

Problem 2.7

Derive the Maclaurin Series expansions for $\cos(t)$ and $\sin(t)$

$$f(t) = \cos(t)$$

Derivative	Value at t=0
$f(t) = \cos(t)$	1
$f'(t) = -\sin(t)$	0
$f''(t) = -\cos(t)$	-1
$f'''(t) = \sin(t)$	0
...	...

$$\cos(t) = f(0) + f'(0)t + \frac{f''(0)t^2}{2!} + \frac{f'''(0)t^3}{3!} + \dots$$

$$\cos(t) = 1 - \frac{1}{2}t^2 + \frac{1}{24}t^4 + \dots$$

$$\cos(t) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$f(t) = \sin(t)$$

Derivative	Value at t=0
$f(t) = \sin(t)$	0
$f'(t) = \cos(t)$	1
$f''(t) = -\sin(t)$	0
$f'''(t) = -\cos(t)$	-1
...	...

$$\sin(t) = f(0) + f'(0)t + \frac{f''(0)t^2}{2!} + \frac{f'''(0)t^3}{3!} + \dots$$

$$\sin(t) = t - \frac{1}{6}t^3 + \frac{1}{120}t^5 + \dots$$

$$\sin(t) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$