DFT Matrix Properties (3B)

- X[1]
- X[2]
- X[3]
- X[4]
- X[5]
- X[6]
- X[7]

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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N=8 DFT

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad\qquad W_8^{kn} = e^{-j(\frac{2\pi}{8})kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} = \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\ W_8^0 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\ W_8^0 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\ W_8^0 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\ W_8^0 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\ W_8^0 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49} \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix}$$

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N=8 IDFT

$$x[n] = \frac{1}{N}\sum_{k=0}^{7} W_8^{-kn} X[k] \qquad W_8^{-kn} = e^{+j(\frac{2\pi}{8})kn}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \frac{1}{N} \begin{bmatrix} W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} \\ W_8^{0} & W_8^{-1} & W_8^{-2} & W_8^{-3} & W_8^{-4} & W_8^{-5} & W_8^{-6} & W_8^{-7} \\ W_8^{0} & W_8^{-2} & W_8^{-4} & W_8^{-6} & W_8^{-8} & W_8^{-10} & W_8^{-12} & W_8^{-14} \\ W_8^{0} & W_8^{-3} & W_8^{-6} & W_8^{-9} & W_8^{-12} & W_8^{-15} & W_8^{-18} & W_8^{-21} \\ W_8^{0} & W_8^{-4} & W_8^{-8} & W_8^{-12} & W_8^{-16} & W_8^{-20} & W_8^{-24} & W_8^{-28} \\ W_8^{0} & W_8^{-5} & W_8^{-10} & W_8^{-15} & W_8^{-20} & W_8^{-25} & W_8^{-30} & W_8^{-35} \\ W_8^{0} & W_8^{-6} & W_8^{-12} & W_8^{-18} & W_8^{-24} & W_8^{-30} & W_8^{-36} & W_8^{-42} \\ W_8^{0} & W_8^{-7} & W_8^{-14} & W_8^{-21} & W_8^{-28} & W_8^{-35} & W_8^{-42} & W_8^{-49} \end{bmatrix} \begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix}$$

N=8 DFT Matrix (1)

$$X[k] = \sum_{n=0}^{7} W_8^{k\,n}\, x[n] \qquad\qquad W_8^{k\,n} = e^{-j\left(\frac{2\pi}{8}\right)k\,n}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} = \underbrace{\begin{bmatrix}
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 7} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 5} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 3} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 1}
\end{bmatrix}}_{A} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix}$$

N=8 IDFT Matrix (1)

$$x[n] = \frac{1}{N}\sum_{k=0}^{7} W_8^{-kn}\, X[k] \qquad\qquad W_8^{-kn} = e^{+j(\frac{2\pi}{8})kn}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \begin{bmatrix}
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 7} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 6} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 7} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 5} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 7} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 3} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 2} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 7} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 1}
\end{bmatrix} \begin{bmatrix} \frac{X[0]}{N} \\ \frac{X[1]}{N} \\ \frac{X[2]}{N} \\ \frac{X[3]}{N} \\ \frac{X[4]}{N} \\ \frac{X[5]}{N} \\ \frac{X[6]}{N} \\ \frac{X[7]}{N} \end{bmatrix}$$

B

Symmetric DFT Matrix – Index (1)

$$X[k] = \sum_{n=0}^{N-1} x[n] \boxed{e^{-j(2\pi/N)kn}} \qquad X[k] = \sum_{n=0}^{N-1} x[n] \boxed{W_N^{kn}}$$

$$\begin{array}{c|ccccc|}
 & n=0 & n=1 & n=2 & \cdots & n=N-1 \\
\hline
k=0 & 0\cdot 0 & 0\cdot 1 & 0\cdot 2 & \cdots & 0\cdot(N-1) \\
k=1 & 1\cdot 0 & 1\cdot 1 & 1\cdot 2 & \cdots & 1\cdot(N-1) \\
k=2 & 2\cdot 0 & 2\cdot 1 & 2\cdot 2 & \cdots & 2\cdot(N-1) \\
\vdots & \vdots & \vdots & \vdots & & \vdots \\
k=N-1 & (N-1)\cdot 0 & (N-1)\cdot 1 & (N-1)\cdot 2 & \cdots & (N-1)\cdot(N-1) \\
\hline
\end{array}$$

Exponents in
DFT matrix **A** and
IDFT matrix **B**

$$\boldsymbol{A} = \boldsymbol{A}^T$$

$$\boldsymbol{B} = \boldsymbol{B}^T$$

Symmetric DFT Matrix – Index (2)

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn} \qquad X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn}$$

	n=0		n=1		n=2		∙ ∙ ∙		n=N-1	
k=0	$0\cdot 0$	$\xrightarrow{+0}$	$0\cdot 1$	$\xrightarrow{+0}$	$0\cdot 2$	$\xrightarrow{+0}$	∙ ∙ ∙	$\xrightarrow{+0}$	$0\cdot(N\text{-}1)$	$+\,0 \pmod N$
k=1	$1\cdot 0$	$\xrightarrow{+1}$	$1\cdot 1$	$\xrightarrow{+1}$	$1\cdot 2$	$\xrightarrow{+1}$	∙ ∙ ∙	$\xrightarrow{+1}$	$1\cdot(N\text{-}1)$	$+\,1 \pmod N$
k=2	$2\cdot 0$	$\xrightarrow{+2}$	$2\cdot 1$	$\xrightarrow{+2}$	$2\cdot 2$	$\xrightarrow{+2}$	∙ ∙ ∙	$\xrightarrow{+2}$	$2\cdot(N\text{-}1)$	$+\,2 \pmod N$
⋮	⋮		⋮		⋮				⋮	
k=N-1	$(N\text{-}1)\cdot 0$	$\xrightarrow{+(N-1)}$	$(N\text{-}1)\cdot 1$	$\xrightarrow{+(N-1)}$	$(N\text{-}1)\cdot 2$	$\xrightarrow{+(N-1)}$	∙ ∙ ∙	$\xrightarrow{+(N-1)}$	$(N\text{-}1)\cdot(N\text{-}1)$	$+\,N\text{-}1 \pmod N$

$$\boldsymbol{A} = \boldsymbol{A}^T$$
$$\boldsymbol{B} = \boldsymbol{B}^T$$

Exponents in **DFT** matrix **A** and **IDFT** matrix **B**

Symmetric DFT Matrix – Index (3)

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn} \qquad\qquad X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn}$$

	n=0	n=1	n=2	• • •	n=N-1
k=0	$0\cdot 0$	$0\cdot 1$	$0\cdot 2$	• • •	$0\cdot(N-1)$
	+0	+1	+2		+(N-1)
k=1	$1\cdot 0$	$1\cdot 1$	$1\cdot 2$	• • •	$1\cdot(N-1)$
	+0	+1	+2		+(N-1)
k=2	$2\cdot 0$	$2\cdot 1$	$2\cdot 2$	• • •	$2\cdot(N-1)$
	+0	+1	+2		+(N-1)
•	•	•	•		•
•	•	•	•		•
•	•	•	•		•
	+0	+1	+2		+(N-1)
k=N-1	$(N-1)\cdot 0$	$(N-1)\cdot 1$	$(N-1)\cdot 2$	• • •	$(N-1)\cdot(N-1)$
	$+0 \pmod{N}$	$+1 \pmod{N}$	$+2 \pmod{N}$		$+N-1 \pmod{N}$

$$A = A^T$$

$$B = B^T$$

Exponents in
DFT matrix **A** and
IDFT matrix **B**

Conjugate Transpose DFT Matrix

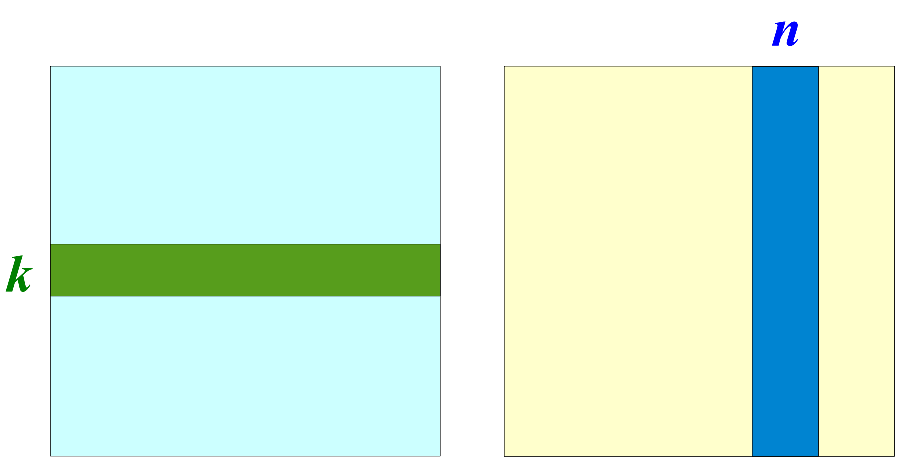
$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn} \quad \Leftrightarrow \quad x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k]\, e^{+j(2\pi/N)kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ \vdots \\ X[N-1] \end{bmatrix} = \begin{array}{c} \quad\quad k \\ n \begin{bmatrix} \quad e^{-j\left(\frac{2\pi}{N}\right)kn} \quad \end{bmatrix} \end{array} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ \vdots \\ x[N-1] \end{bmatrix} \quad \Leftrightarrow \quad \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ \vdots \\ x[N-1] \end{bmatrix} = \frac{1}{N} \begin{array}{c} \quad\quad n \\ k \begin{bmatrix} \quad e^{+j\left(\frac{2\pi}{N}\right)kn} \quad \end{bmatrix} \end{array} \begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ \vdots \\ X[N-1] \end{bmatrix}$$

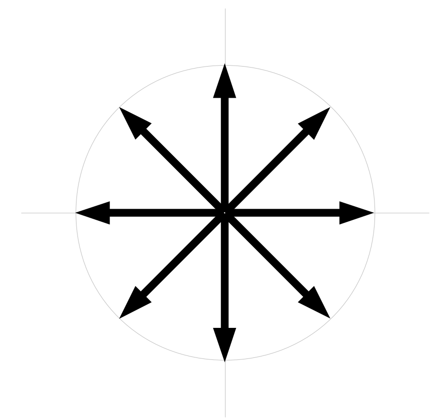
$$\begin{cases} \boldsymbol{A} = \boldsymbol{A}^T \\ \boldsymbol{B} = \boldsymbol{B}^T \end{cases} \quad \begin{cases} \boldsymbol{A}^* = \boldsymbol{B} \\ \boldsymbol{B}^* = \boldsymbol{A} \end{cases} \quad \Rightarrow \quad \begin{cases} \boldsymbol{A}^H = \boldsymbol{B} \\ \boldsymbol{B}^H = \boldsymbol{A} \end{cases}$$

Product of DFT & IDFT Matrix

$$X[k] = \sum_{n=0}^{N-1} x[n]\, \boxed{e^{-j(2\pi/N)kn}} \qquad X[k] = \sum_{n=0}^{N-1} x[n]\, \boxed{W_N^{kn}}$$



$$\begin{Bmatrix} e^{+j\left(\frac{2\pi}{N}\right)0\cdot n} \\ e^{+j\left(\frac{2\pi}{N}\right)1\cdot n} \\ \vdots \\ e^{+j\left(\frac{2\pi}{N}\right)(N-1)\cdot n} \end{Bmatrix}$$



$$\underline{\left\{ e^{-j\left(\frac{2\pi}{N}\right)k\cdot 0}, \quad e^{-j\left(\frac{2\pi}{N}\right)k\cdot 1}, \quad \cdots \quad , \quad e^{-j\left(\frac{2\pi}{N}\right)k\cdot (N-1)} \right\}}$$

Inner product

$$e^{-j\left(\frac{2\pi}{N}\right)(n-k)\cdot 0} + e^{-j\left(\frac{2\pi}{N}\right)(n-k)\cdot 1} + \cdots + e^{-j\left(\frac{2\pi}{N}\right)(n-k)\cdot (N-1)} = \begin{cases} 0 & (n \neq k) \\ N & (n = k) \end{cases}$$

N=8 DFT & IDFT Matrix (1)

	0	1	2	3	4	5	6	7	
Row 0	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	DFT
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	IDFT
Row 1	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 1}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 3}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 5}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 7}$	DFT
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 1}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 3}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 5}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 7}$	IDFT
Row 2	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	DFT
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	IDFT
Row 3	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 3}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 1}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 7}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 5}$	DFT
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 3}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 1}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 7}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 5}$	IDFT

N=8 DFT & IDFT Matrix (2)

	0	1	2	3	4	5	6	7	
Row 4	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	DFT
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	IDFT
Row 5	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 5}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 7}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 1}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 3}$	DFT
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 5}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 7}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 1}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 3}$	IDFT
Row 6	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	DFT
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	IDFT
Row 7	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 7}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 5}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 3}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 1}$	DFT
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 7}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 5}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 3}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 1}$	IDFT

Product AB – Diagonal Elements

$$C = A \cdot B \qquad [C]_{(i,j)} = [A]_{(row\ \ i)} \cdot [B]_{(col\ \ j)}$$

$$C_{(i,\, i)} = N$$

$$C_{(1,\,1)} \quad \begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 7} \end{pmatrix} \bullet \begin{pmatrix} e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 7} \end{pmatrix}^T$$

+1 +1 +1 +1 +1 +1 +1 +1 = N

Product AB – Off-Diagonal Elements

$C = A \cdot B$

$$[\boldsymbol{C}]_{(i,j)} = [\boldsymbol{A}]_{(row\ \ i)} \cdot [\boldsymbol{B}]_{(col\ \ j)}$$

$$C_{(i,j)} = 0$$

$$C_{(1,2)} \quad \begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 7} \end{pmatrix} \bullet$$

$$\begin{pmatrix} e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 6} \end{pmatrix}^T$$

$$e^{+j\cdot\frac{\pi}{4}\cdot 0} + e^{+j\cdot\frac{\pi}{4}\cdot 1} + e^{+j\cdot\frac{\pi}{4}\cdot 2} + e^{+j\cdot\frac{\pi}{4}\cdot 3} + e^{+j\cdot\frac{\pi}{4}\cdot 4} + e^{+j\cdot\frac{\pi}{4}\cdot 5} + e^{+j\cdot\frac{\pi}{4}\cdot 6} + e^{+j\cdot\frac{\pi}{4}\cdot 7} = 0$$

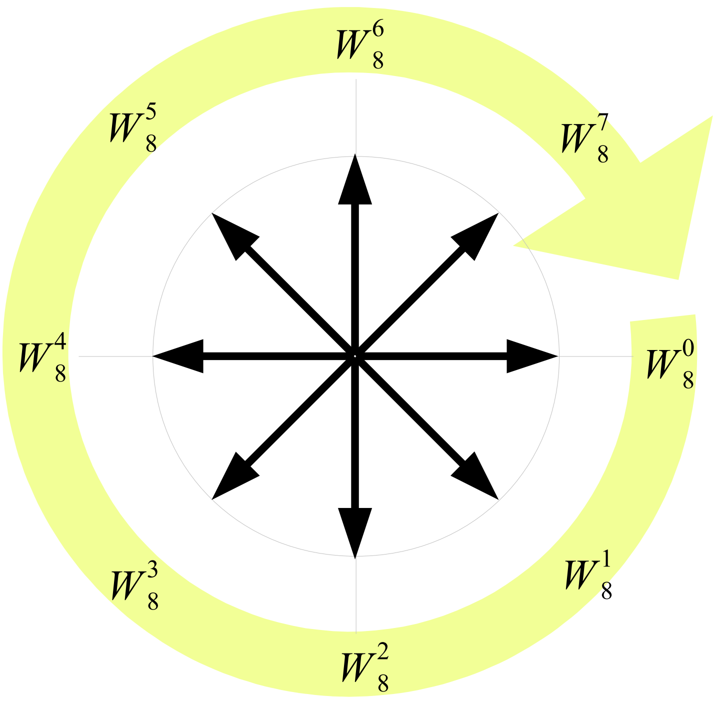
Root of Unity

$$\boxed{\sum_{k=0}^{N-1} W_N^k = \sum_{k=0}^{N-1} e^{-j\left(\frac{2\pi}{N}\right)k} = 0}$$

$$z \equiv e^{-j\left(\frac{2\pi}{N}\right)}$$

$$z^N = e^{-j\left(\frac{2\pi}{N}\right)N} = 1$$

$$\sum_{k=0}^{N-1} e^{-j\left(\frac{2\pi}{N}\right)k} = \frac{z^N - 1}{z - 1} = 0$$



$$W_8^0 + W_8^1 + W_8^2 + W_8^3 + W_8^4 + W_8^5 + W_8^6 + W_8^7 = 0$$

Cauchy-Schwartz Inequality

For all vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ of an inner product space

$$\left|\langle \boldsymbol{x}, \boldsymbol{y}\rangle\right|^2 \le \langle \boldsymbol{x}, \boldsymbol{x}\rangle \cdot \langle \boldsymbol{y}, \boldsymbol{y}\rangle$$

$$\left|\langle \boldsymbol{x}, \boldsymbol{y}\rangle\right| \le \|\boldsymbol{x}\| \cdot \|\boldsymbol{y}\|$$

The equality holds if and only if $\boldsymbol{x}$ and $\boldsymbol{y}$ are linearly dependent $\Rightarrow$ maximum

$$\boldsymbol{x}^H \cdot \boldsymbol{y} \le \|\boldsymbol{x}\| \cdot \|\boldsymbol{y}\|$$

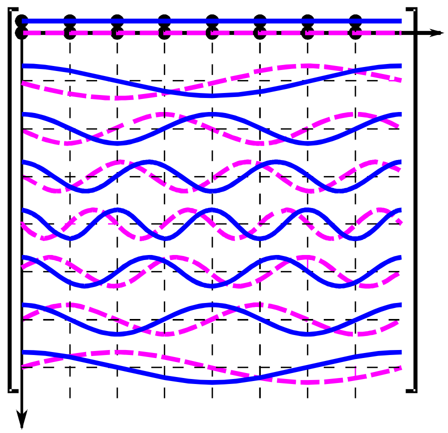
$$\boldsymbol{x} = \begin{pmatrix} a_1 + j b_1 \\ a_2 + j b_2 \\ \vdots \\ a_n + j b_n \end{pmatrix} \qquad \boldsymbol{y} = \begin{pmatrix} c_1 + j d_1 \\ c_2 + j d_2 \\ \vdots \\ c_n + j d_n \end{pmatrix}$$

Inner product is maximum when $\boldsymbol{y} = k\boldsymbol{x}^*$

$$\boldsymbol{y} = k \begin{pmatrix} a_1 - j b_1 \\ a_2 - j b_2 \\ \vdots \\ a_n - j b_n \end{pmatrix}$$

N=8 DFT : Inner Product X[0]

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} =$$



$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix}$$

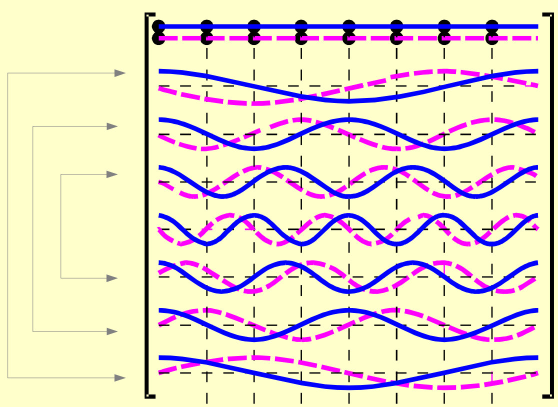
$\boldsymbol{x}$

X[0] measures "0 cycle" component in x

$$\boldsymbol{r}_0^H \cdot \boldsymbol{x} \le \|\boldsymbol{r}_0\| \cdot \|\boldsymbol{x}\|$$

maximum when $\boldsymbol{x} = k\,\boldsymbol{r}_0^*$

When $\boldsymbol{x}$ *looks like this,* $\boldsymbol{X}[0]$ *is max.*



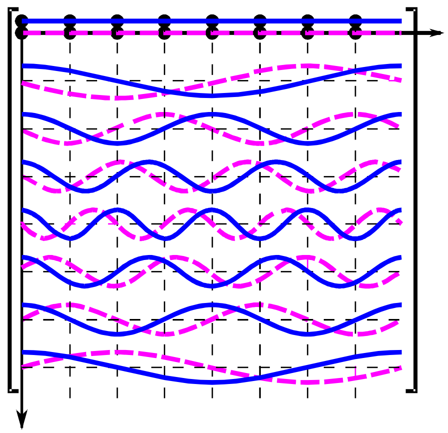
0 cycle
-1 cycles
-2 cycles
-3 cycles
-4 cycles
+3 cycles
+2 cycles
+1 cycles

$$Re\left\{e^{-j\frac{2\pi}{8}kn}\right\} = \cos\left(-\frac{2\pi}{8}kn\right)$$

$$Im\left\{e^{-j\frac{2\pi}{8}kn}\right\} = \sin\left(-\frac{2\pi}{8}kn\right)$$

N=8 DFT : Inner Product X[1]

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} =$$



$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix}$$

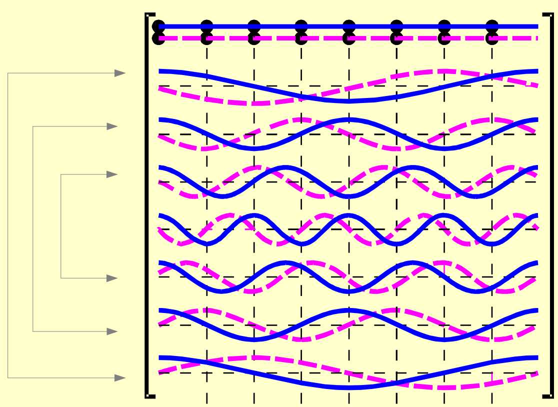
$\boldsymbol{x}$

X[1] measures "+1 cycle" component in x

$$\boldsymbol{r}_1^H \cdot \boldsymbol{x} \;\le\; \|\boldsymbol{r}_1\| \cdot \|\boldsymbol{x}\|$$

maximum when $\boldsymbol{x} = k\,\boldsymbol{r}_1^*$

*When **x** looks like this, **X**[1] is max.*

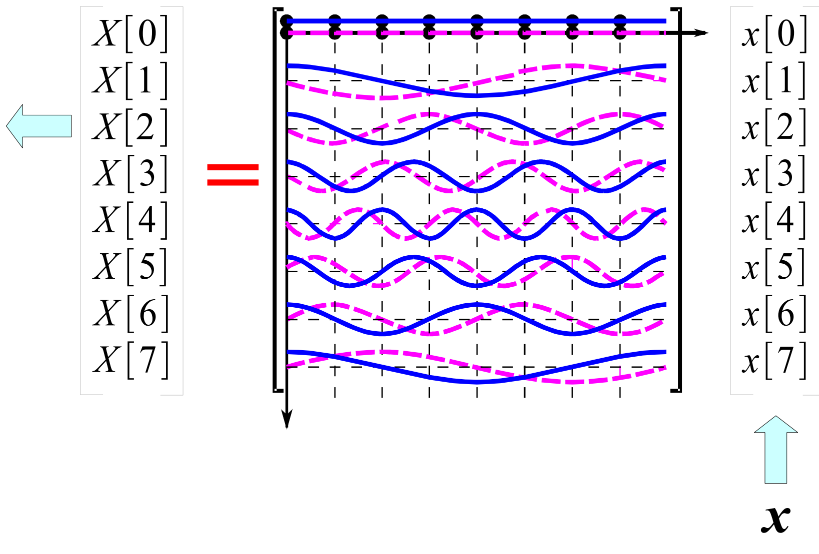


0 cycle
-1 cycles
-2 cycles
-3 cycles
-4 cycles
+3 cycles
+2 cycles
+1 cycles

$$Re\left\{ e^{-j\frac{2\pi}{8}kn} \right\} = \cos\left(-\frac{2\pi}{8}kn\right)$$

$$Im\left\{ e^{-j\frac{2\pi}{8}kn} \right\} = \sin\left(-\frac{2\pi}{8}kn\right)$$

N=8 DFT : Inner Product X[2]

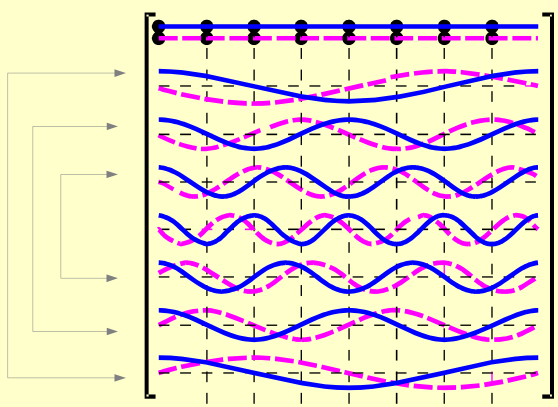


X[2] measures "+2 cycle" component in x

$$\boldsymbol{r}_2^H \cdot \boldsymbol{x} \le \|\boldsymbol{r}_2\| \cdot \|\boldsymbol{x}\|$$

maximum when $\boldsymbol{x} = k\,\boldsymbol{r}_2^*$

When $\boldsymbol{x}$ *looks like this,* $\boldsymbol{X}[2]$ *is max.*

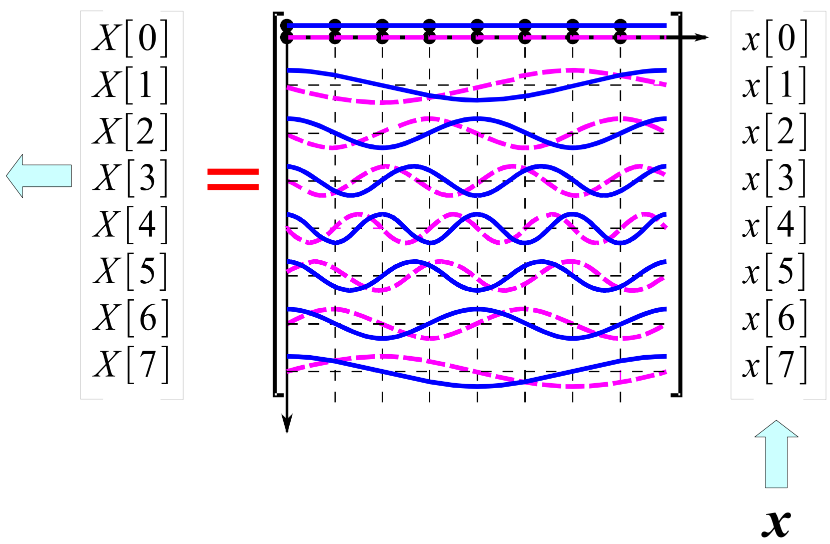


0 cycle
-1 cycles
-2 cycles
-3 cycles
-4 cycles
+3 cycles
+2 cycles
+1 cycles

$$Re\left\{e^{-j\frac{2\pi}{8}kn}\right\} = \cos\left(-\frac{2\pi}{8}kn\right)$$

$$Im\left\{e^{-j\frac{2\pi}{8}kn}\right\} = \sin\left(-\frac{2\pi}{8}kn\right)$$

N=8 DFT : Inner Product X[3]

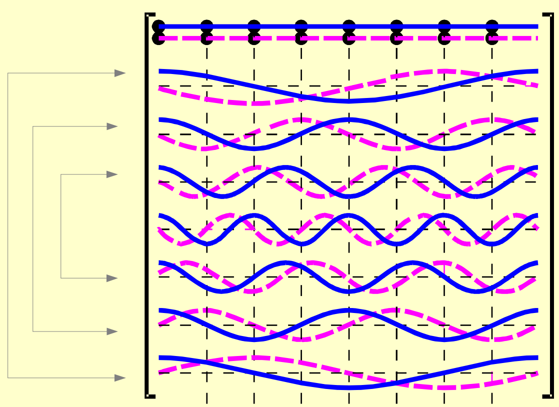


X[3] measures "+3 cycle" component in x

$$\boldsymbol{r}_3^H \cdot \boldsymbol{x} \le \|\boldsymbol{r}_3\| \cdot \|\boldsymbol{x}\|$$

maximum when $\boldsymbol{x} = k\,\boldsymbol{r}_3^*$

*When **x** looks like this, **X**[3] is max.*

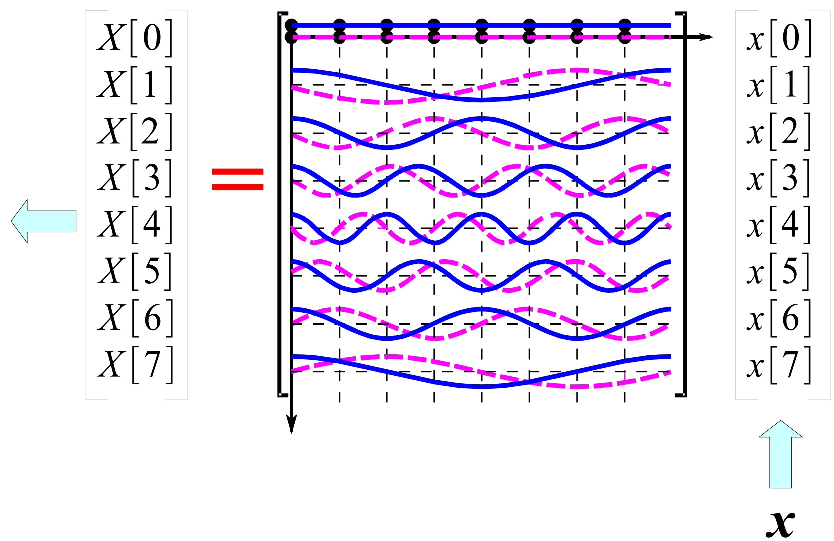


0 cycle
-1 cycles
-2 cycles
-3 cycles
-4 cycles
+3 cycles
+2 cycles
+1 cycles

$$Re\left\{e^{-j\frac{2\pi}{8}kn}\right\} = \cos\left(-\frac{2\pi}{8}kn\right)$$

$$Im\left\{e^{-j\frac{2\pi}{8}kn}\right\} = \sin\left(-\frac{2\pi}{8}kn\right)$$

N=8 DFT : Inner Product X[4]

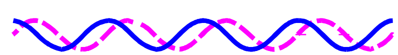


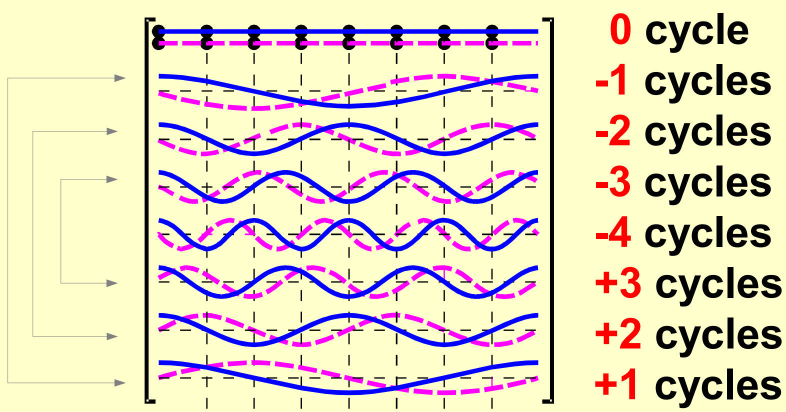
X[4] measures "+4 cycle" component in x

$$\boldsymbol{r}_4^H \cdot \boldsymbol{x} \le \|\boldsymbol{r}_4\| \cdot \|\boldsymbol{x}\|$$

maximum when $\boldsymbol{x} = k\,\boldsymbol{r}_4^*$

When $\boldsymbol{x}$ *looks like this,* $\boldsymbol{X}[4]$ *is max.*

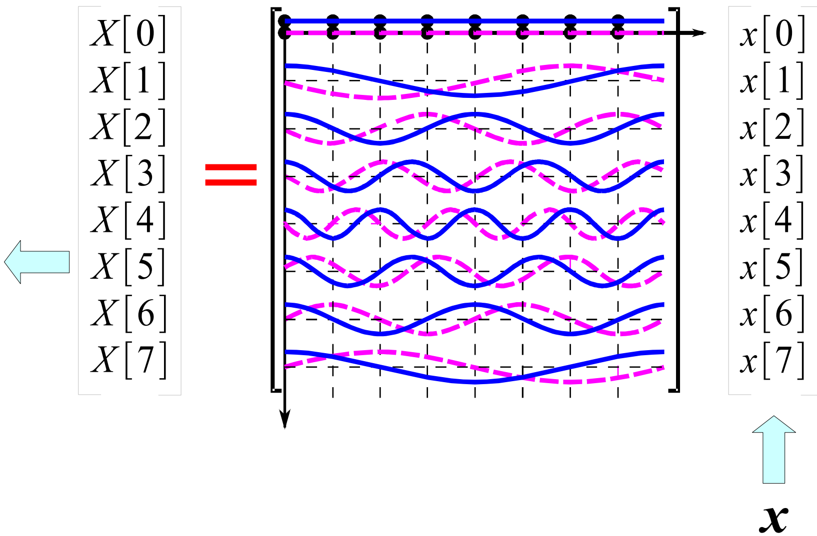




$$Re\left\{e^{-j\frac{2\pi}{8}kn}\right\} = \cos\left(-\frac{2\pi}{8}kn\right)$$

$$Im\left\{e^{-j\frac{2\pi}{8}kn}\right\} = \sin\left(-\frac{2\pi}{8}kn\right)$$

N=8 DFT : Inner Product X[5]

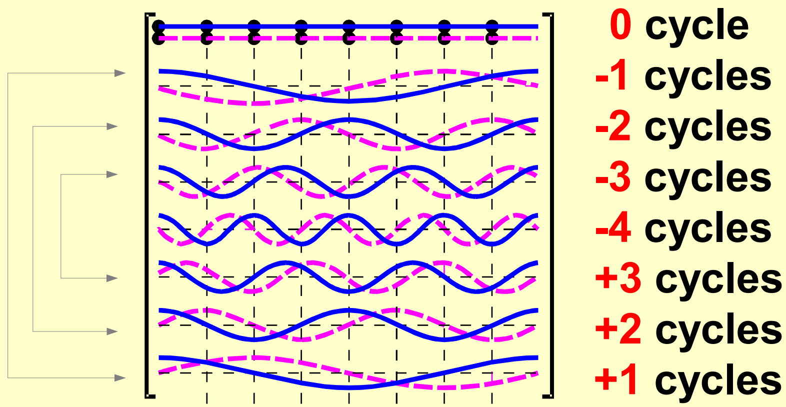


X[5] measures "-3 cycle" component in x

$$\boldsymbol{r}_5^H \cdot \boldsymbol{x} \;\le\; \|\boldsymbol{r}_5\| \cdot \|\boldsymbol{x}\|$$

maximum when $\boldsymbol{x} = k\,\boldsymbol{r}_5^*$

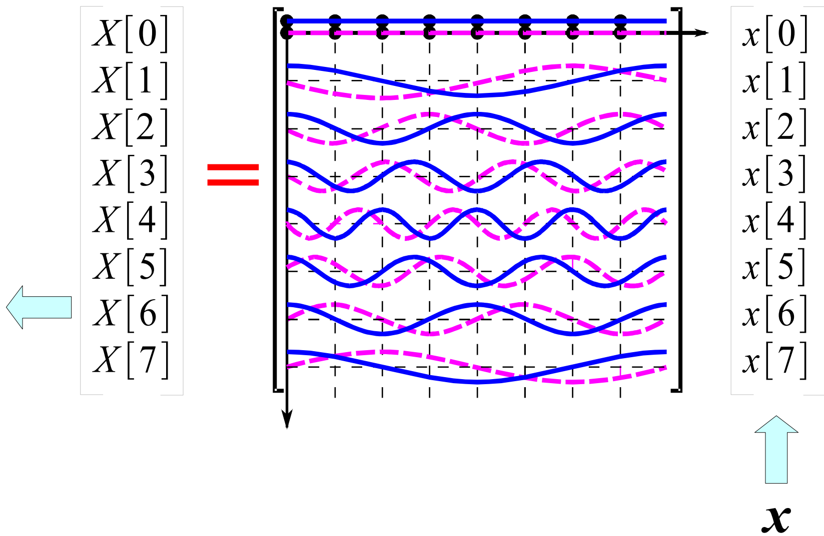
*When **x** looks like this, **X**[5] is max.*



$$Re\left\{e^{-j\frac{2\pi}{8}kn}\right\} = \cos\left(-\frac{2\pi}{8}kn\right)$$

$$Im\left\{e^{-j\frac{2\pi}{8}kn}\right\} = \sin\left(-\frac{2\pi}{8}kn\right)$$

N=8 DFT : Inner Product X[6]

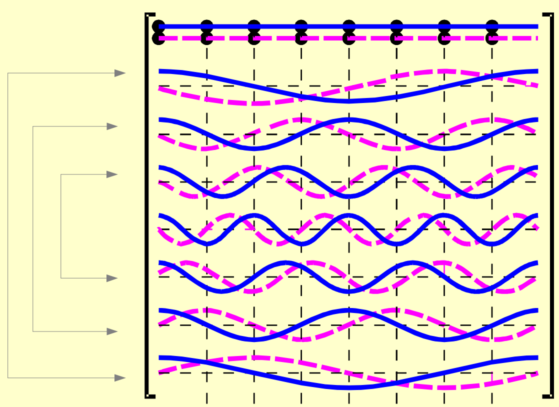


X[6] measures "-2 cycle" component in x

$$\boldsymbol{r}_6^H \cdot \boldsymbol{x} \le \|\boldsymbol{r}_6\| \cdot \|\boldsymbol{x}\|$$

maximum when $\boldsymbol{x} = k\,\boldsymbol{r}_6^*$

*When **x** looks like this, **X**[6] is max.*



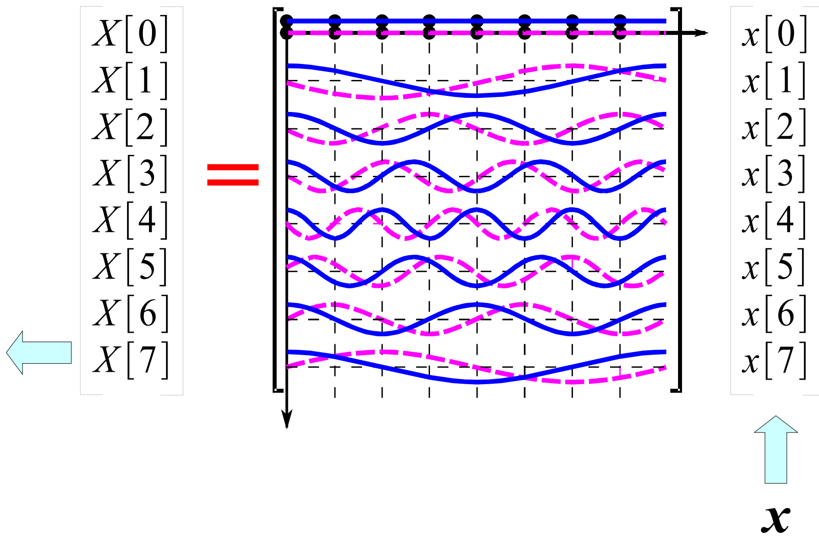
0 cycle
-1 cycles
-2 cycles
-3 cycles
-4 cycles
+3 cycles
+2 cycles
+1 cycles

$$Re\left\{e^{-j\frac{2\pi}{8}kn}\right\} = \cos\left(-\frac{2\pi}{8}kn\right)$$

$$Im\left\{e^{-j\frac{2\pi}{8}kn}\right\} = \sin\left(-\frac{2\pi}{8}kn\right)$$

Young Won Lim
5/14/11

N=8 DFT : Inner Product X[7]

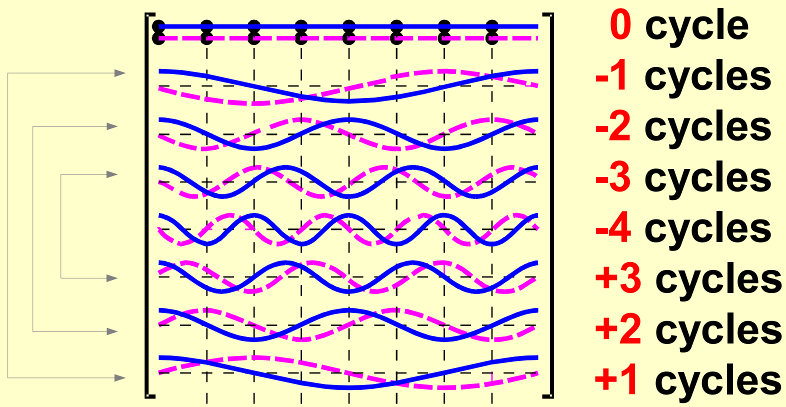


X[7] measures "-1 cycle" component in x

$$\boldsymbol{r}_7^H \cdot \boldsymbol{x} \le \|\boldsymbol{r}_7\| \cdot \|\boldsymbol{x}\|$$

maximum when $\boldsymbol{x} = k\,\boldsymbol{r}_7^*$

*When **x** looks like this, **X**[7] is max.*

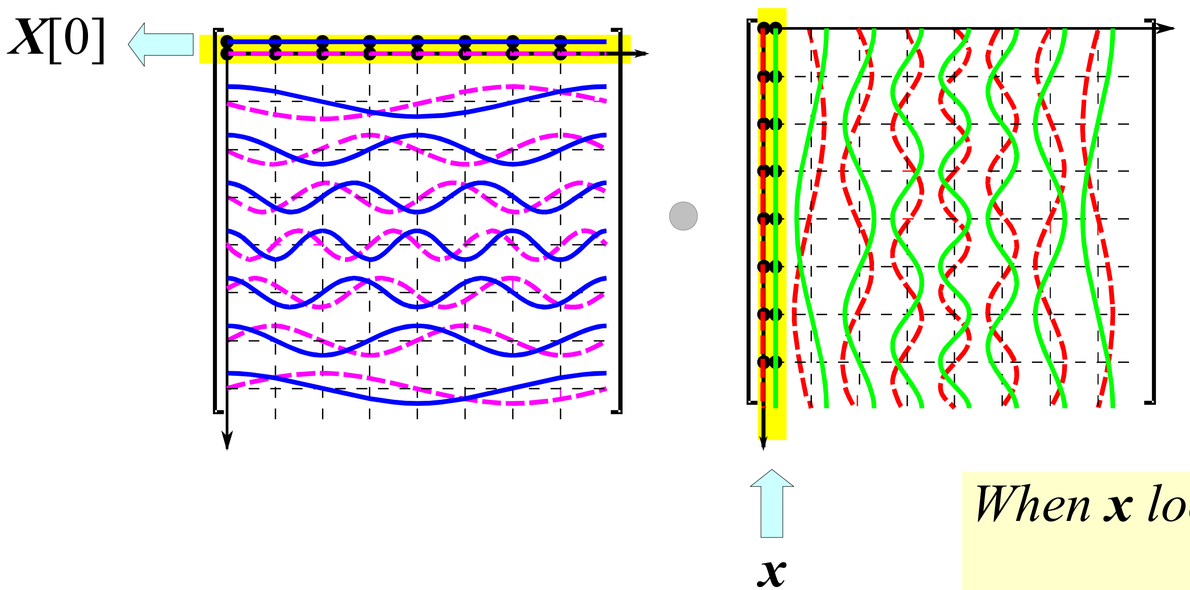


$$Re\left\{e^{-j\frac{2\pi}{8}kn}\right\} = \cos\left(-\frac{2\pi}{8}kn\right)$$

$$Im\left\{e^{-j\frac{2\pi}{8}kn}\right\} = \sin\left(-\frac{2\pi}{8}kn\right)$$

N=8 DFT : X[0] in A·B

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad W_8^{kn} = e^{-j(\frac{2\pi}{8})kn}$$



$$\boldsymbol{A} \cdot \boldsymbol{B} = \boldsymbol{A} \cdot \boldsymbol{A}^H = N\,\boldsymbol{I}$$

$$\boldsymbol{U} \cdot \boldsymbol{U}^H = \boldsymbol{I}$$

Unitary Matrix

When $\boldsymbol{x}$ *looks like this,* $\boldsymbol{X}[0]$ *is max.* $(=\boldsymbol{N})$

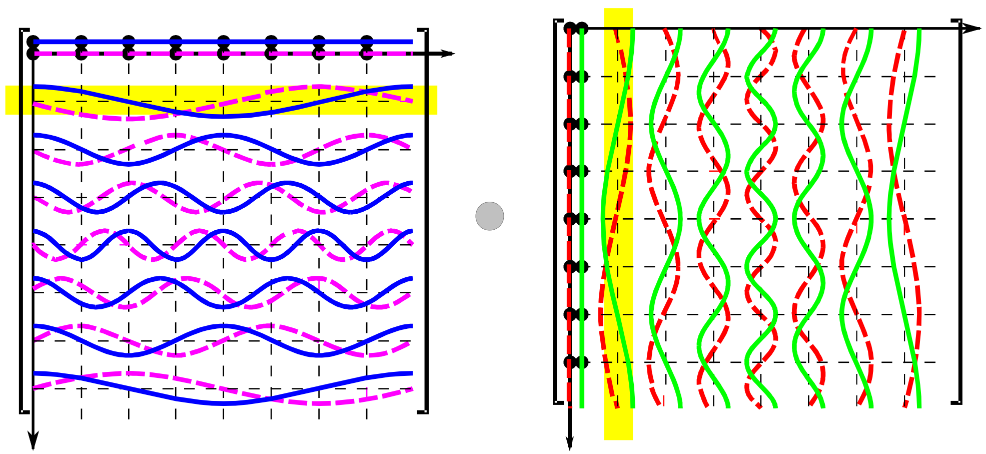
$\boldsymbol{X}[k] = 0 \quad \text{for } k \neq 0$

$$\boldsymbol{X}[0] = \begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} \end{pmatrix} \bullet \begin{pmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] & x[6] & x[7] \end{pmatrix}^T$$

N=8 DFT : X[1] in A·B

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$

$\boldsymbol{X}[1]$



$\boldsymbol{x}$

$$\boldsymbol{A}\cdot\boldsymbol{B} = \boldsymbol{A}\cdot\boldsymbol{A}^H = N\,\boldsymbol{I}$$

$$\boldsymbol{U}\cdot\boldsymbol{U}^H = \boldsymbol{I}$$

Unitary Matrix

When $\boldsymbol{x}$ *looks like this,* $\boldsymbol{X}[1]$ *is max.* $(=\boldsymbol{N})$

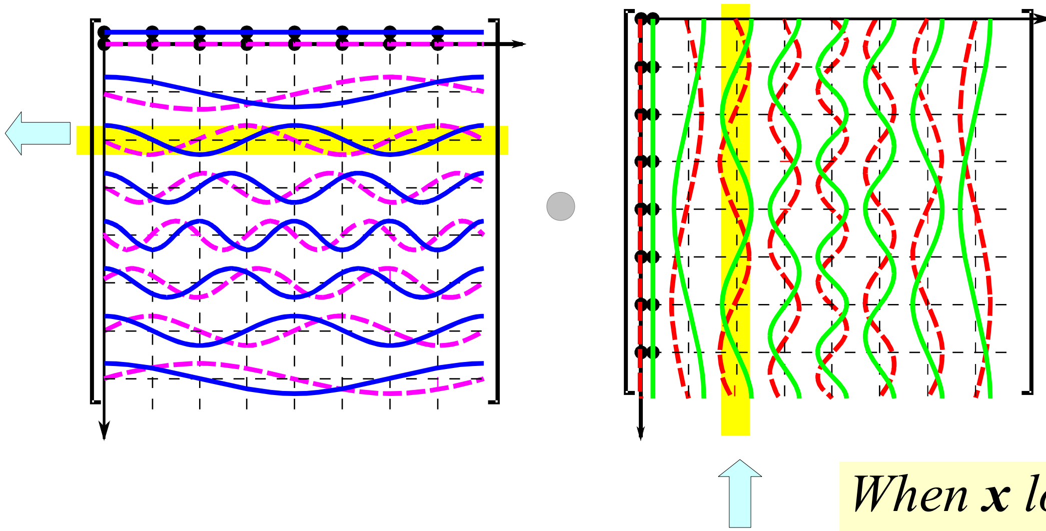
$\boldsymbol{X}[k] = 0 \quad for \;\; k \neq 1$

$$X[1] = \begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 7} \end{pmatrix} \bullet \begin{pmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] & x[6] & x[7] \end{pmatrix}^T$$

N=8 DFT : X[2] in A·B

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$

$\boldsymbol{X}[2]$



$\boldsymbol{x}$

$$\boldsymbol{A}\cdot\boldsymbol{B} = \boldsymbol{A}\cdot\boldsymbol{A}^H = N\,\boldsymbol{I}$$

$$\boldsymbol{U}\cdot\boldsymbol{U}^H = \boldsymbol{I}$$

Unitary Matrix

When $\boldsymbol{x}$ *looks like this,* $\boldsymbol{X}[2]$ *is max.* $(=\boldsymbol{N})$

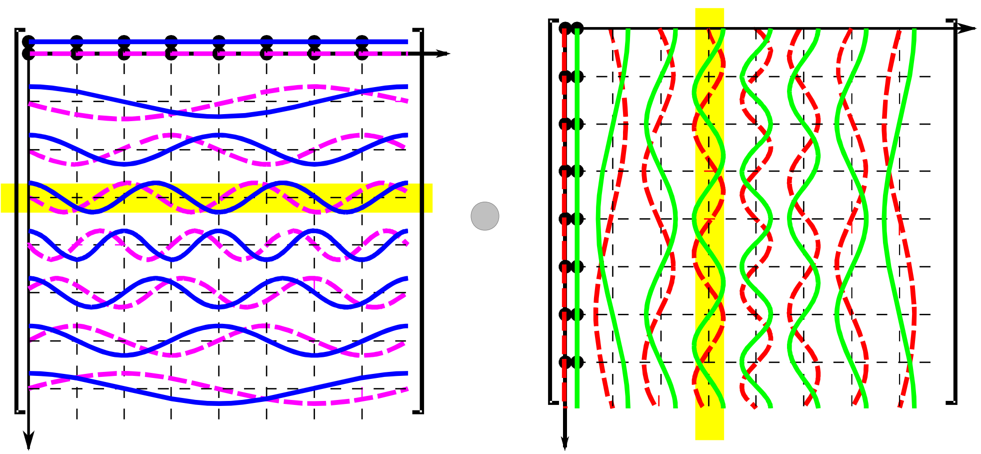
$\boldsymbol{X}[k] = 0 \quad for \ \ k \neq 2$

$$\boldsymbol{X}[2] = \begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} \end{pmatrix} \bullet \begin{pmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] & x[6] & x[7] \end{pmatrix}^T$$

N=8 DFT : X[3] in A·B

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$

$\boldsymbol{X}[3]$



$$\boldsymbol{A} \cdot \boldsymbol{B} = \boldsymbol{A} \cdot \boldsymbol{A}^H = N\,\boldsymbol{I}$$

$$\boldsymbol{U} \cdot \boldsymbol{U}^H = \boldsymbol{I}$$

Unitary Matrix

$\boldsymbol{x}$

When $\boldsymbol{x}$ *looks like this,* $\boldsymbol{X}[3]$ *is max.* (= $\boldsymbol{N}$)

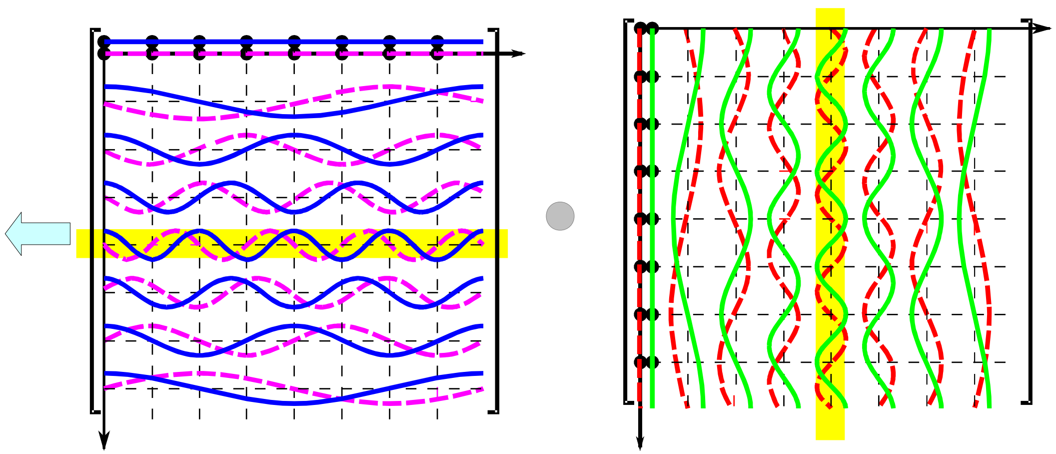
$\boldsymbol{X}[k] = 0 \quad \text{for } k \neq 3$

$$\boldsymbol{X}[3] = \begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 5} \end{pmatrix} \bullet \begin{pmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] & x[6] & x[7] \end{pmatrix}^T$$

N=8 DFT : X[4] in A·B

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$

X[4]



$$\boldsymbol{A}\cdot\boldsymbol{B} = \boldsymbol{A}\cdot\boldsymbol{A}^H = N\,\boldsymbol{I}$$

$$\boldsymbol{U}\cdot\boldsymbol{U}^H = \boldsymbol{I}$$

Unitary Matrix

*When **x** looks like this, **X**[4] is max. (=**N**)*

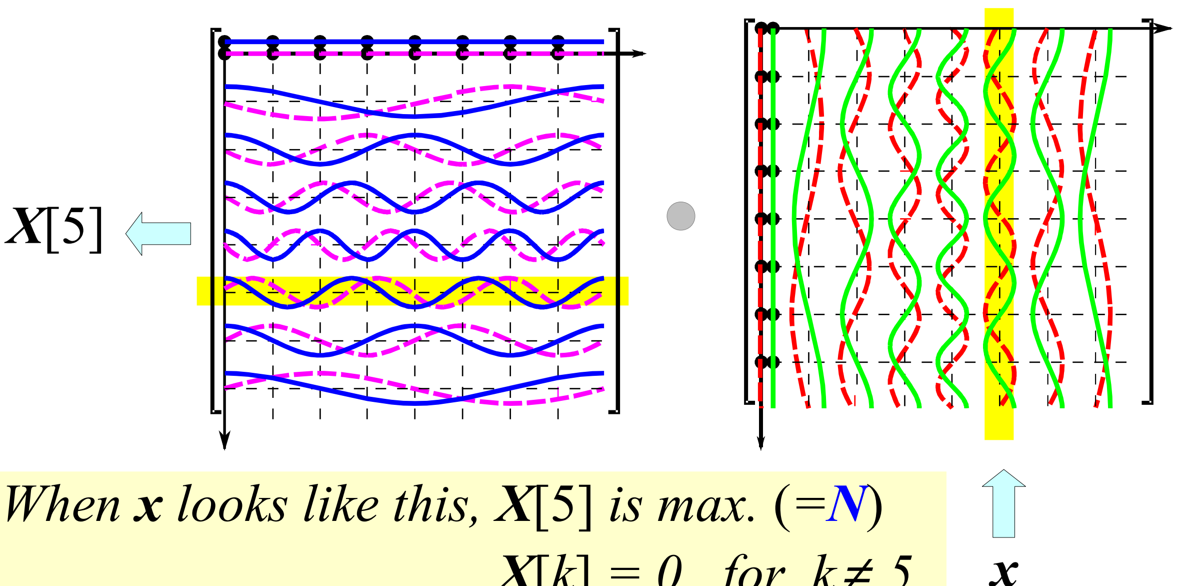
$\boldsymbol{X}[k] = 0$ for $k \neq 4$

x

$$X[4] = \begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} \end{pmatrix} \bullet \begin{pmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] & x[6] & x[7] \end{pmatrix}^T$$

N=8 DFT : X[5] in A·B

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad W_8^{kn} = e^{-j(\frac{2\pi}{8})kn}$$



$$\boldsymbol{A} \cdot \boldsymbol{B} = \boldsymbol{A} \cdot \boldsymbol{A}^H = N\,\boldsymbol{I}$$

$$\boldsymbol{U} \cdot \boldsymbol{U}^H = \boldsymbol{I}$$

Unitary Matrix

*When **x** looks like this, **X**[5] is max. (=**N**)*

$\boldsymbol{X}[k] = 0 \quad for \;\; k \neq 5$

$$X[5] = \begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 3} \end{pmatrix} \bullet \begin{pmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] & x[6] & x[7] \end{pmatrix}^T$$

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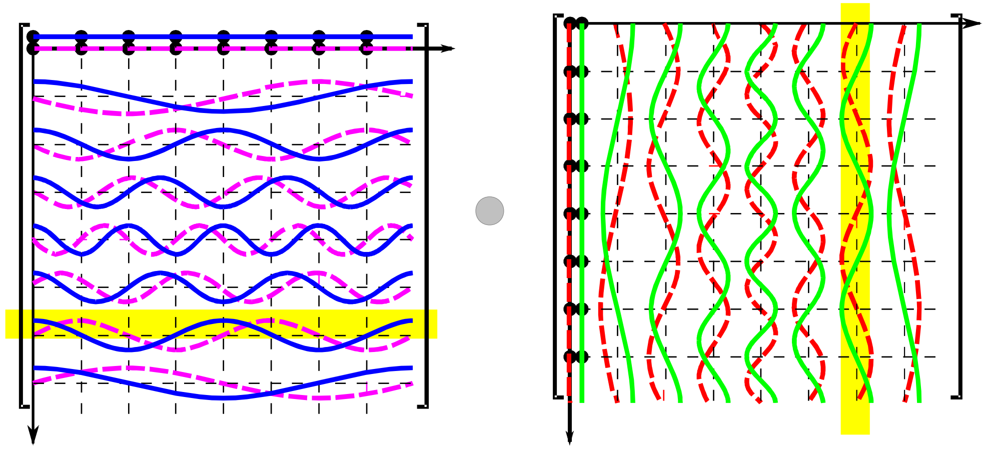
N=8 DFT : X[6] in A·B

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad W_8^{kn} = e^{-j(\frac{2\pi}{8})kn}$$

$X[6]$



$$A \cdot B = A \cdot A^H = N\, I$$

$$U \cdot U^H = I$$

Unitary Matrix

When $\boldsymbol{x}$ *looks like this,* $\boldsymbol{X}[6]$ *is max. (=*$\boldsymbol{N}$*)*

$\boldsymbol{X}[k] = 0 \quad$ *for* $\; k \neq 6$

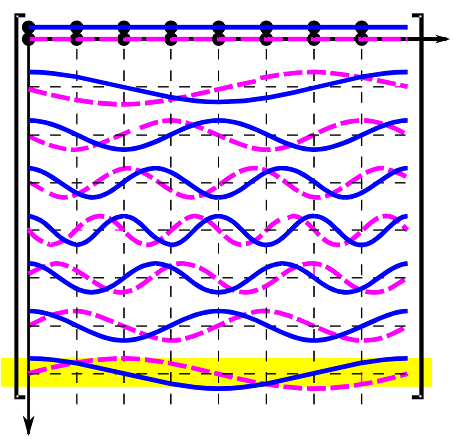
$\boldsymbol{x}$

$$X[6] = \begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} \end{pmatrix} \bullet \begin{pmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] & x[6] & x[7] \end{pmatrix}^T$$

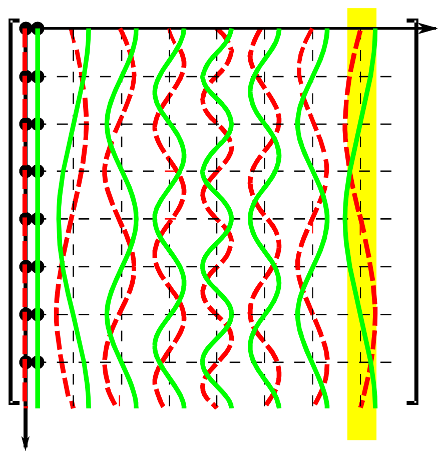
N=8 DFT : X[7] in A·B

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$

$\boldsymbol{X}[7]$



$\bullet$



$\boldsymbol{x}$

$$\boldsymbol{A}\cdot\boldsymbol{B} = \boldsymbol{A}\cdot\boldsymbol{A}^H = N\,\boldsymbol{I}$$

$$\boldsymbol{U}\cdot\boldsymbol{U}^H = \boldsymbol{I}$$

Unitary Matrix

When $\boldsymbol{x}$ *looks like this,* $\boldsymbol{X}[7]$ *is max. (=* $\boldsymbol{N}$ *)*

$\boldsymbol{X}[k] = 0 \quad for \;\; k \neq 7$

$$\boldsymbol{X}[7] = \begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 1} \end{pmatrix} \bullet \begin{pmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] & x[6] & x[7] \end{pmatrix}^T$$

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References

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