CORDIC Background (4A)

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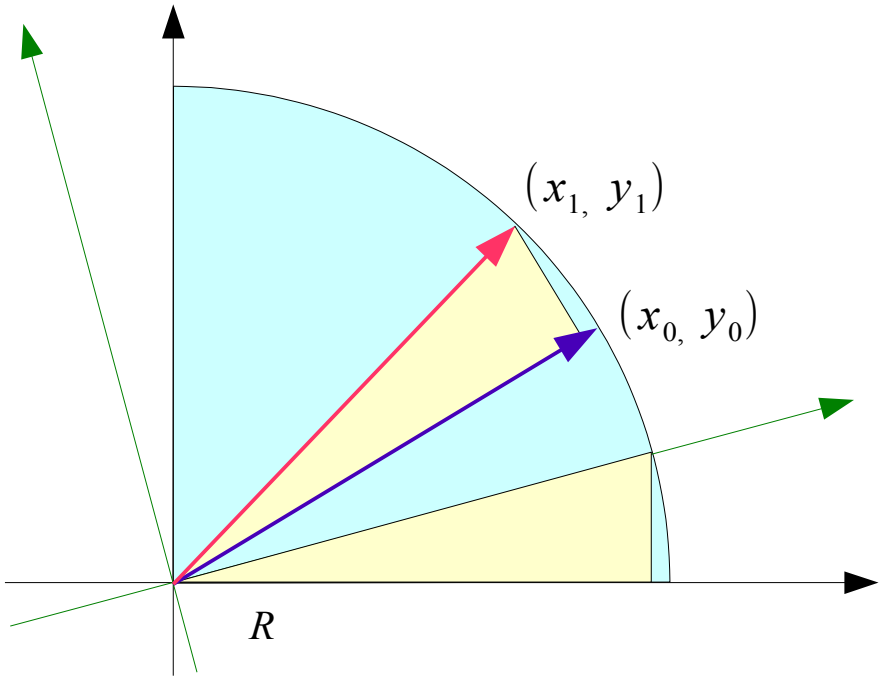
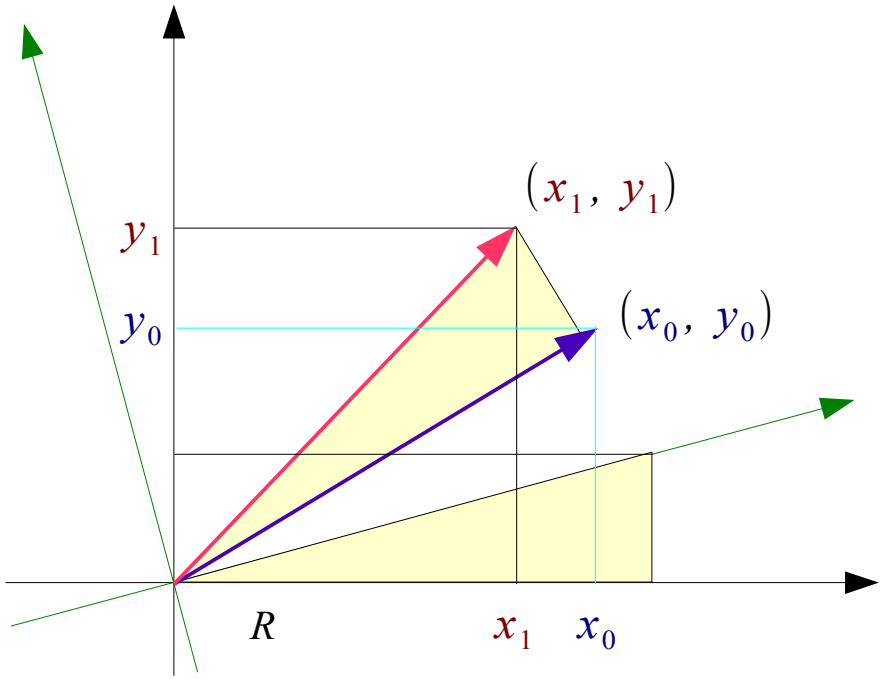
CORDIC Background

J. P. Deschamps, G. A. Bioul, G.D. Sutter, Synthesis of Arithmetic Circuits

Vector Rotation (1)

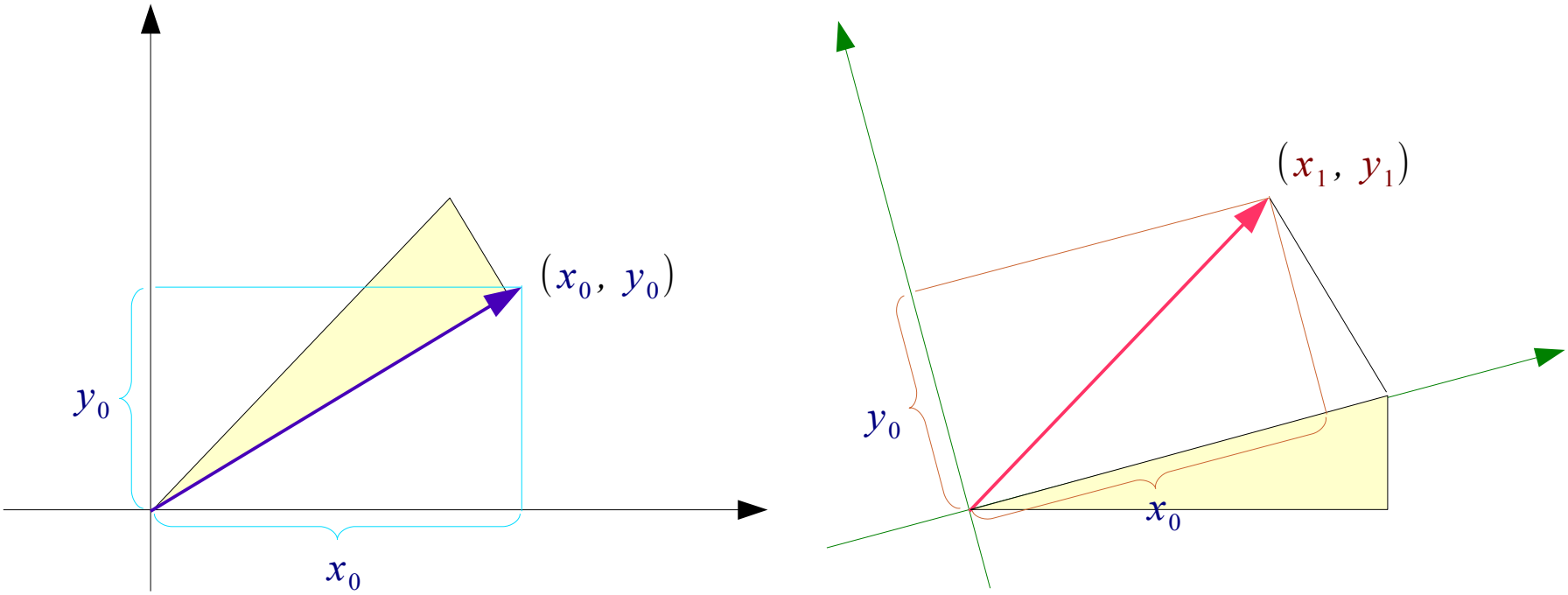
$$y_1 = x_0 \sin\alpha + y_0 \cos\alpha$$



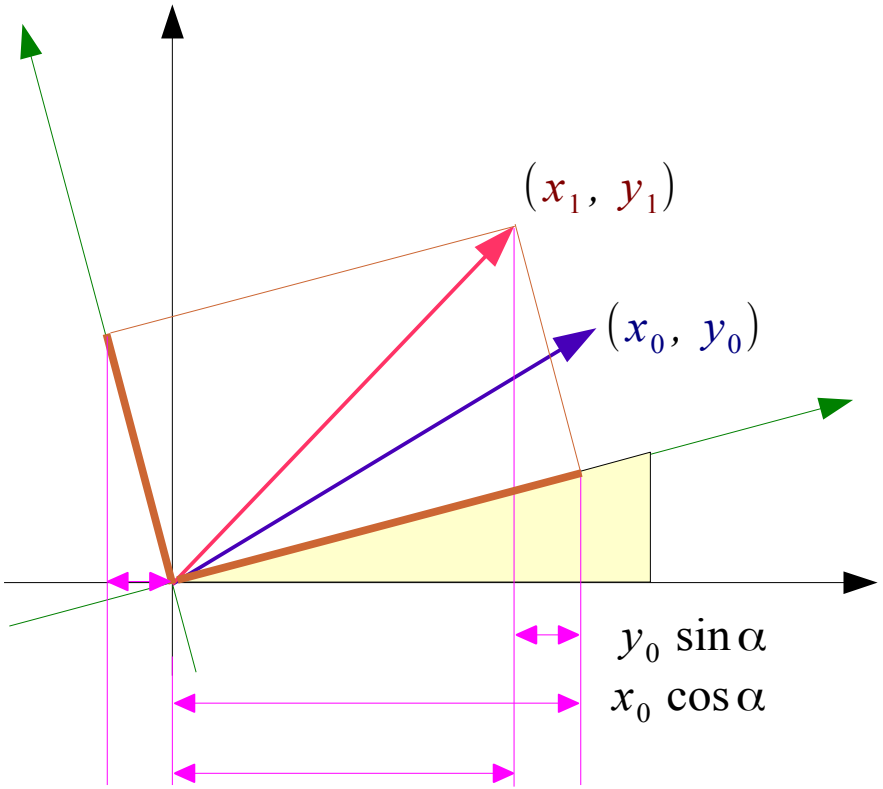


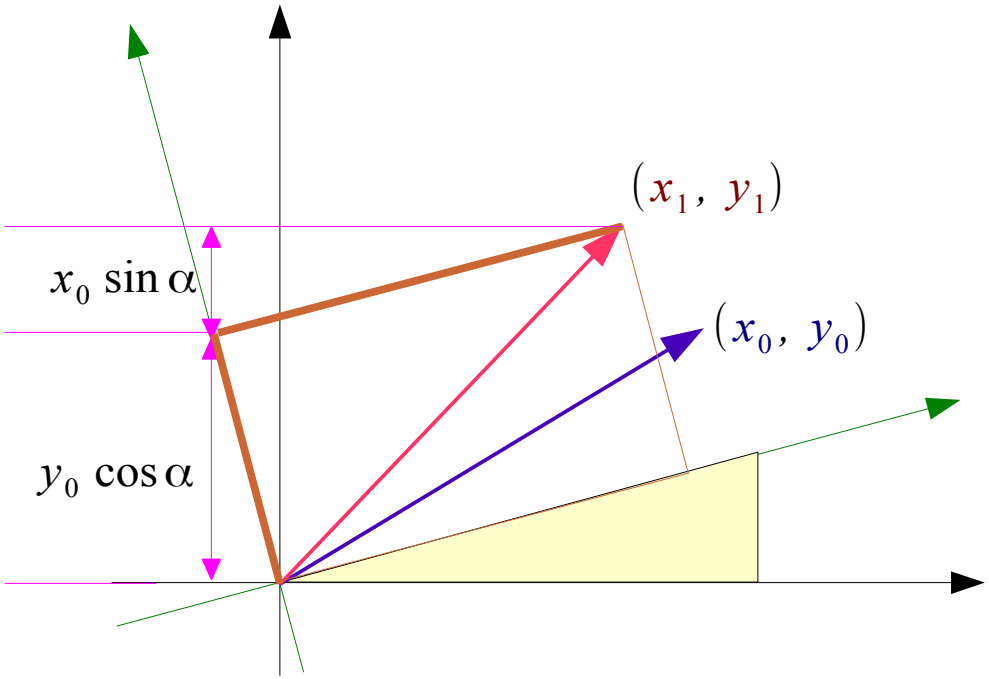
$$x_1 = x_0 \cos\alpha - y_0 \sin\alpha$$

Vector Rotation (2)



Vector Rotation (3)

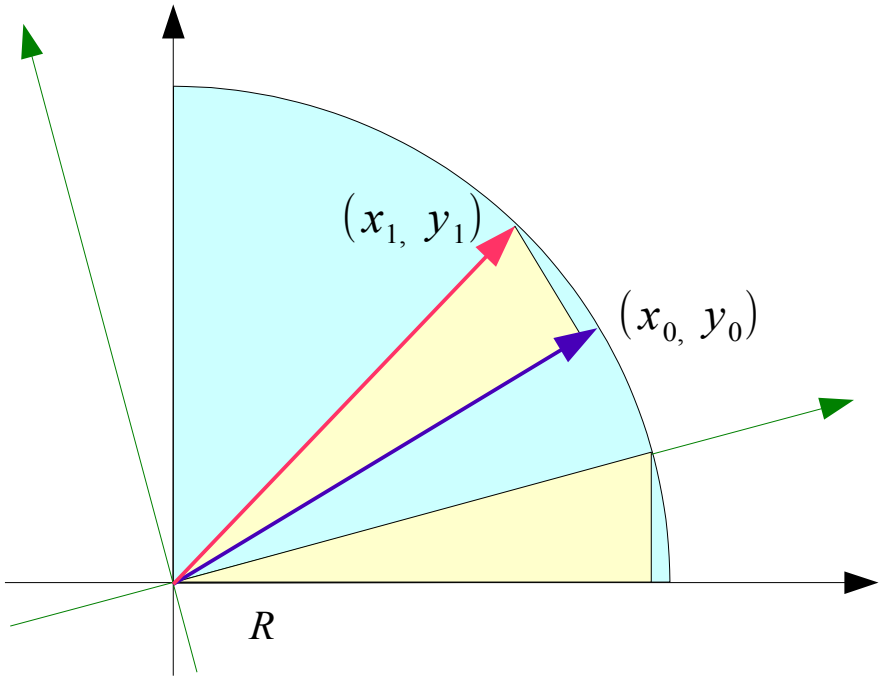


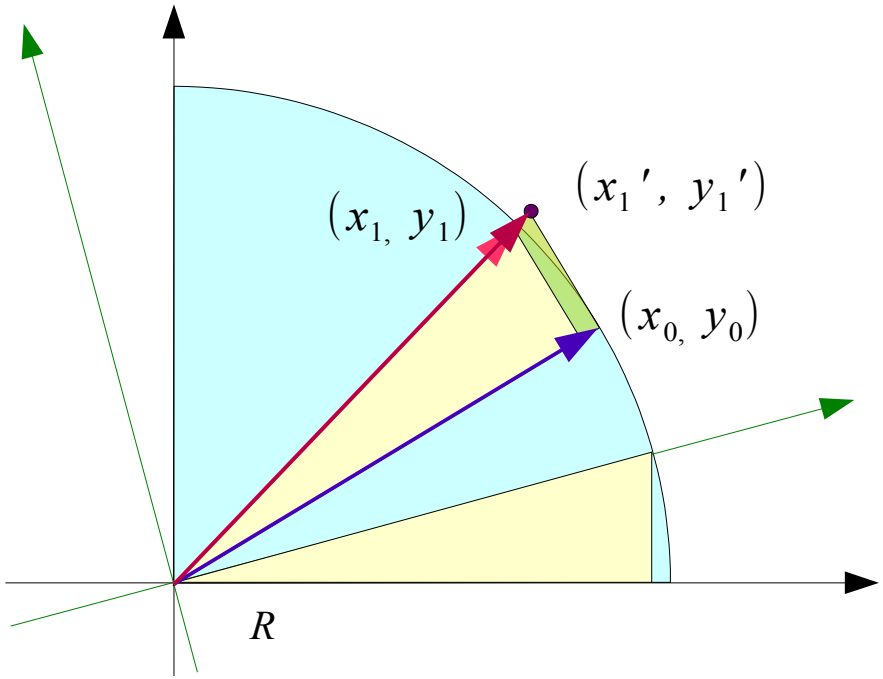


$$x_1 = x_0 \cos\alpha - y_0 \sin\alpha$$

$$y_1 = x_0 \sin\alpha + y_0 \cos\alpha$$

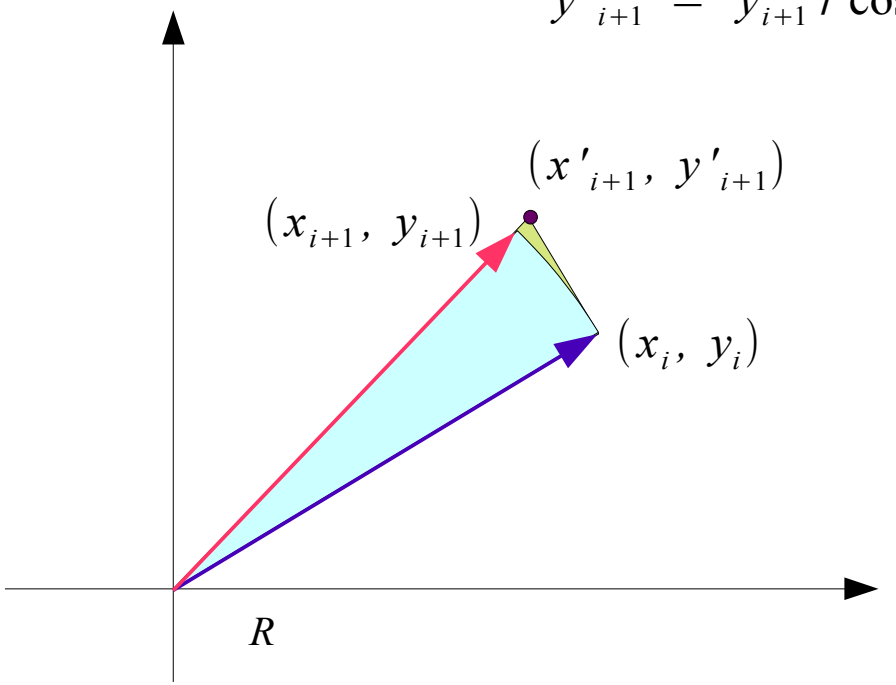
Pseudo-rotation (4)

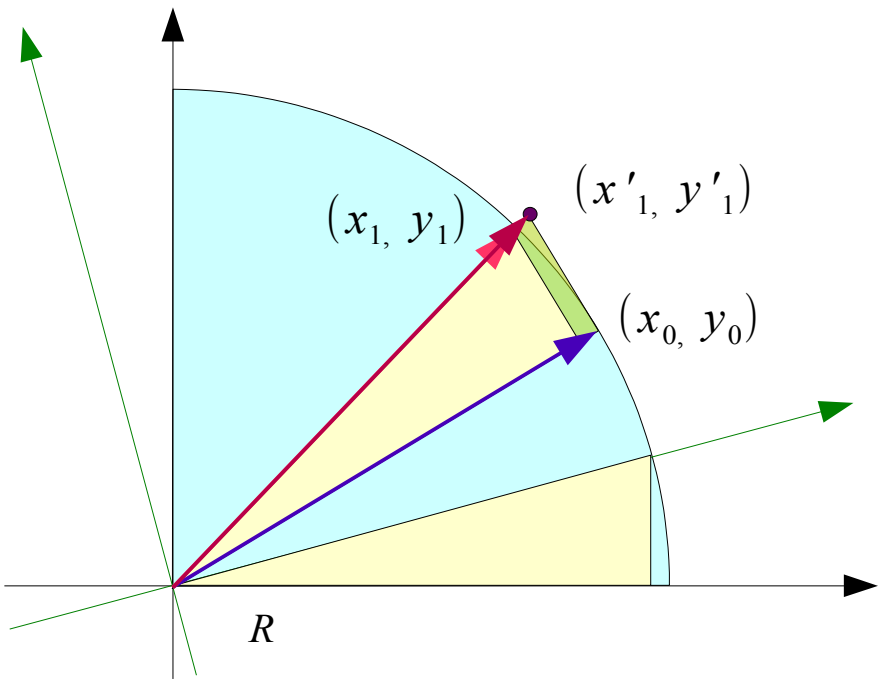




Pseudo-rotation

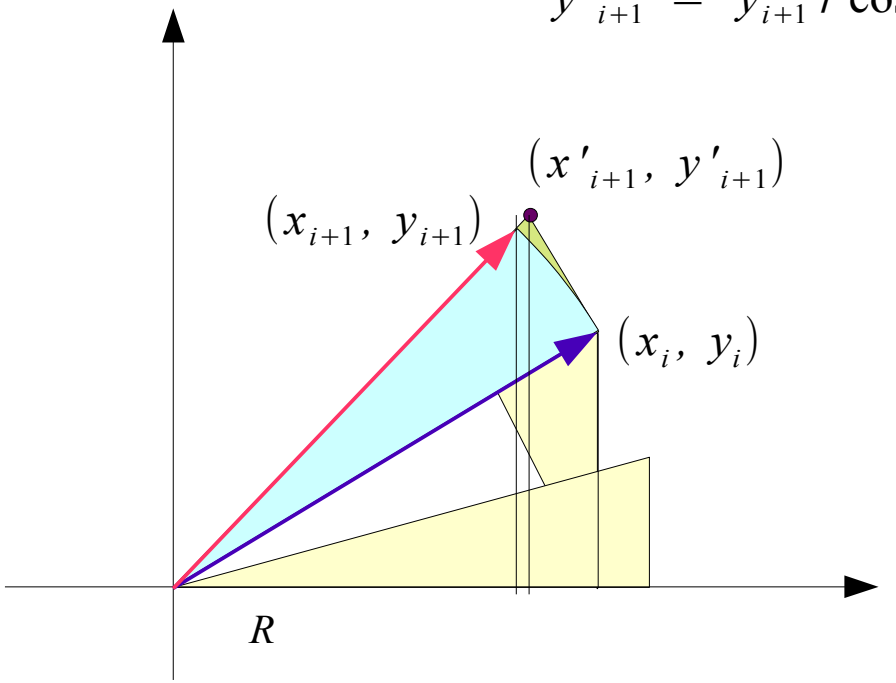
$$x'_{i+1} = x_{i+1} / \cos\alpha_i$$
$$y'_{i+1} = y_{i+1} / \cos\alpha_i$$

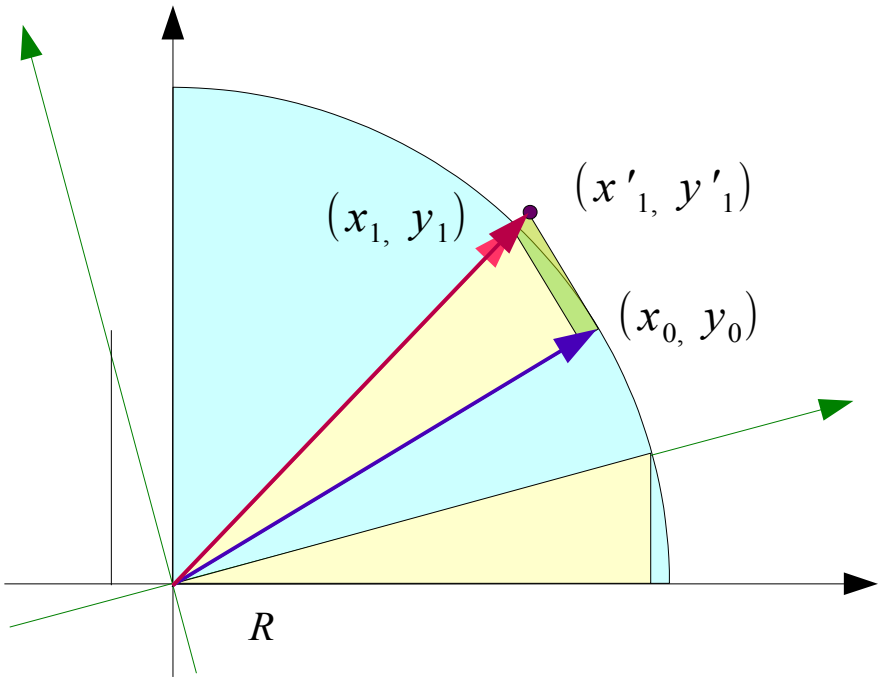




Pseudo-rotation

$$x'_{i+1} = x_{i+1} / \cos\alpha_i$$
$$y'_{i+1} = y_{i+1} / \cos\alpha_i$$





Unified CORDIC Iteration Eq

Given Vector
Given Angle

Rotate

Computing
new coordinates

Given Unit Vector
Given Angle α

Rotate

$$x = \cos\alpha$$
$$y = \sin\alpha$$

Given Vector $(x_0,\ y_0)$
Given Angle α

Rotate

$$x_1 = x_0 \cos\alpha - y_0 \sin\alpha$$
$$y_1 = x_0 \sin\alpha + y_0 \cos\alpha$$

$$\alpha_0 \rightarrow \cdots \rightarrow \alpha_i \rightarrow \alpha_{i+1} \rightarrow \cdots \rightarrow 0$$

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \rightarrow \cdots \rightarrow \begin{pmatrix} x_i \\ y_i \end{pmatrix} \rightarrow \begin{pmatrix} x_{i+1} \\ y_{i+1} \end{pmatrix} \rightarrow \cdots \rightarrow \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

Unified CORDIC Iteration Eq

$$\alpha_0 \rightarrow \cdots \rightarrow \alpha_i \rightarrow \alpha_{i+1} \rightarrow \cdots \rightarrow 0$$

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \rightarrow \cdots \rightarrow \begin{pmatrix} x_i \\ y_i \end{pmatrix} \rightarrow \begin{pmatrix} x_{i+1} \\ y_{i+1} \end{pmatrix} \rightarrow \cdots \rightarrow \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

$$\begin{aligned} x_{i+1} &= x_i \cos\alpha_i - y_i \sin\alpha_i \\ y_{i+1} &= x_i \sin\alpha_i + y_i \cos\alpha_i \end{aligned}$$

Pseudo-rotation

$$\begin{aligned} x'_{i+1} &= (x_i - y_i \tan\alpha_i) \\ y'_{i+1} &= (x_i \tan\alpha_i + y_i) \end{aligned}$$

$$\begin{aligned} x_{i+1} &= \cos\alpha_i (x_i - y_i \tan\alpha_i) \\ y_{i+1} &= \cos\alpha_i (x_i \tan\alpha_i + y_i) \end{aligned}$$

$$x_{i+1} = \frac{1}{\sqrt{1 + \tan^2\alpha_i}} (x_i - y_i \tan\alpha_i)$$

$$y_{i+1} = \frac{1}{\sqrt{1 + \tan^2\alpha_i}} (x_i \tan\alpha_i + y_i)$$

References

[1] http://en.wikipedia.org/
[2] CORDIC FAQ, www.dspguru.com
[3] R. Andraka, A survey of CORDIC algorithms for FPGA based computers
[4] J. S. Walther, A Unified Algorithm for Elementary Functions
[5] J. P. Deschamps, G. A. Bioul, G.D. Sutter, Synthesis of Arithmetic Circuits