

Homework 1- problem 2

Given: $C_0(Y^1, t) = -F^1[(1 - \bar{R} U_{,ss}(Y^1, t))] - F^2 U_{,s}^2 - \frac{T}{R} + M[[1 - \bar{R} U_{,ss}^2(Y^1, t)][U_{,tt}^1(Y^1, t) - \bar{R} U_{,stt}^2(Y^1, t)] + U_{,s}^2(Y^1, t) U_{,tt}^2(Y^1, t)]$

the subscripts "s" and "t" indicate partial differentiation with respect to space and time.

Find: do dimensional analysis of all terms in the above equation and determine physical meaning.

Solution:

Dimensional Analysis of Terms

$$[F^1] = \frac{m * L}{t^2} = F = force$$

$$[F^2] = \frac{m * L}{t^2} = force$$

$$[\bar{R}] = L = length$$

$$[U_{,s}^2] = \frac{L}{L} = 1$$

$$[U^2] = L = length$$

$$[T] = F * L = torque$$

$$[U_{,ss}^2] = \frac{\partial^2 U^2}{\partial S^2} = \frac{L}{L^2} = L^{-1}$$

$$[R] = L = length$$

$$[\bar{R} U_{,ss}^2] = L * \frac{L}{L^2} = 1$$

$$[\frac{T}{R}] = F * \frac{L}{L} = F = force$$

$$[M] = m = mass of wheel$$

$$[U_{,tt}^1] = \frac{\partial^2 U^2}{\partial t^2} = \frac{L}{t^2} = acceleration$$

$$[U_{,stt}^2] = \frac{\partial}{\partial s} (\frac{\partial^2 U^2}{\partial t^2}) = \frac{L}{L * t^2} = t^{-2}$$

$$[U_{,s}^2] = \frac{\partial U^2}{\partial s} = \frac{L}{L} = 1$$

$$[U_{,tt}^2] = \frac{\partial^2 U^2}{\partial t^2} = \frac{L}{t^2} = acceleration$$

Physical Meaning

- 1) C_0 = sum of horizontal forces acting on the wheel
- 2) Each Term on the right hand side of the equation is a force component acting in the horizontal direction.
- 3) The terms with " F^1 " are reaction forces on the wheel due to " F^1 " in the horizontal direction.
- 4) The terms with " F^2 " are reaction forces due to " F^1 " in the horizontal direction.
- 5) The term " $\frac{T}{R}$ " is the horizontal force component caused by the torque " T " acting on the wheel.
- 6) The terms with " M " are the horizontal force components due to the acceleration of the mass of the wheel.