

CORDIC Background (4A)

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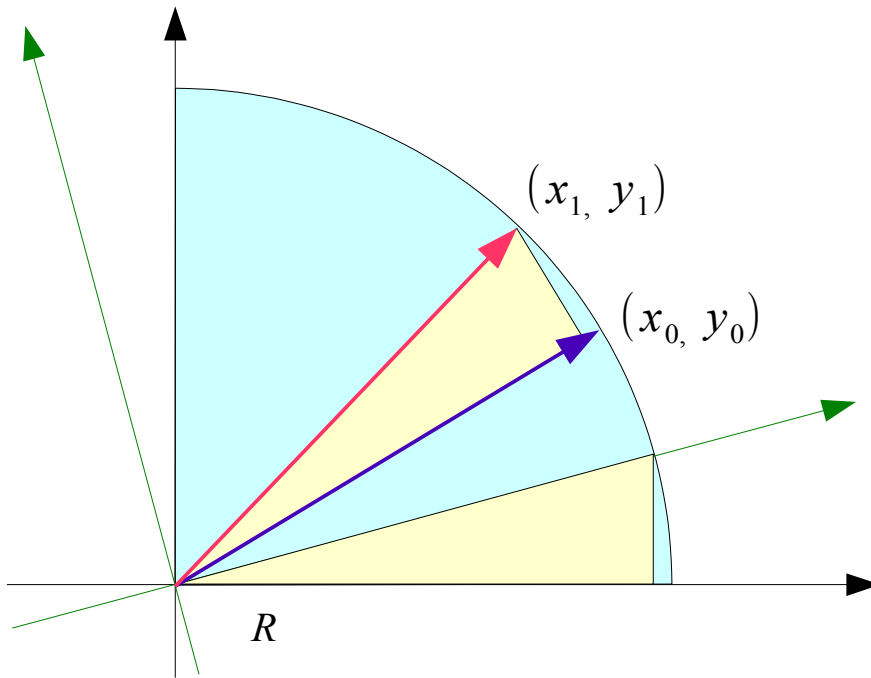
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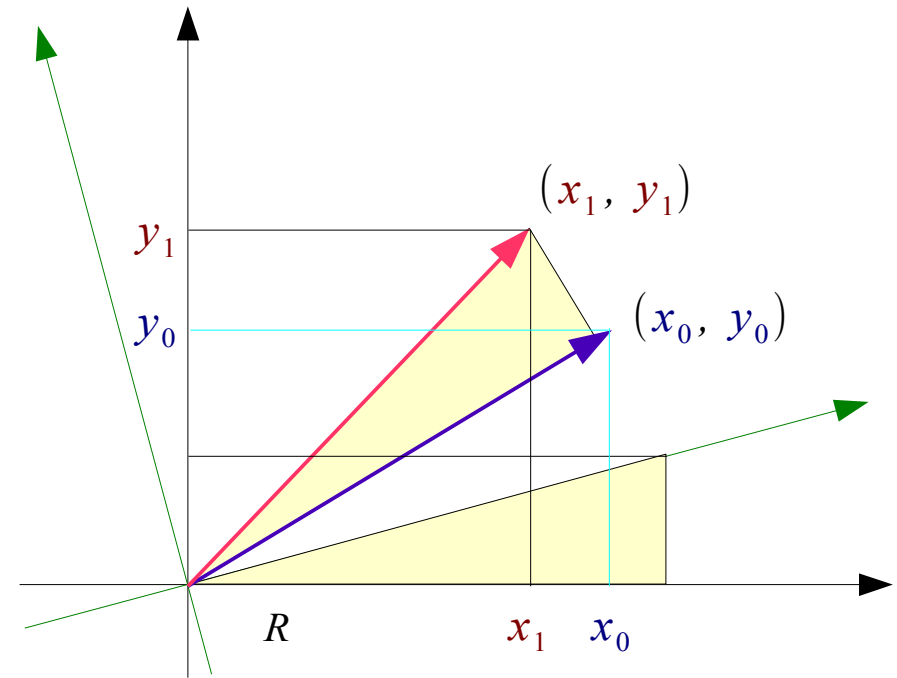
CORDIC Background

J. P. Deschamps, G. A. Bioul, G.D. Sutter, Synthesis of Arithmetic Circuits

Vector Rotation (1)

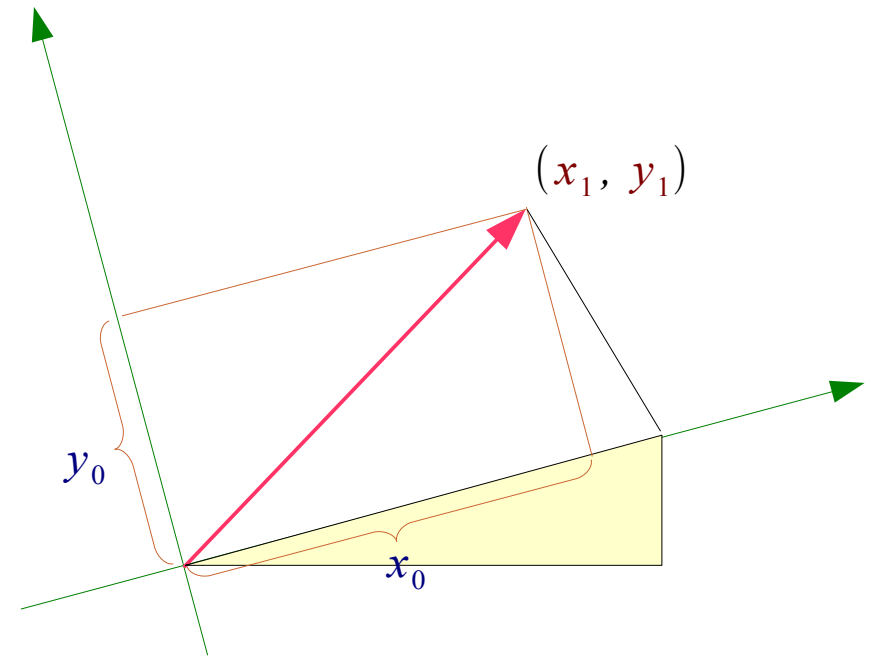
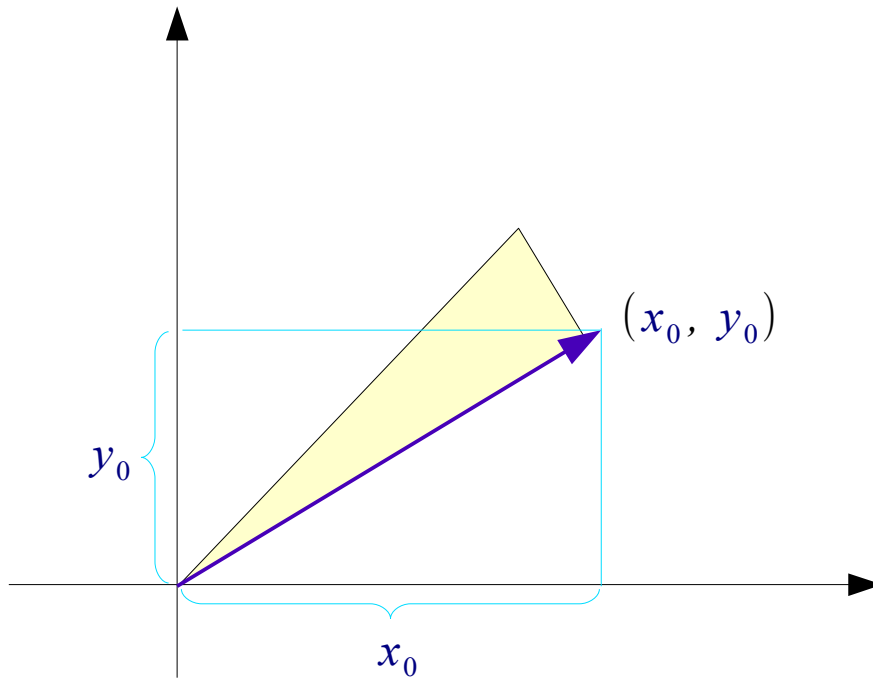


$$y_1 = x_0 \sin \alpha + y_0 \cos \alpha$$

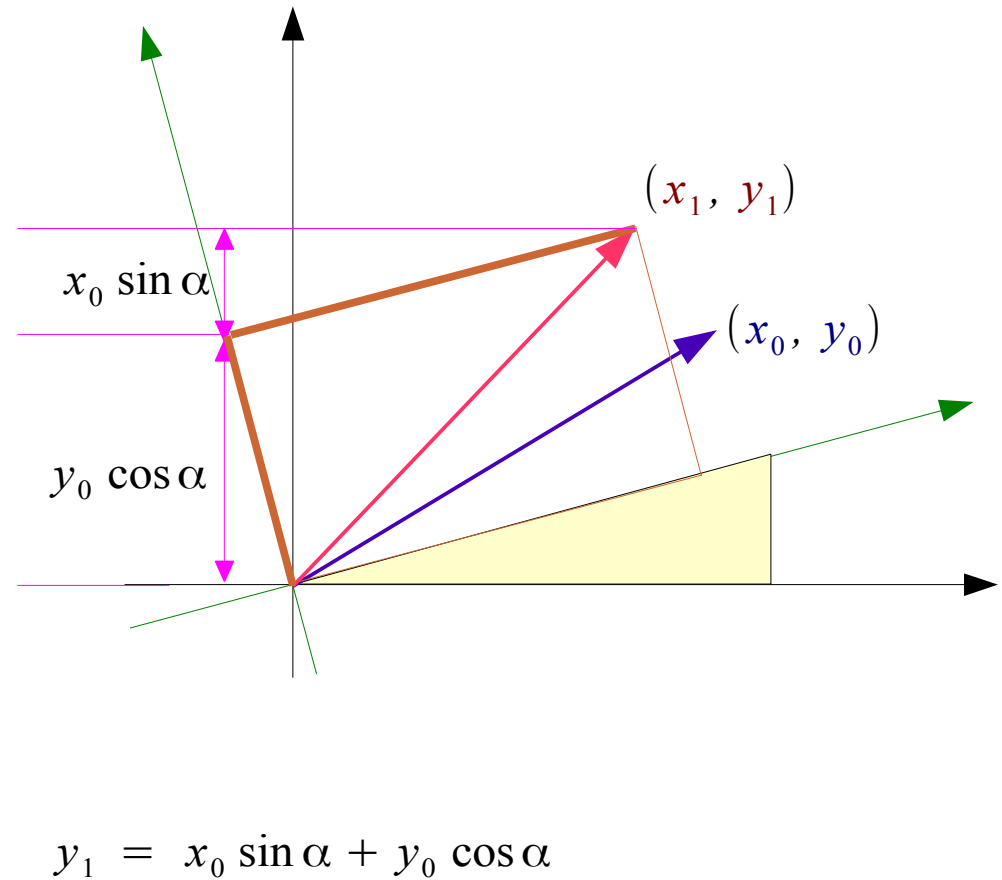
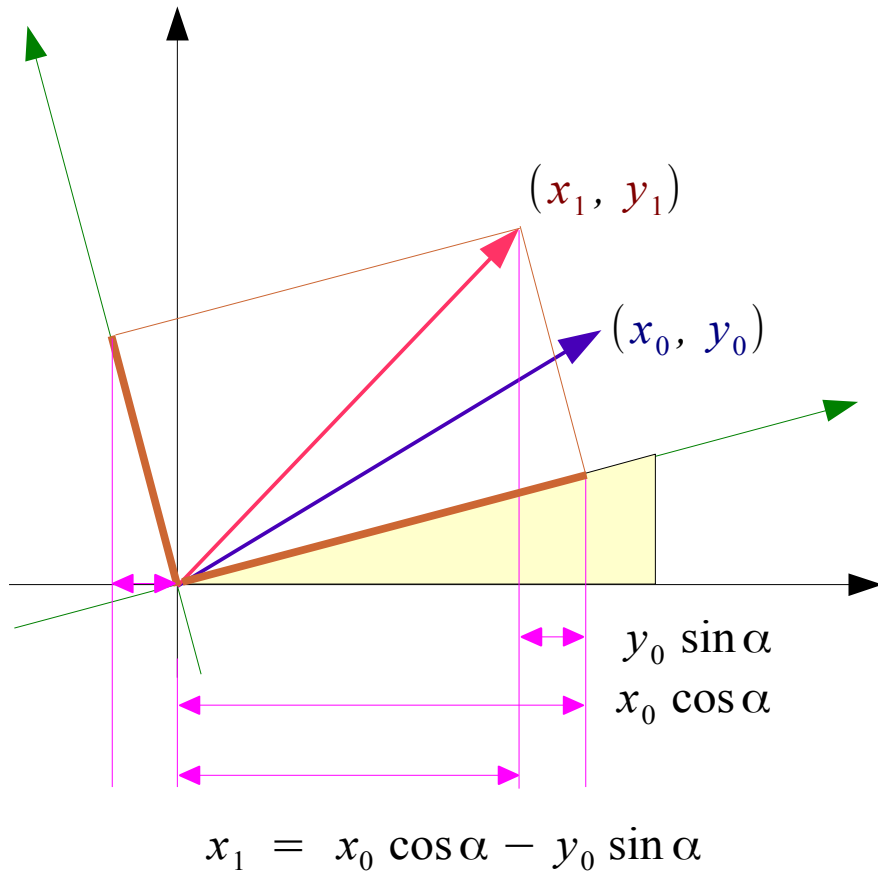


$$x_1 = x_0 \cos \alpha - y_0 \sin \alpha$$

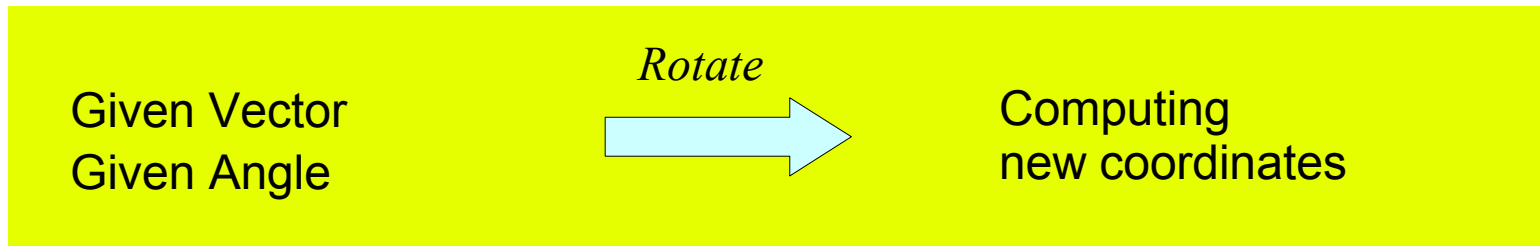
Vector Rotation (2)



Vector Rotation (3)



CORDIC Iteration Equations



Given Unit Vector
Given Angle α

Rotate

→

$$x = \cos \alpha$$

$$y = \sin \alpha$$

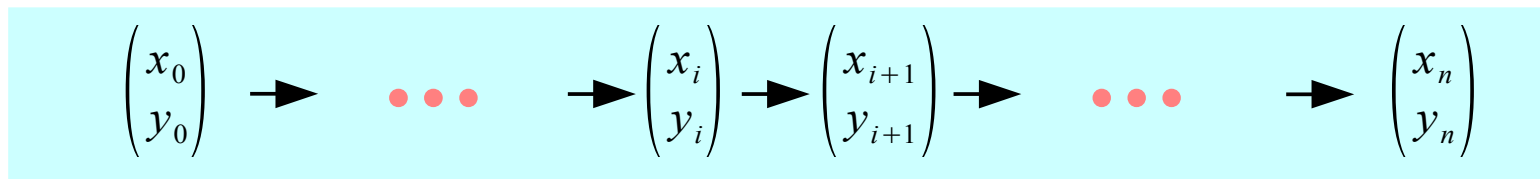
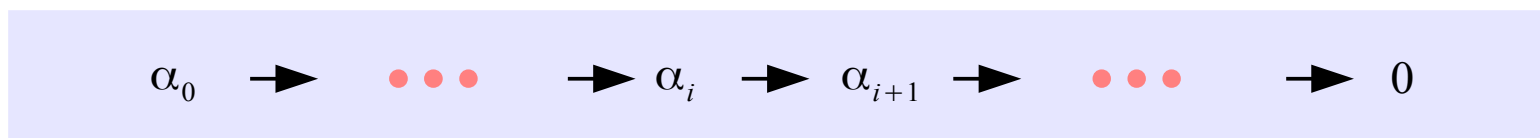
Given Vector (x_0, y_0)
Given Angle α

Rotate

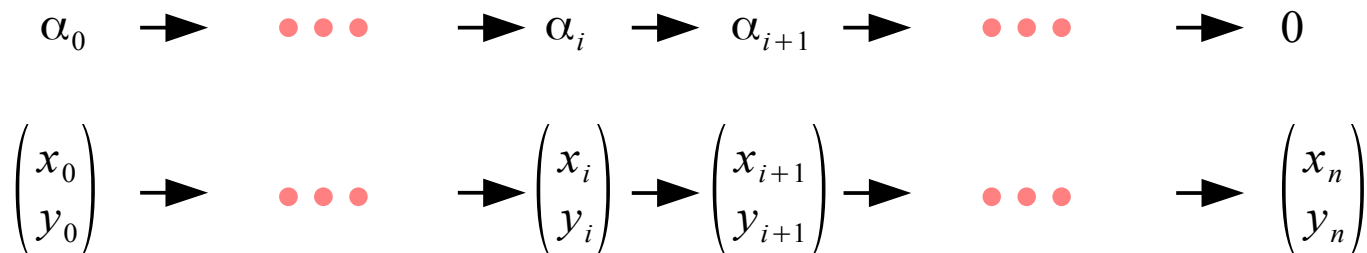
→

$$x_n = x_0 \cos \alpha - y_0 \sin \alpha$$

$$y_n = x_0 \sin \alpha + y_0 \cos \alpha$$



CORDIC Iteration Equations – Pseudo-Rotation



Pseudo-rotation

$$x'_{i+1} = (x_i - y_i \tan \alpha_i)$$

$$y'_{i+1} = (x_i \tan \alpha_i + y_i)$$

$$x_{i+1} = x_i \cos \alpha_i - y_i \sin \alpha_i$$

$$y_{i+1} = x_i \sin \alpha_i + y_i \cos \alpha_i$$

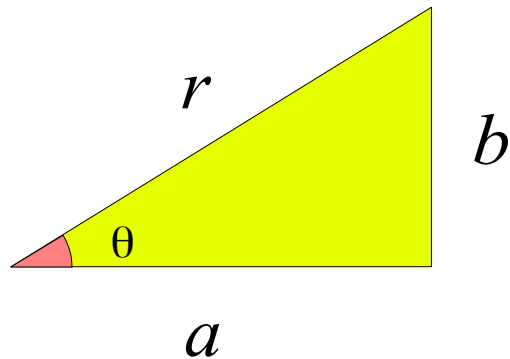
$$x_{i+1} = \cos \alpha_i (x_i - y_i \tan \alpha_i)$$

$$y_{i+1} = \cos \alpha_i (x_i \tan \alpha_i + y_i)$$

$$x_{i+1} = \frac{1}{\sqrt{1 + \tan^2 \alpha_i}} (x_i - y_i \tan \alpha_i)$$

$$y_{i+1} = \frac{1}{\sqrt{1 + \tan^2 \alpha_i}} (x_i \tan \alpha_i + y_i)$$

COS θ



$$r = \sqrt{a^2 + b^2}$$

$$\cos \theta = \frac{a}{r} = \frac{a}{\sqrt{a^2 + b^2}}$$

$$\cos \theta = \frac{a}{r}$$

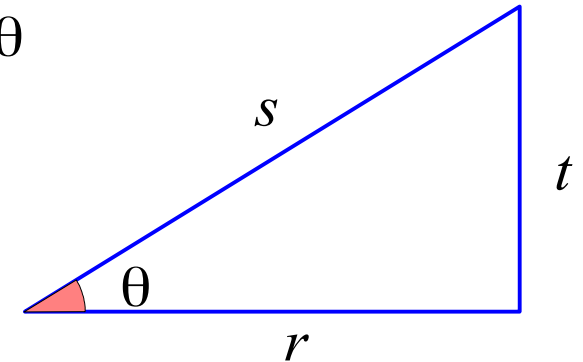
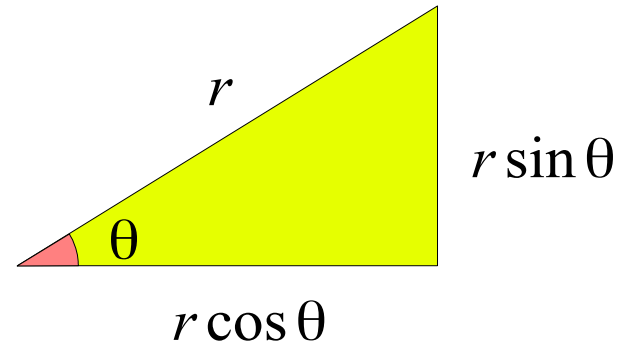
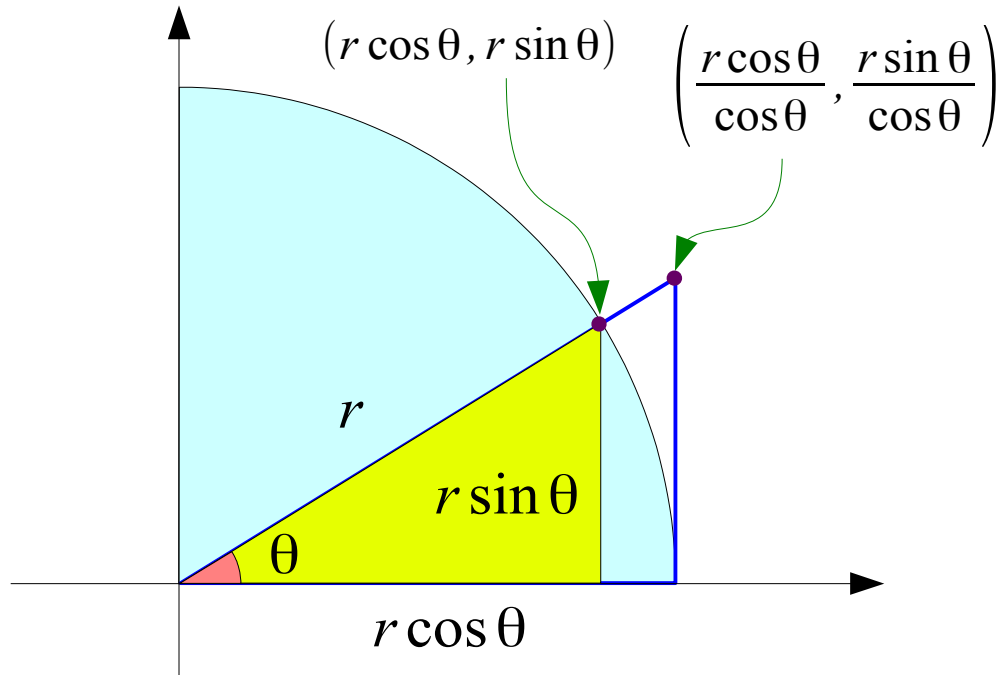
$$= \frac{1}{\sqrt{1 + (b/a)^2}}$$

$$\sin \theta = \frac{b}{r}$$

$$= \frac{1}{\sqrt{1 + \tan^2 \theta}}$$

$$\tan \theta = \frac{b}{a}$$

Pseudo-rotation



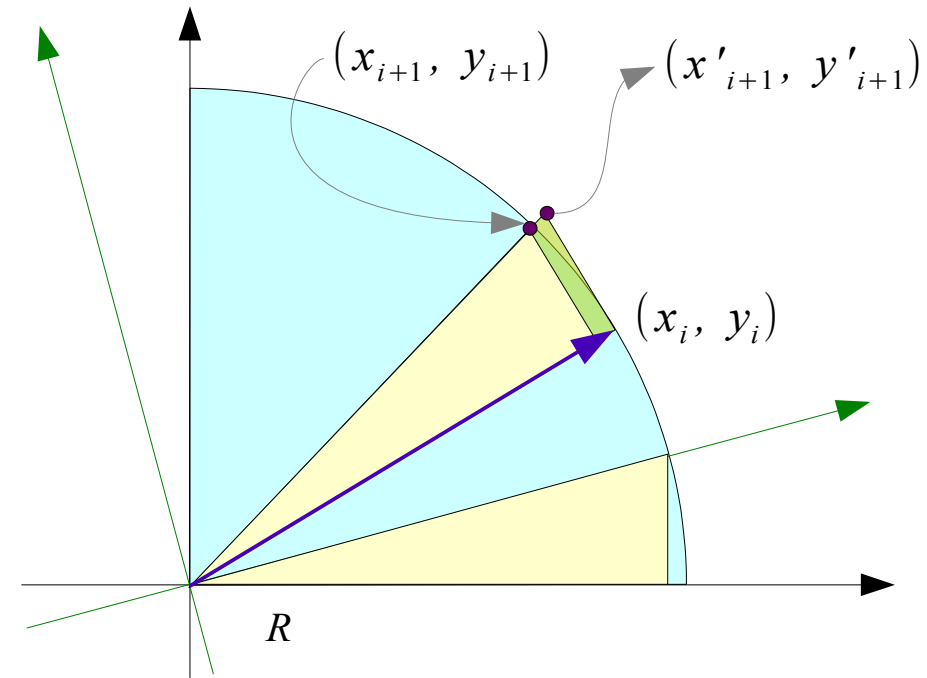
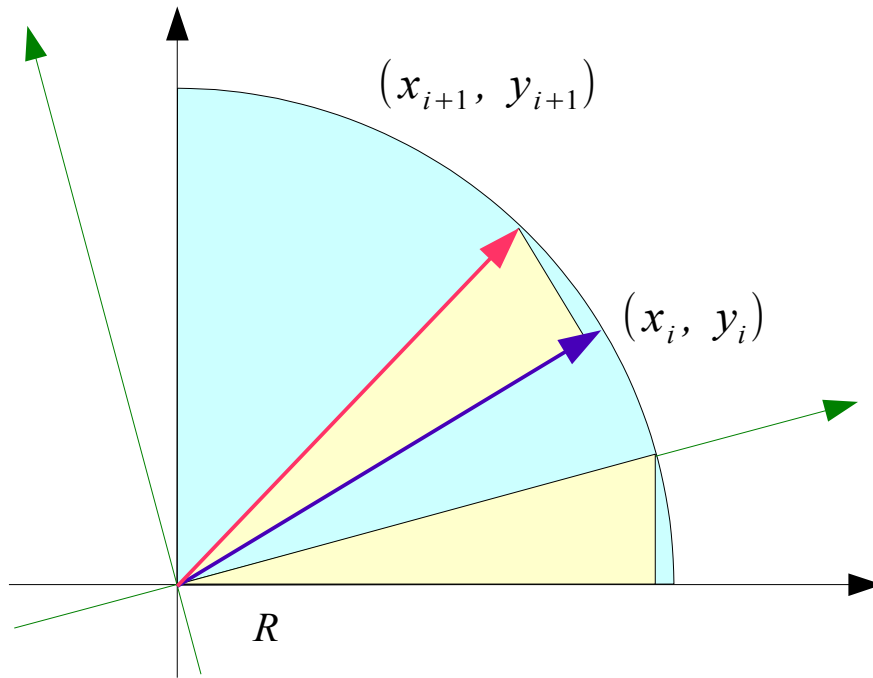
$$r : r \cos \theta = s : r$$

$$r \cos \theta : r \sin \theta = r : t$$

$$s = \frac{r}{\cos \theta}$$

$$s = \frac{r \sin \theta}{\cos \theta}$$

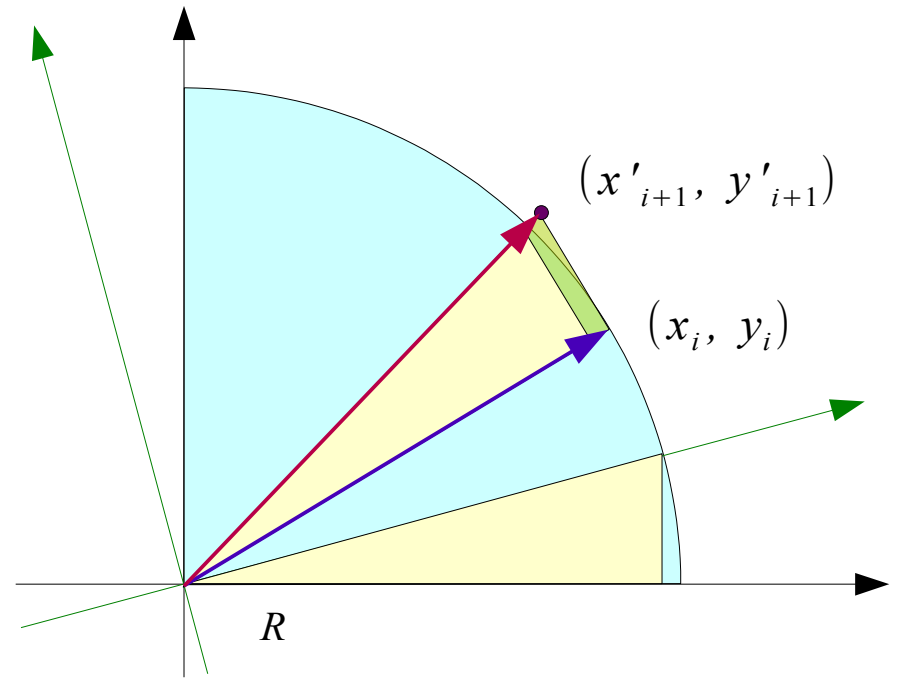
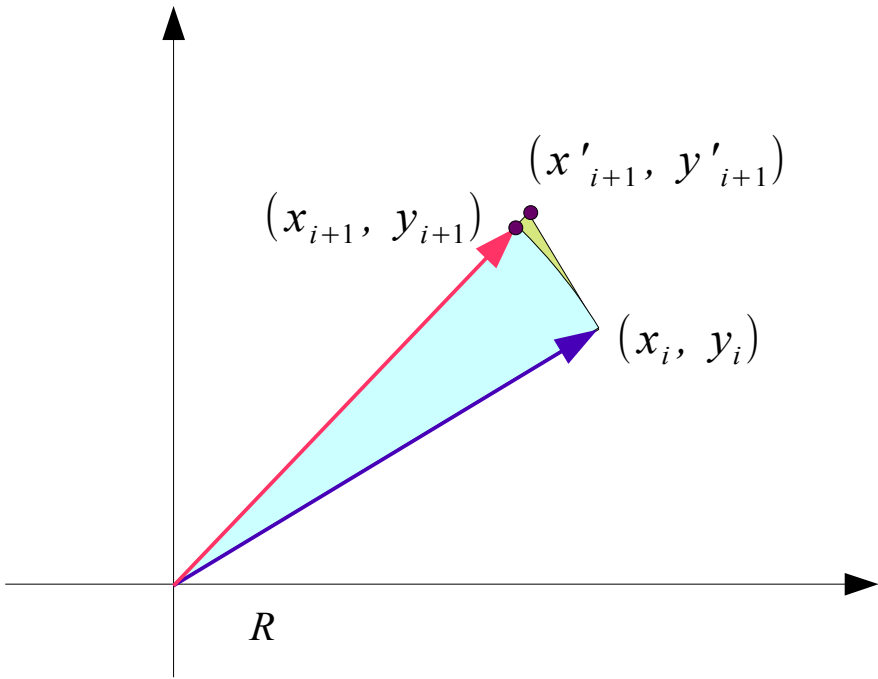
Pseudo-rotation (1)



Pseudo-rotation (2)

$$\begin{aligned}x'_{i+1} &= x_{i+1} / \cos \alpha_i \\y'_{i+1} &= y_{i+1} / \cos \alpha_i\end{aligned}$$

$$\begin{aligned}x'_{i+1} &> x_{i+1} \\y'_{i+1} &> y_{i+1}\end{aligned}$$



CORDIC Iteration Equations – Pseudo-Rotation

$$\alpha_0 \rightarrow \dots \rightarrow \alpha_i \rightarrow \alpha_{i+1} \rightarrow \dots \rightarrow 0$$

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \rightarrow \dots \rightarrow \begin{pmatrix} x_i \\ y_i \end{pmatrix} \rightarrow \begin{pmatrix} x_{i+1} \\ y_{i+1} \end{pmatrix} \rightarrow \dots \rightarrow \begin{pmatrix} x_n \\ y_n \end{pmatrix}$$

$$x_{i+1} = x_i \cos \alpha_i - y_i \sin \alpha_i = \cos \alpha_i (x_i - y_i \tan \alpha_i) = (1/\sqrt{1 + \tan^2 \alpha_i}) (x_i - y_i \tan \alpha_i)$$

$$y_{i+1} = x_i \sin \alpha_i + y_i \cos \alpha_i = \cos \alpha_i (x_i \tan \alpha_i + y_i) = (1/\sqrt{1 + \tan^2 \alpha_i}) (x_i \tan \alpha_i + y_i)$$

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \rightarrow \dots \rightarrow \begin{pmatrix} x'_i \\ y'_i \end{pmatrix} \rightarrow \begin{pmatrix} x'_{i+1} \\ y'_{i+1} \end{pmatrix} \rightarrow \dots \rightarrow \begin{pmatrix} x'_n \\ y'_n \end{pmatrix}$$

Pseudo-rotation

$$x'_{i+1} = (x_i - y_i \tan \alpha_i)$$

$$y'_{i+1} = (x_i \tan \alpha_i + y_i)$$

References

- [1] <http://en.wikipedia.org/>
- [2] CORDIC FAQ, www.dspguru.com
- [3] R. Andraka, A survey of CORDIC algorithms for FPGA based computers
- [4] J. S. Walther, A Unified Algorithm for Elementary Functions
- [5] J. P. Deschamps, G. A. Bioul, G.D. Sutter, Synthesis of Arithmetic Circuits