Elementary Matrix

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A General m x n Matrix
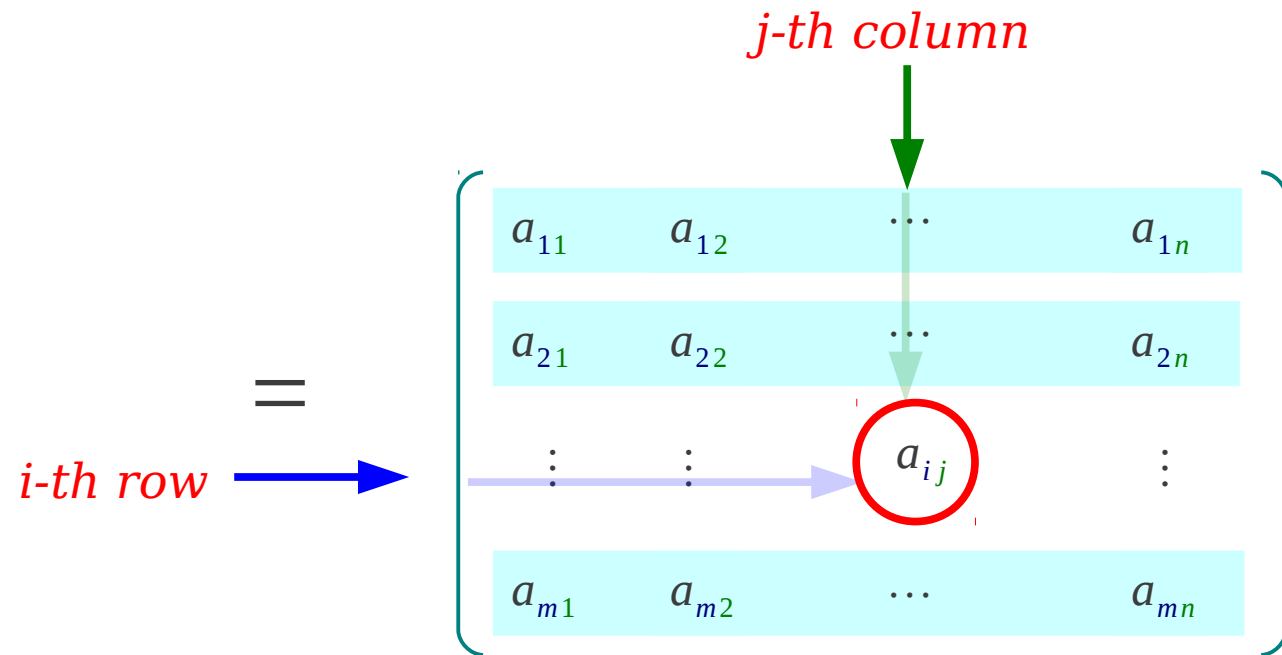
$$A \;=\; \left[a_{ij}\right]_{m\times n} \;=\; \left[a_{ij}\right]$$

$$=\;
\begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & & \vdots \\
a_{m1} & a_{m2} & \cdots & a_{mn}
\end{bmatrix}$$

A Element of a Matrix

$$A = [a_{ij}]_{m \times n} = [a_{ij}]$$

j-th column

$$= \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & a_{ij} & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

i-th row

$$(A)_{m \times n} = a_{ij}$$

Matrix Multiplication (1)

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

1st row →

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

2nd row →

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

m-th row →

$$A = \left[a_{ij} \right]_{m \times n}$$

$$x = \left[x_{ij} \right]_{n \times 1}$$

$$b = \left[b_{ij} \right]_{m \times 1}$$

$$A \quad x = \quad b$$

$$m \times n \quad n \times 1 \qquad m \times 1$$

Matrix Multiplication (2)

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ & \\ \vdots & \vdots \\ & \\ b_{n1} & b_{n2} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ = & \\ c_{21} & c_{22} \\ \vdots & \vdots \\ = & \\ c_{m1} & c_{m2} \end{pmatrix}$$

1st row *1st column*

$$A = \left[a_{ij}\right]_{m \times n}$$

$$B = \left[b_{ij}\right]_{n \times 2}$$

$$C = \left[c_{ij}\right]_{m \times 2}$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ & \\ \vdots & \vdots \\ & \\ b_{n1} & b_{n2} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ = & \\ c_{21} & c_{22} \\ \vdots & \vdots \\ = & \\ c_{m1} & c_{m2} \end{pmatrix}$$

2nd row *1st column*

$$A \quad B = C$$

$$m \times n \quad n \times 2 \quad m \times 2$$

Matrix Multiplication (3)

$$A = \begin{bmatrix} a_{ij} \end{bmatrix}_{m \times n}$$

$$B = \begin{bmatrix} b_{ij} \end{bmatrix}_{n \times 2}$$

$$C = \begin{bmatrix} c_{ij} \end{bmatrix}_{m \times 2}$$

$$A \quad B = \quad C$$

$$m \times n \quad n \times 2 \quad m \times 2$$

Matrix Multiplication (4)

1st column

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ \vdots & \vdots \\ b_{n1} & b_{n2} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ \vdots & \vdots \\ c_{m1} & c_{m2} \end{bmatrix}$$

m-th row

$$A = \left[a_{ij} \right]_{m \times n}$$

$$B = \left[b_{ij} \right]_{n \times 2}$$

$$C = \left[c_{ij} \right]_{m \times 2}$$

2nd column

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ \vdots & \vdots \\ b_{n1} & b_{n2} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ \vdots & \vdots \\ c_{m1} & c_{m2} \end{bmatrix}$$

m-th row

$$A \quad B = C$$

$$m \times n \quad n \times 2 \quad m \times 2$$

Matrix Multiplication (4)

1st column

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ \vdots & \vdots \\ \\ b_{n1} & b_{n2} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ \vdots & \vdots \\ c_{m1} & c_{m2} \end{bmatrix}$$

m-th row

$$A = \left[a_{ij} \right]_{m \times n}$$

$$B = \left[b_{ij} \right]_{n \times 2}$$

$$C = \left[c_{ij} \right]_{m \times 2}$$

2nd column

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ \vdots & \vdots \\ \\ b_{n1} & b_{n2} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ \vdots & \vdots \\ c_{m1} & c_{m2} \end{bmatrix}$$
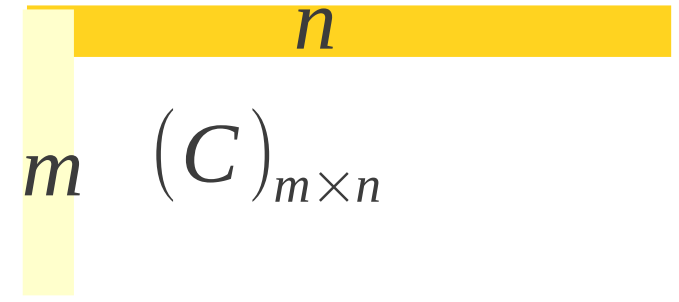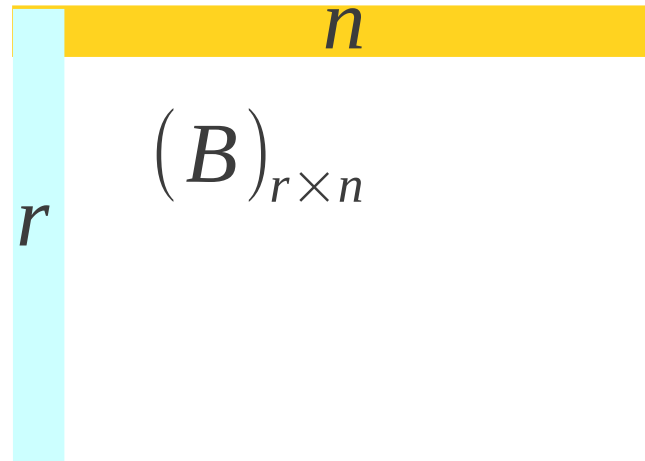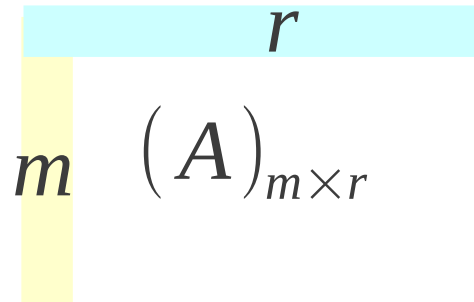
m-th row

$$A \quad B = C$$

$$m \times n \quad n \times 2 \quad m \times 2$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1r} \\ a_{21} & a_{22} & \cdots & a_{2r} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mr} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{r1} & b_{r2} & \cdots & b_{rn} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & c_{ii} & \vdots \\ c_{r1} & c_{r2} & \cdots & c_{rn} \end{bmatrix}$$

$$m \; (A)_{m \times r} \qquad r \quad (B)_{r \times n} \qquad m \; (C)_{m \times n}$$

r $\qquad$ n $\qquad$ n

Multiplication of Matrices (2)

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1r} \\ a_{21} & a_{22} & \cdots & a_{2r} \\ \vdots & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ir} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mr} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ b_{r1} & b_{r2} & \cdots & b_{r2} & \cdots & b_{rn} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & & c_{1n} \\ c_{21} & c_{22} & \cdots & & c_{2n} \\ \vdots & \vdots & & c_{ii} & \vdots \\ c_{r1} & c_{r2} & \cdots & & c_{rn} \end{bmatrix}$$

$$(AB)_{ij} = c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} \quad \cdots \quad + a_{ir}b_{rj}$$

$$= \sum_{k=1}^{r} a_{ik}b_{kj}$$

Partitioned Matrix

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ \vdots & \vdots \\ b_{n1} & b_{n2} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ \vdots & \vdots \\ c_{m1} & c_{m2} \end{bmatrix}$$

$$A = [a_{ij}]_{m \times n}$$

$$B = [b_{ij}]_{n \times 2}$$

$$C = [c_{ij}]_{m \times 2}$$

$$\boldsymbol{b}_1 \qquad \boldsymbol{b}_2$$

$$B = [\boldsymbol{b}_1 \ \boldsymbol{b}_2] \qquad\qquad AB = [A\boldsymbol{b}_1 \ A\boldsymbol{b}_2]$$

$$n \times 2 \qquad n \times 1 \ n \times 1 \qquad\qquad m \times 2 \qquad m \times 1 \qquad m \times 1$$

$$m \times n \ n \times 2 \qquad m \times n \ n \times 1 \ m \times n \ n \times 1$$

$$\boldsymbol{a}_1 \rightarrow$$
$$\boldsymbol{a}_2 \rightarrow$$
$$\boldsymbol{a}_m \rightarrow$$

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ \vdots & \vdots \\ \vdots & \vdots \\ b_{n1} & b_{n2} \end{pmatrix} \begin{matrix} = \\ = \\ \\ = \end{matrix} \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ \vdots & \vdots \\ c_{m1} & c_{m2} \end{pmatrix}$$

$$A = \left[a_{ij} \right]_{m \times n}$$

$$B = \left[b_{ij} \right]_{n \times 2}$$

$$C = \left[c_{ij} \right]_{m \times 2}$$

$$A = \begin{bmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \\ \vdots \\ \boldsymbol{a}_m \end{bmatrix} \begin{matrix} 1 \times n \\ 1 \times n \\ \\ 1 \times n \end{matrix}$$

$$m \times n$$

$$AB = \begin{bmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \\ \vdots \\ \boldsymbol{a}_m \end{bmatrix} \begin{matrix} 1 \times 2 & 1 \times n\ n \times 2 \\ 1 \times 2 & 1 \times n\ n \times 2 \\ \\ 1 \times 2 & 1 \times n\ n \times 2 \end{matrix}$$

$$m \times 2$$

$$m \times n\ n \times 2$$

Pulse

Pulse

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References

[1] http://en.wikipedia.org/
[2] Anton & Busby, "Contemporary Linear Algebra"