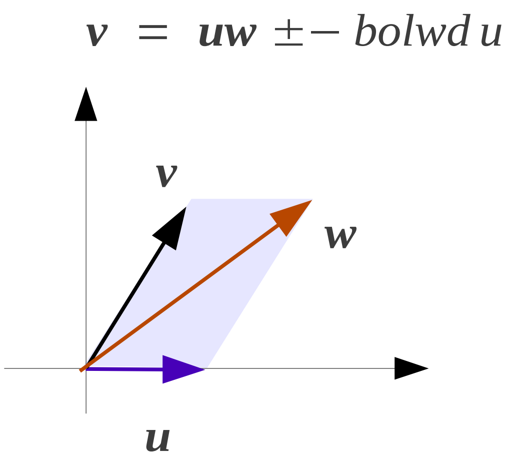
Dimension

In vector space R^2

any one vector	line R^1	linearly independent
any two non-collinear vectors	plane R^2	linearly independent
any three or more vectors		linearly dependent



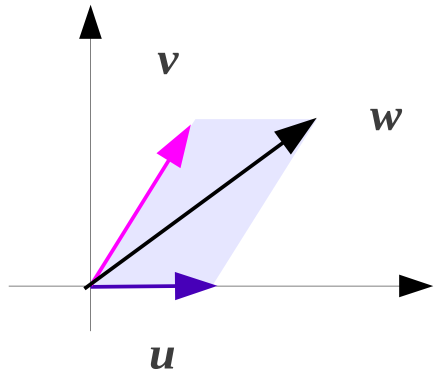




Linear Dependent (1)

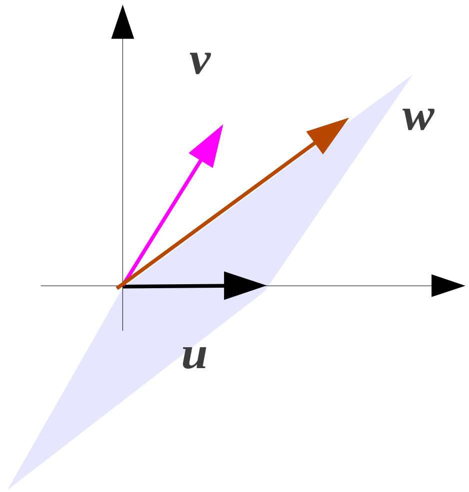
$\{\boldsymbol{u}\ ,\ \boldsymbol{v}\ ,\ \boldsymbol{w}\}$ linearly dependent

$\boldsymbol{w} = \boldsymbol{u} + \boldsymbol{v}$



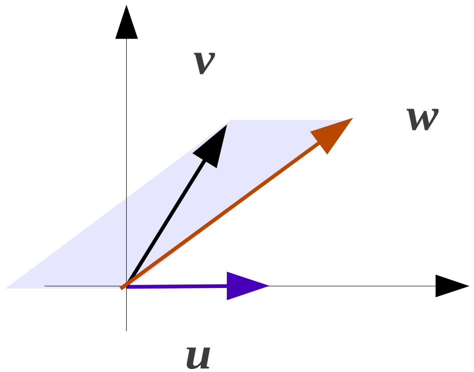
$\boldsymbol{u} + \boldsymbol{v} - \boldsymbol{w} = \boldsymbol{0}$

$\boldsymbol{u} = \boldsymbol{w} - \boldsymbol{v}$



$\boldsymbol{u} + \boldsymbol{v} - \boldsymbol{w} = \boldsymbol{0}$

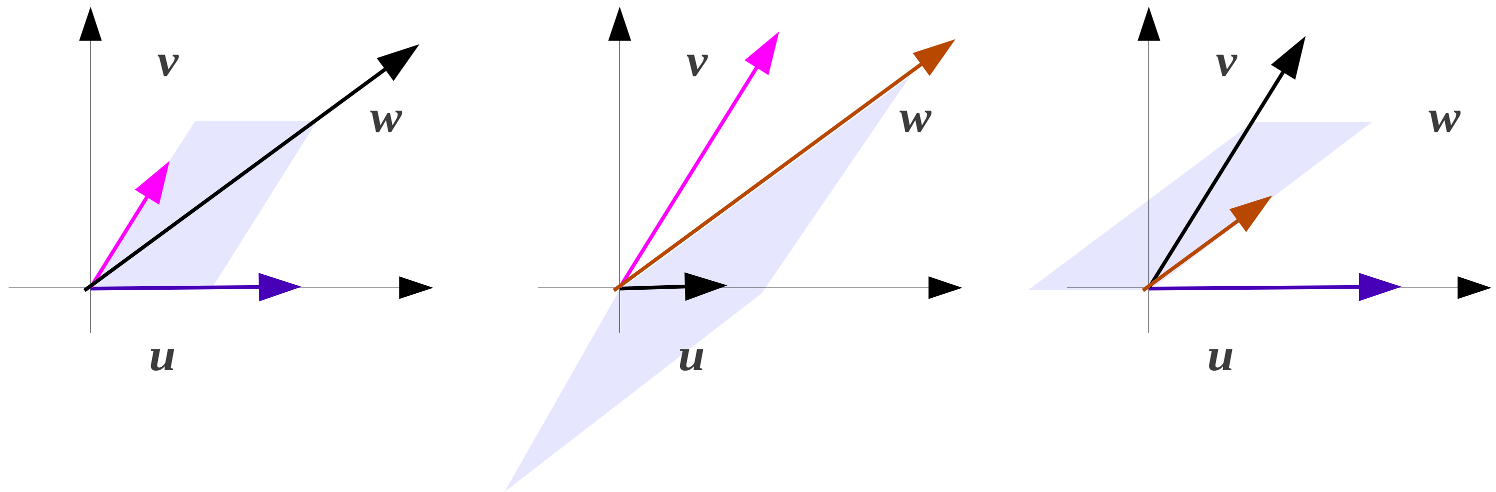
$\boldsymbol{v} = \boldsymbol{w} - \boldsymbol{u}$



$\boldsymbol{u} + \boldsymbol{v} - \boldsymbol{w} = \boldsymbol{0}$

Linear Dependent (2)

$\{\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}\}$ linearly dependent



$k_1\boldsymbol{u} + k_2\boldsymbol{v} + k_3\boldsymbol{w} = \boldsymbol{0}$

$(k_1 = 0)\wedge(k_2 = 0)\wedge(k_3 = 0)$
$(k_1 \neq 0)\vee(k_2 \neq 0)\vee(k_3 \neq 0)$

$m_1\boldsymbol{u} + m_2\boldsymbol{v} + m_3\boldsymbol{w} = \boldsymbol{0}$

$(m_1 = 0)\wedge(m_2 = 0)\wedge(m_3 = 0)$
$(m_1 \neq 0)\vee(m_2 \neq 0)\vee(m_3 \neq 0)$

$n_1\boldsymbol{u} + n_2\boldsymbol{v} + n_3\boldsymbol{w} = \boldsymbol{0}$

$(n_1 = 0)\wedge(n_2 = 0)\wedge(n_3 = 0)$
$(n_1 \neq 0)\vee(n_2 \neq 0)\vee(n_3 \neq 0)$

Linear Independent

$S = \{\boldsymbol{v_1}, \boldsymbol{v_2}, \cdots, \boldsymbol{v_n}\}$ non-empty set of vectors in V

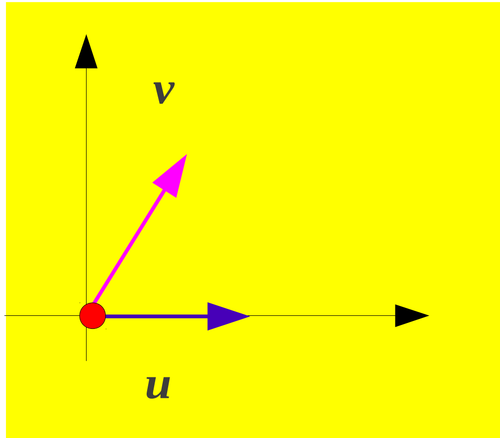
$k_1\boldsymbol{v_1} + k_2\boldsymbol{v_2} + \cdots + k_n\boldsymbol{v_n} = \boldsymbol{0}$

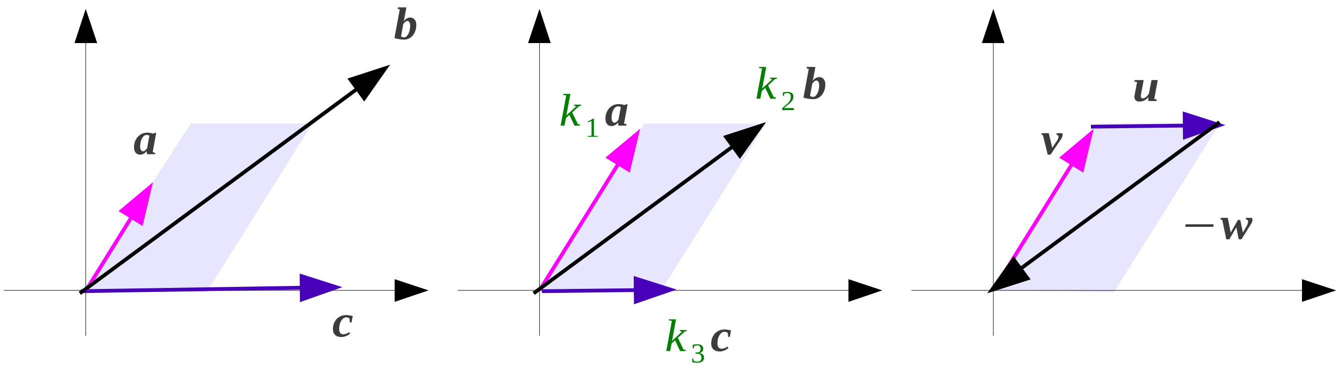
the solution of the above equation

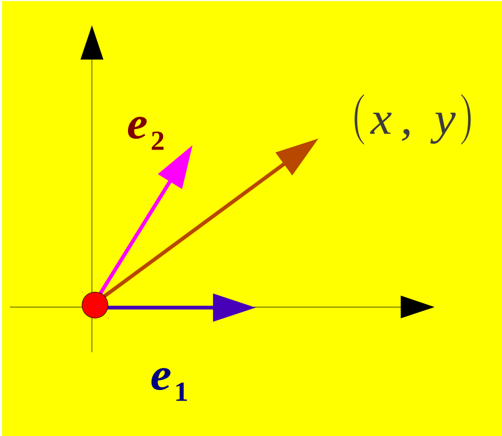
trivial solution: $k_1 = k_2 = \cdots = k_n = 0$

if other solution exists — S linearly dependent

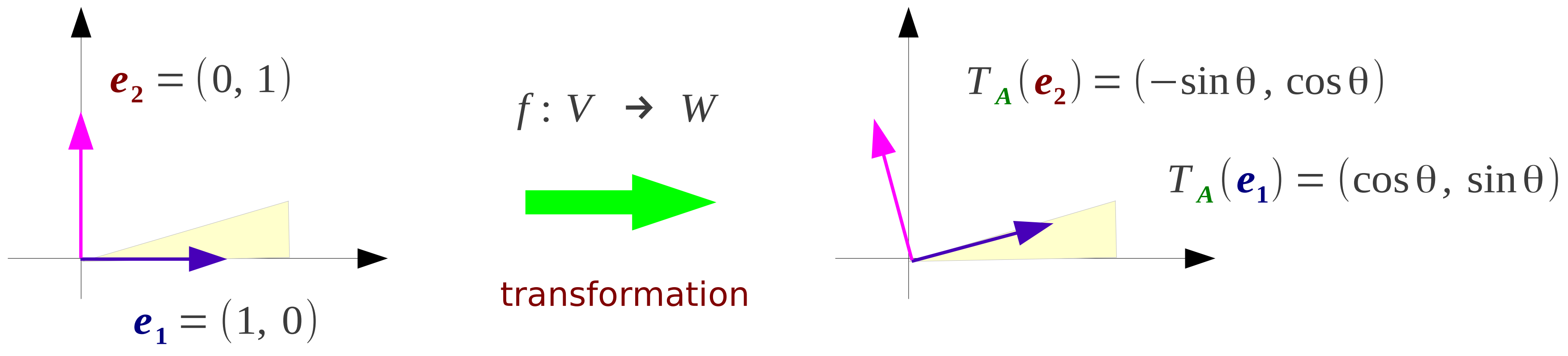
if no other solution exists — S linearly independent







Change of Basis



Old Basis **New Basis**

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_0 \\ y_1 \end{bmatrix}$$

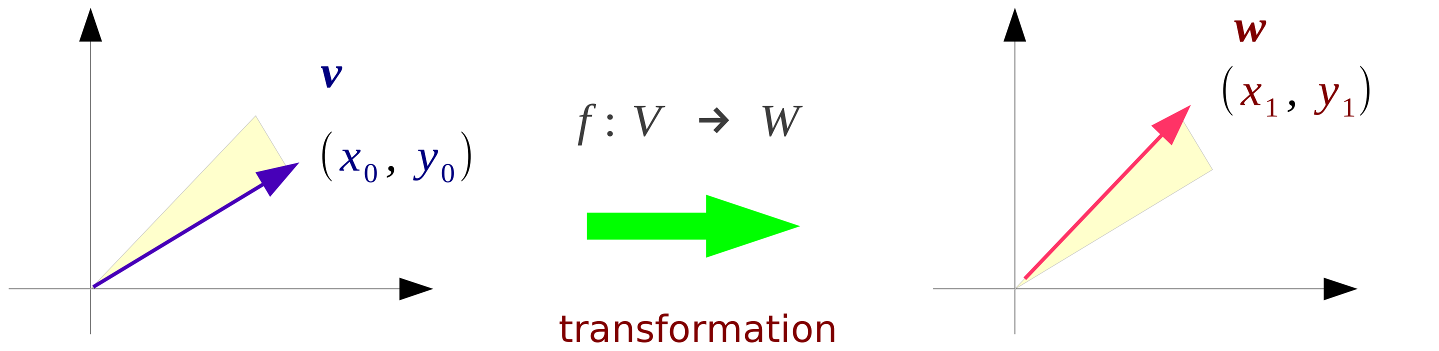
$$\boldsymbol{w} = \boldsymbol{A} \quad \boldsymbol{x}$$

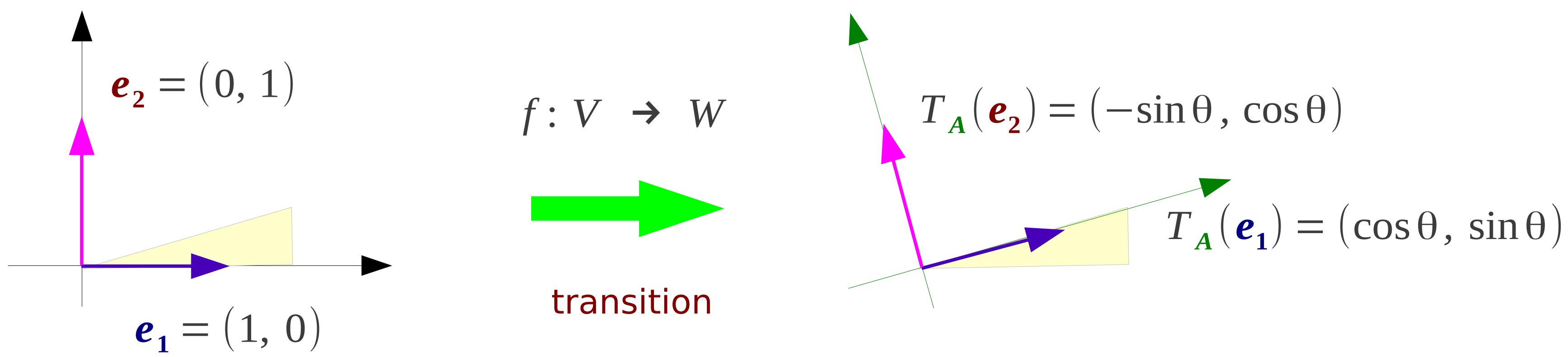
$$\boldsymbol{w} = T_A(\boldsymbol{x})$$

$$\boldsymbol{x} \xrightarrow{T_A} \boldsymbol{w}$$

$$\boldsymbol{A} = \begin{pmatrix} T_A(\boldsymbol{e}_1) & T_A(\boldsymbol{e}_2) & & T_A(\boldsymbol{e}_n) \end{pmatrix}$$

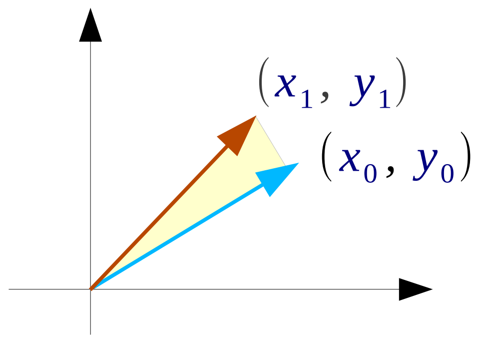
Transformation

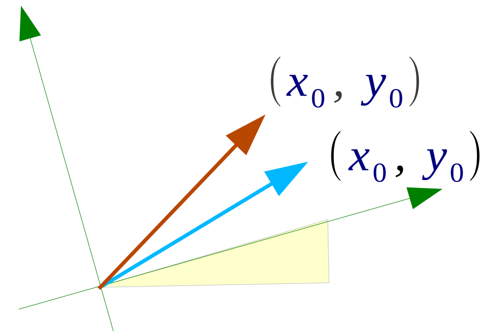


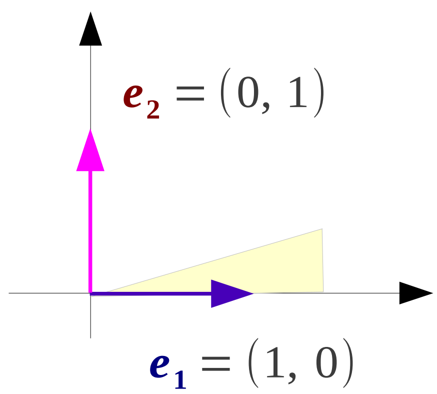


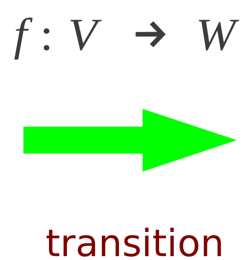
Old Basis **New Basis**

Transformation





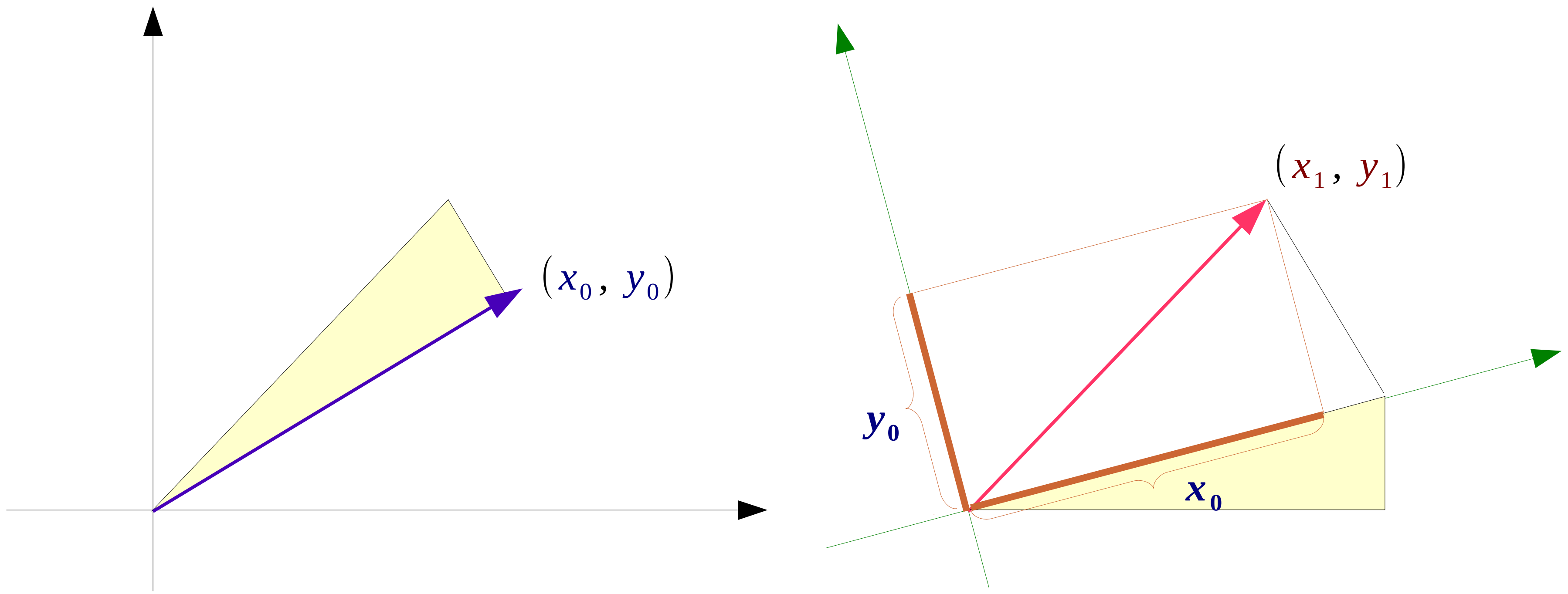




Old Basis

New Basis

Vector Rotation (2)



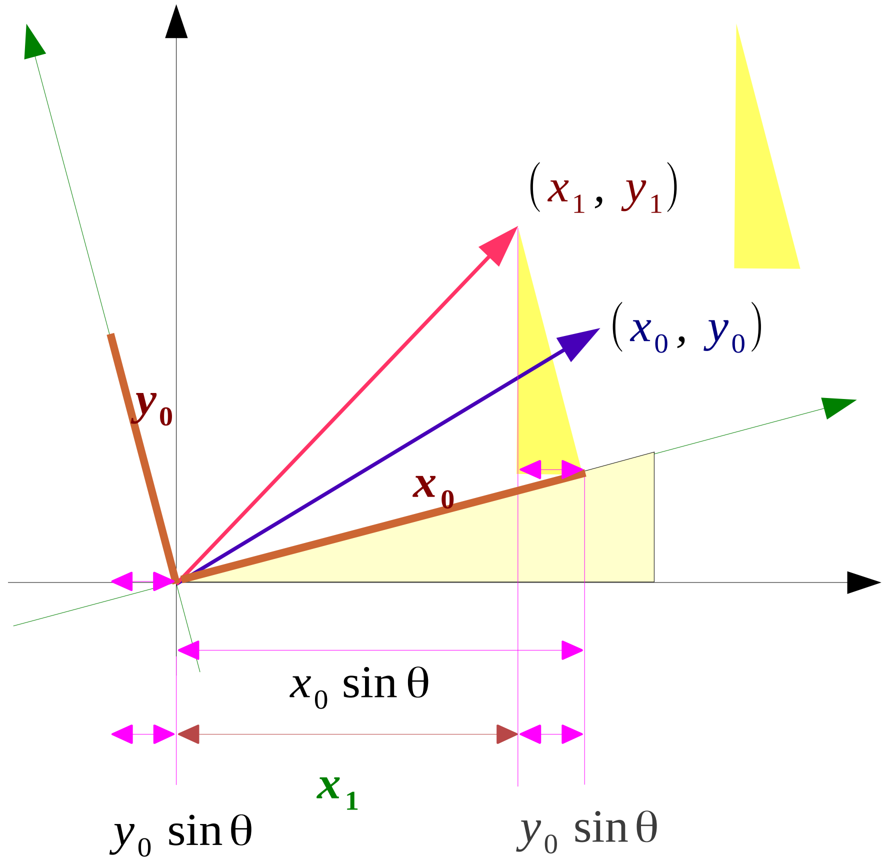
In the rotated coordinate

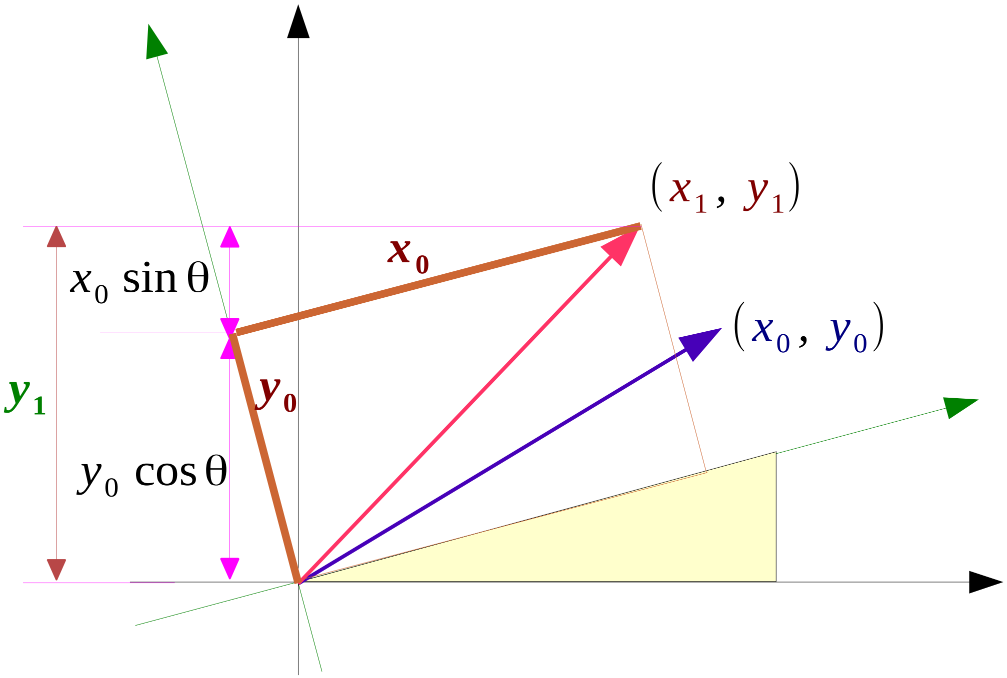
invariant length x_0, y_0

Trasformation

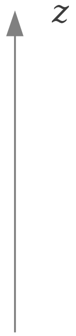
$$x_1 = x_0 \cos\theta - y_0 \sin\theta$$

$$y_1 = x_0 \sin\theta + y_0 \cos\theta$$





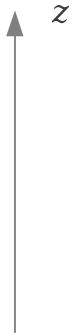
Normal Vector & 3 Points



$$\boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \boldsymbol{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \boldsymbol{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \boldsymbol{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

Normal Vector & 3 Points

Non-collinear
3 points



References

[1] http://en.wikipedia.org/
[2] http://planetmath.org/
[3] M.L. Boas, “Mathematical Methods in the Physical Sciences”
[4] D.G. Zill, “Advanced Engineering Mathematics”