

CORDIC Background (4A)

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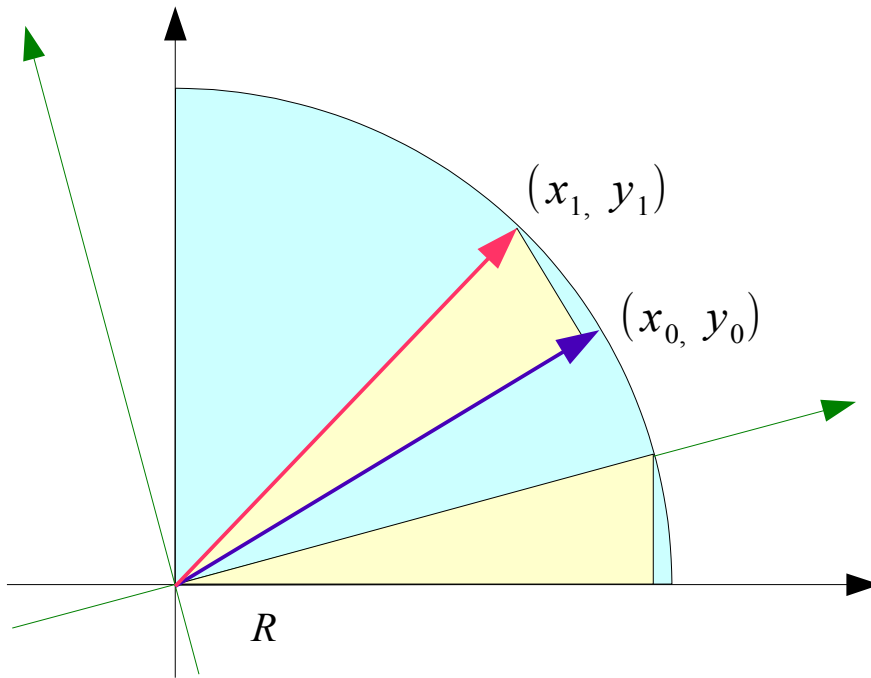
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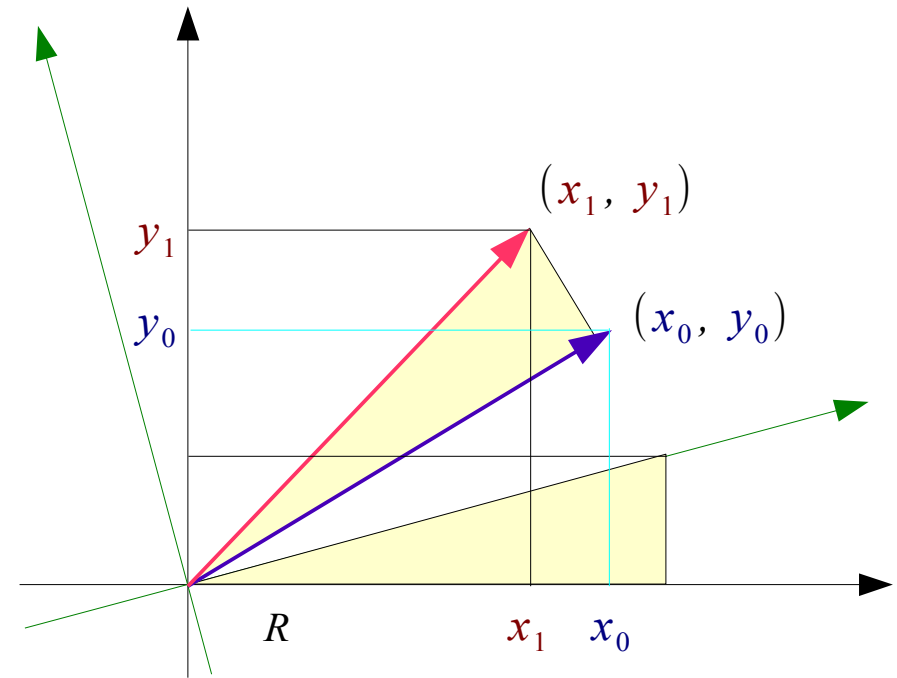
CORDIC Background

J. P. Deschamps, G. A. Bioul, G.D. Sutter, Synthesis of Arithmetic Circuits

Vector Rotation (1)

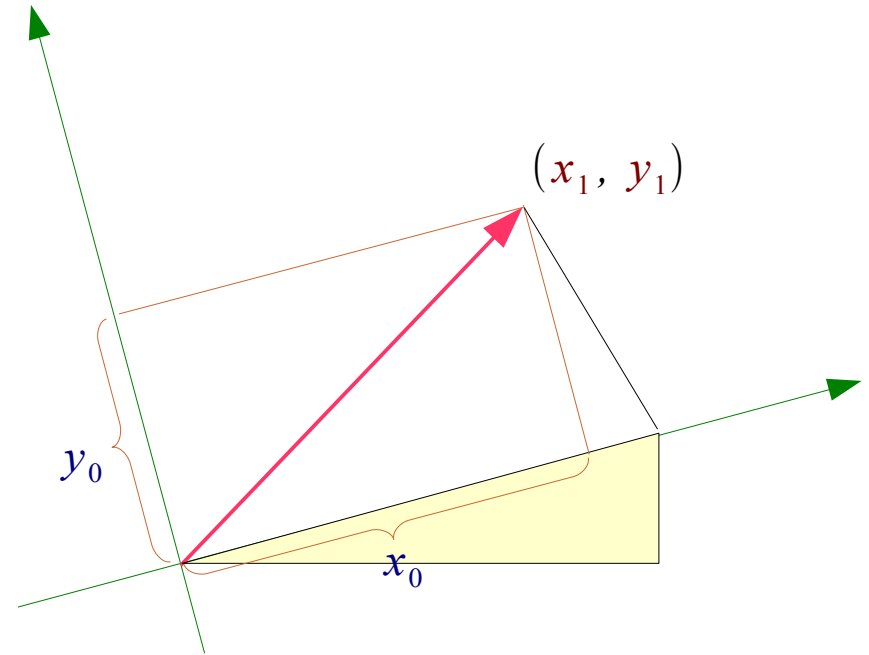
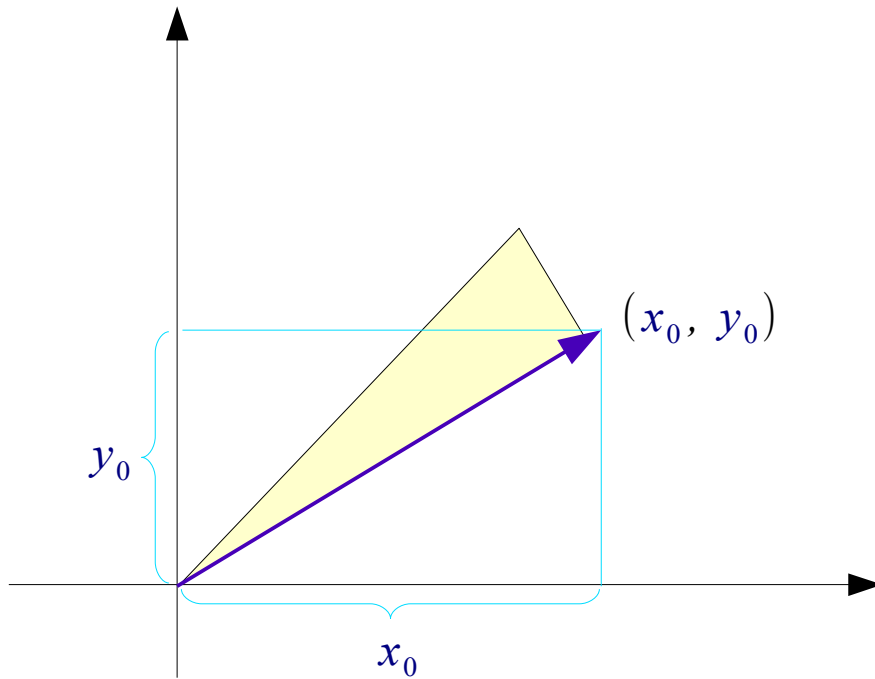


$$y_1 = x_0 \sin \alpha + y_0 \cos \alpha$$

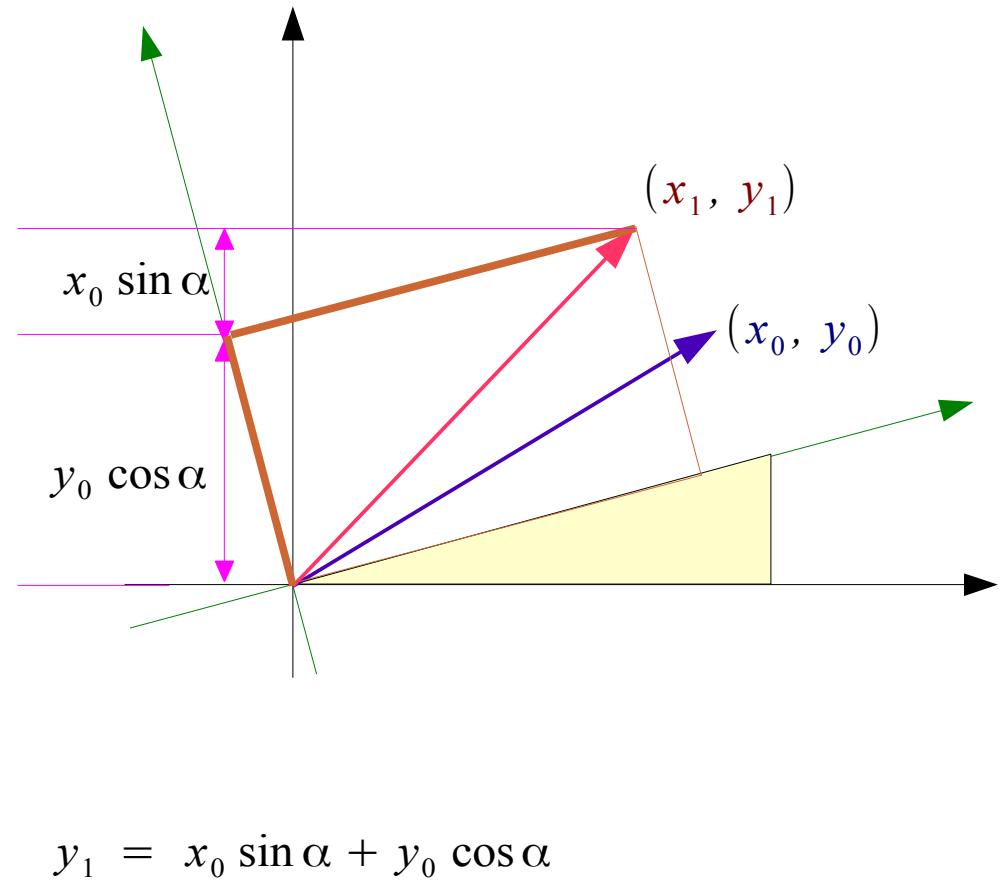
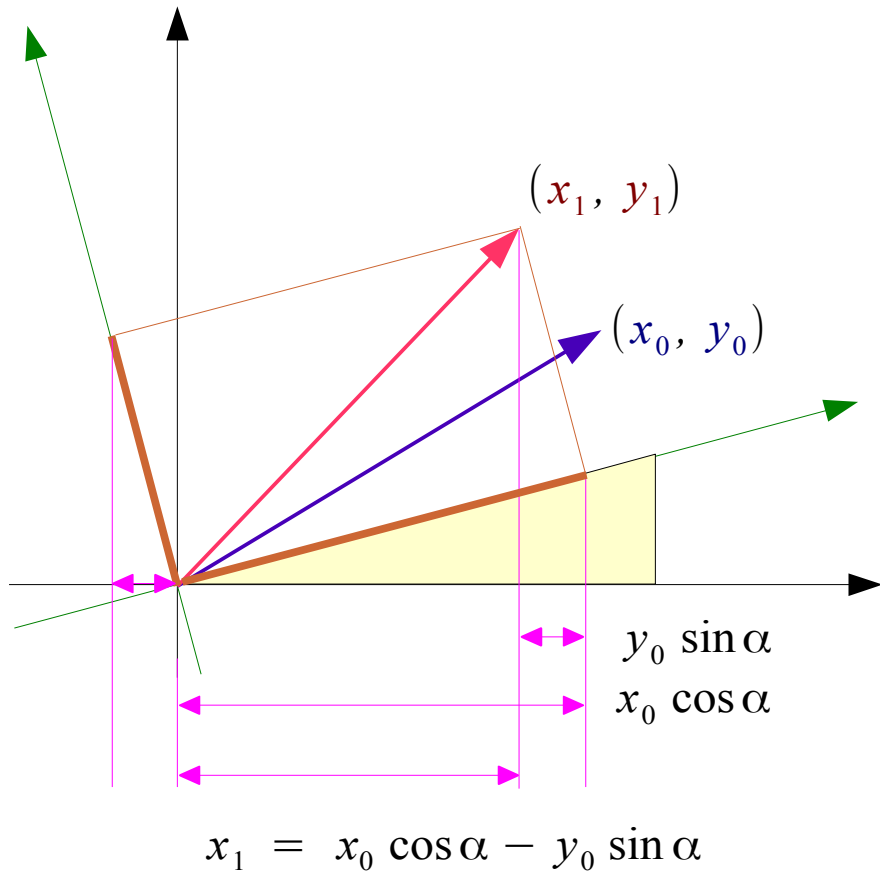


$$x_1 = x_0 \cos \alpha - y_0 \sin \alpha$$

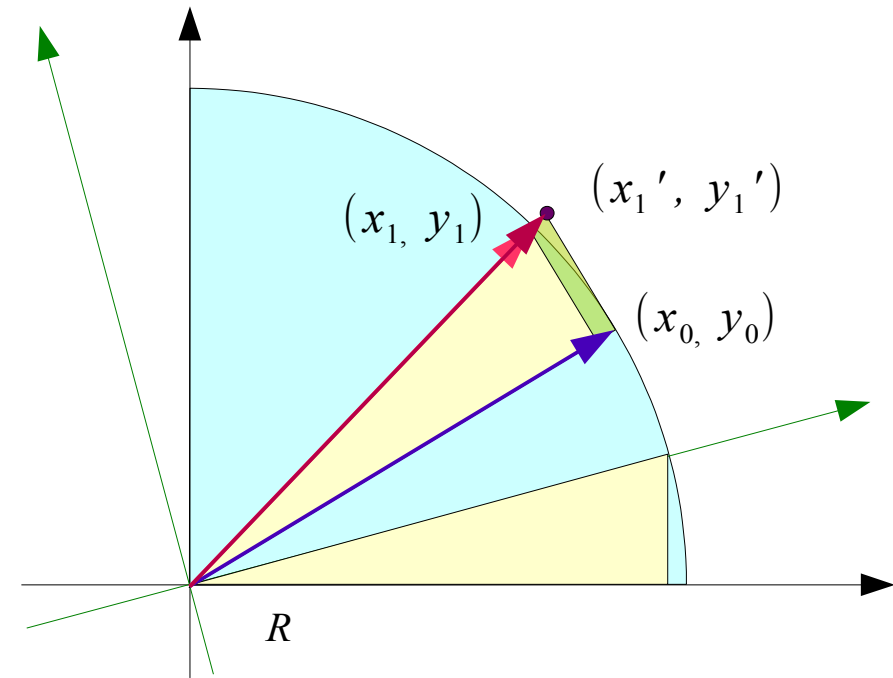
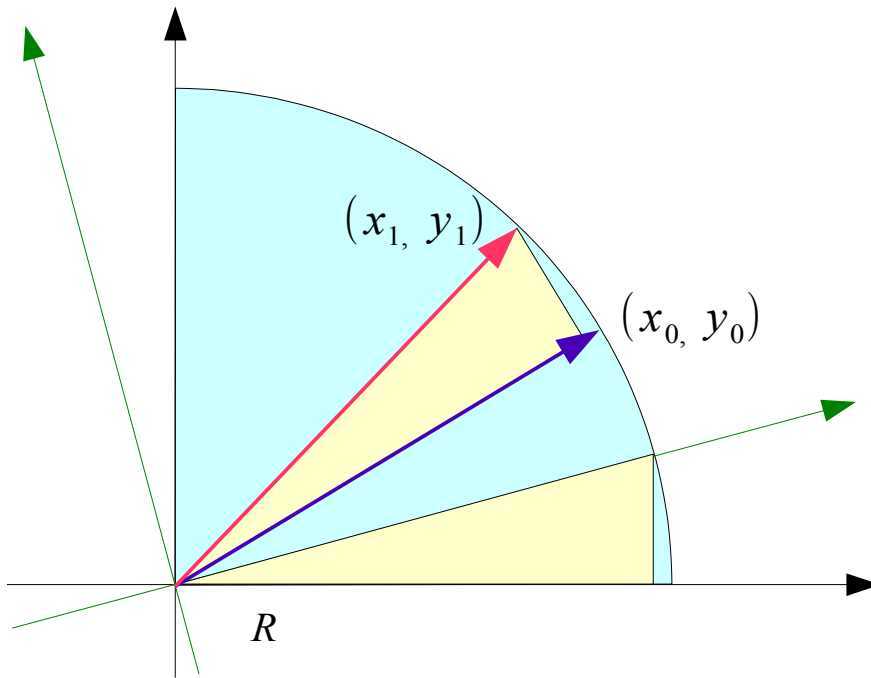
Vector Rotation (2)



Vector Rotation (3)



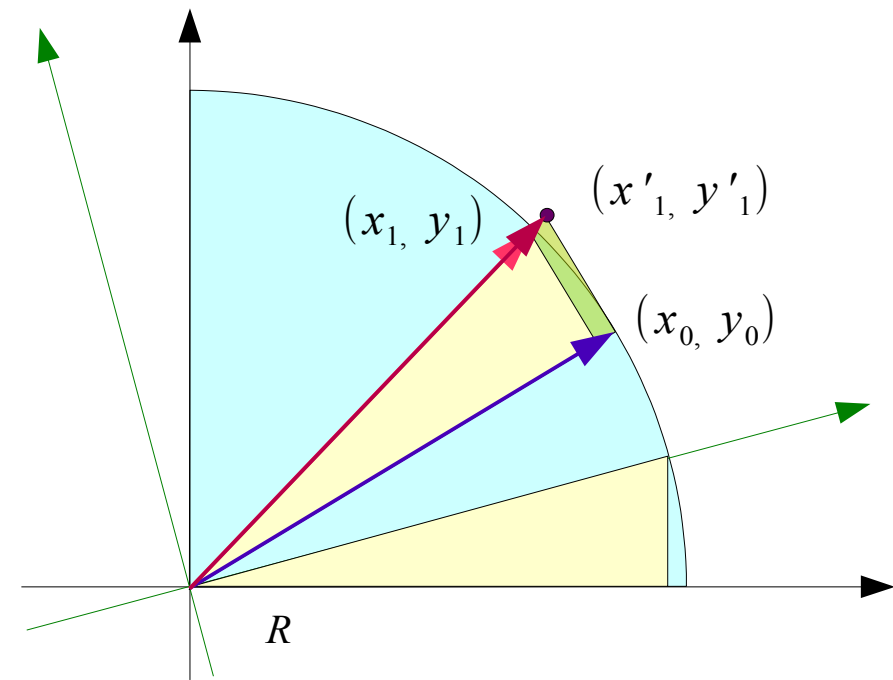
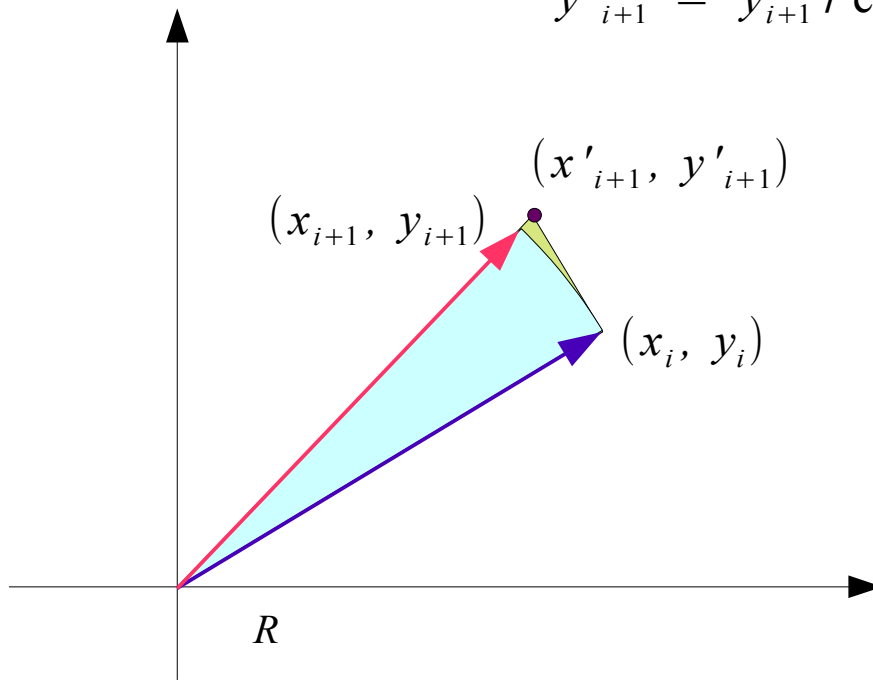
Pseudo-rotation (4)



Pseudo-rotation

$$x'_{i+1} = x_{i+1} / \cos \alpha_i$$

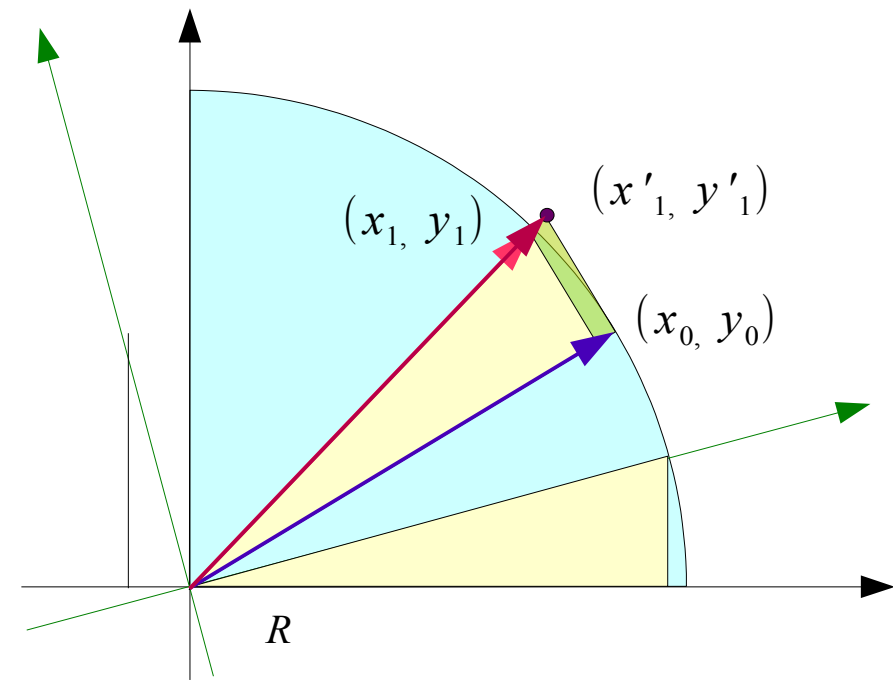
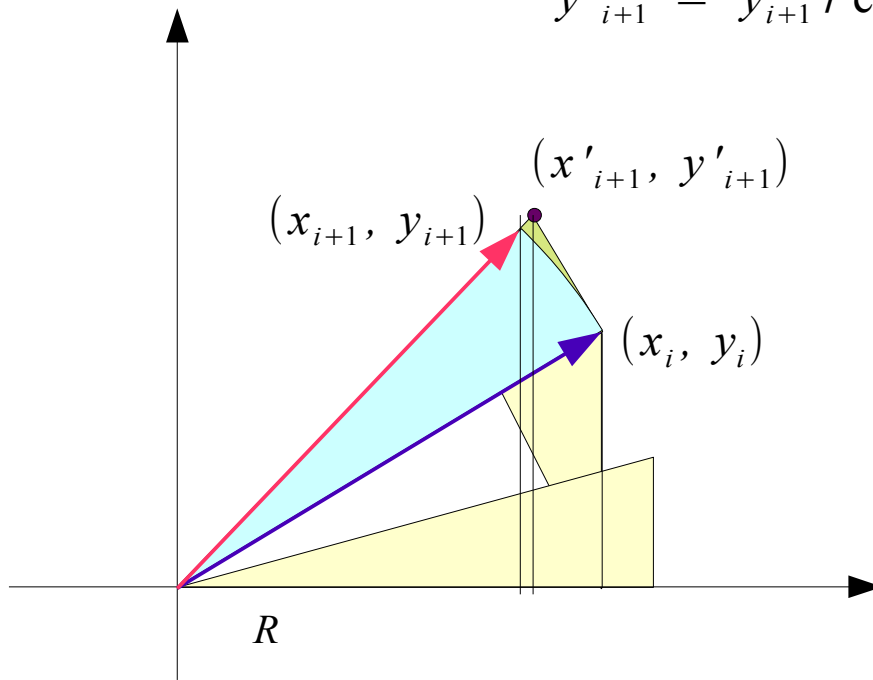
$$y'_{i+1} = y_{i+1} / \cos \alpha_i$$



Pseudo-rotation

$$x'_{i+1} = x_{i+1} / \cos \alpha_i$$

$$y'_{i+1} = y_{i+1} / \cos \alpha_i$$



Unified CORDIC Iteration Eq

Given Vector
Given Angle

Rotate

Computing
new coordinates

Given Unit Vector
Given Angle α

Rotate

$x = \cos \alpha$
 $y = \sin \alpha$

Given Vector (x_0, y_0)
Given Angle α

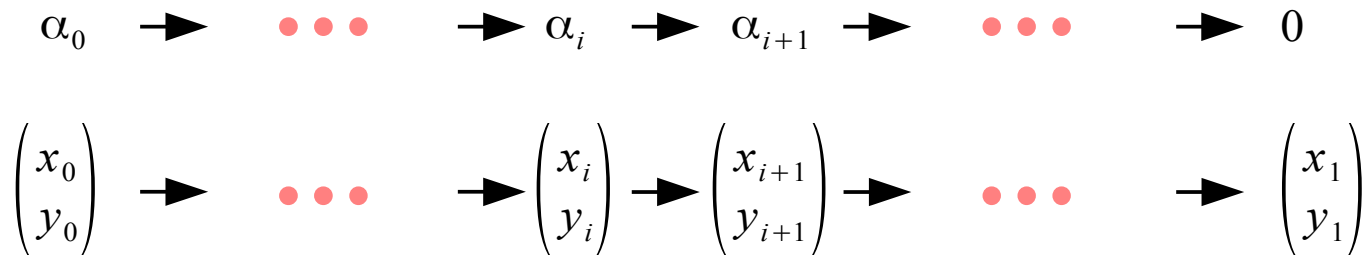
Rotate

$x_1 = x_0 \cos \alpha - y_0 \sin \alpha$
 $y_1 = x_0 \sin \alpha + y_0 \cos \alpha$

$\alpha_0 \rightarrow \dots \rightarrow \alpha_i \rightarrow \alpha_{i+1} \rightarrow \dots \rightarrow 0$

$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \rightarrow \dots \rightarrow \begin{pmatrix} x_i \\ y_i \end{pmatrix} \rightarrow \begin{pmatrix} x_{i+1} \\ y_{i+1} \end{pmatrix} \rightarrow \dots \rightarrow \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$

Unified CORDIC Iteration Eq



Pseudo-rotation

$$\begin{aligned}
 x'_{i+1} &= (x_i - y_i \tan \alpha_i) \\
 y'_{i+1} &= (x_i \tan \alpha_i + y_i)
 \end{aligned}$$

$$\begin{aligned}
 x_{i+1} &= x_i \cos \alpha_i - y_i \sin \alpha_i \\
 y_{i+1} &= x_i \sin \alpha_i + y_i \cos \alpha_i
 \end{aligned}$$

$$\begin{aligned}
 x_{i+1} &= \cos \alpha_i (x_i - y_i \tan \alpha_i) \\
 y_{i+1} &= \cos \alpha_i (x_i \tan \alpha_i + y_i)
 \end{aligned}$$

$$x_{i+1} = \frac{1}{\sqrt{1 + \tan^2 \alpha_i}} (x_i - y_i \tan \alpha_i)$$

$$y_{i+1} = \frac{1}{\sqrt{1 + \tan^2 \alpha_i}} (x_i \tan \alpha_i + y_i)$$

References

- [1] <http://en.wikipedia.org/>
- [2] CORDIC FAQ, www.dspguru.com
- [3] R. Andraka, A survey of CORDIC algorithms for FPGA based computers
- [4] J. S. Walther, A Unified Algorithm for Elementary Functions
- [5] J. P. Deschamps, G. A. Bioul, G.D. Sutter, Synthesis of Arithmetic Circuits