CORDIC Background (4A)

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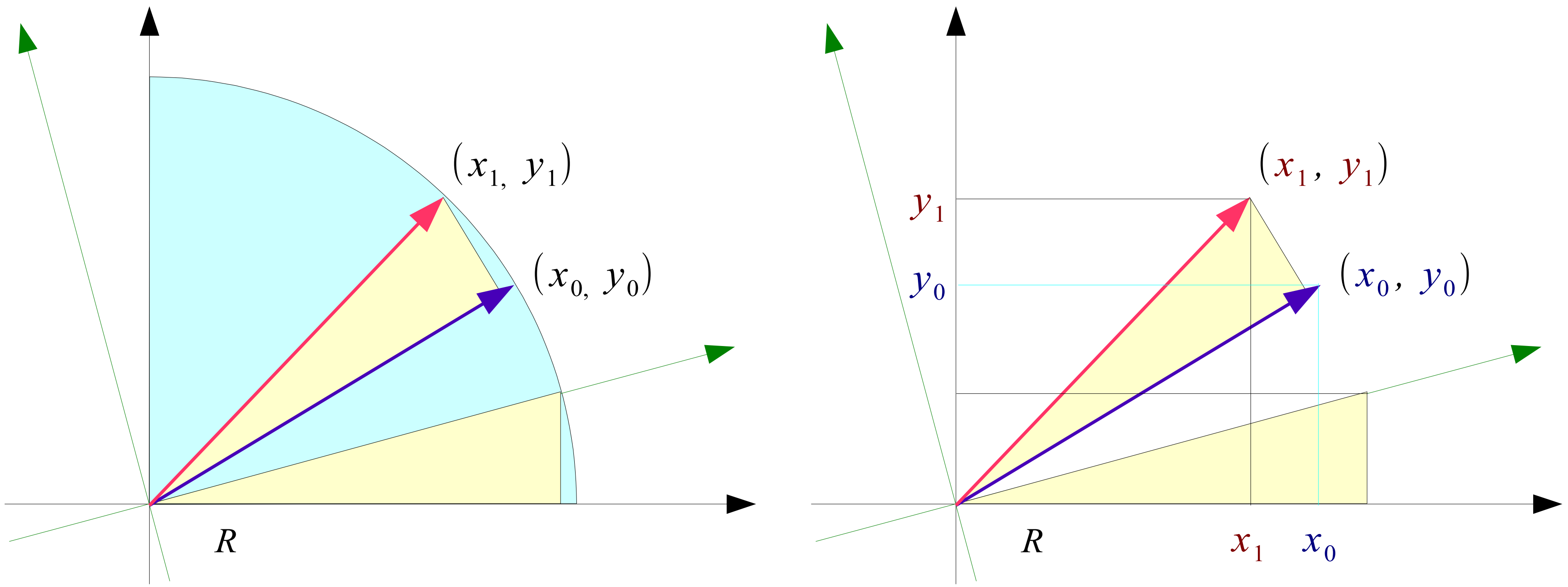
CORDIC Background

J. P. Deschamps, G. A. Bioul, G.D. Sutter, Synthesis of Arithmetic Circuits

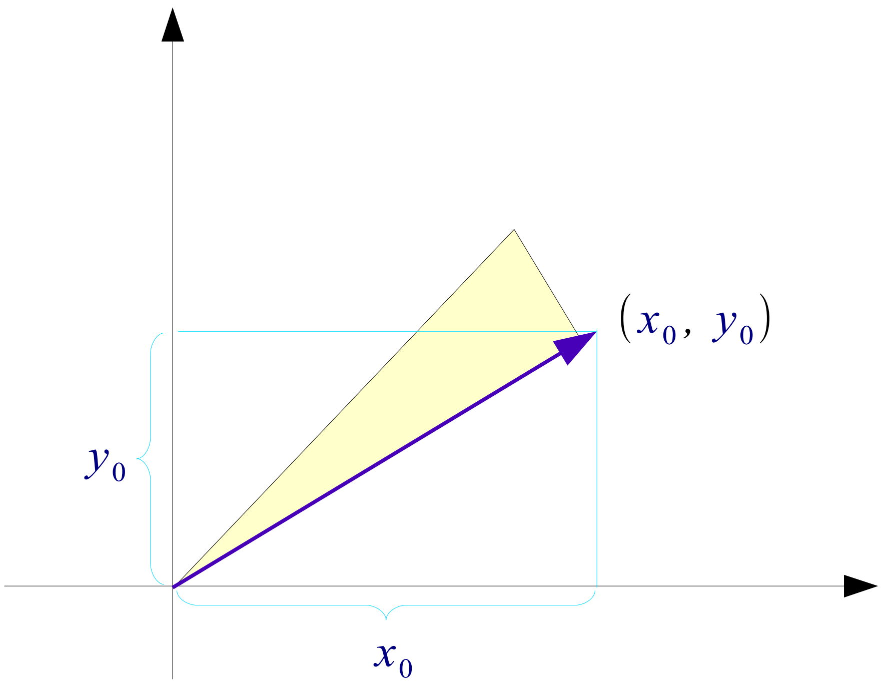
Vector Rotation (1)

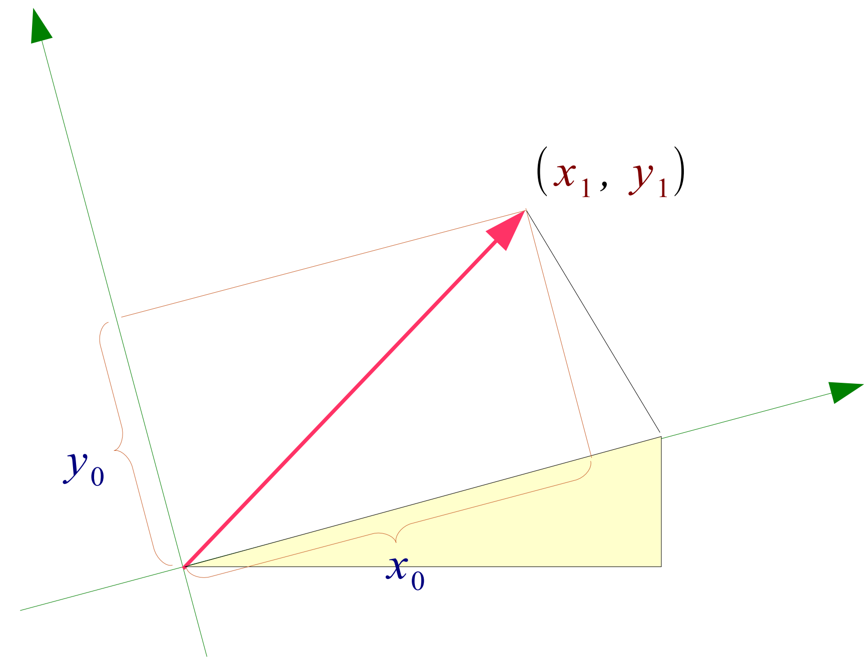
$$y_1 = x_0 \sin\alpha + y_0 \cos\alpha$$



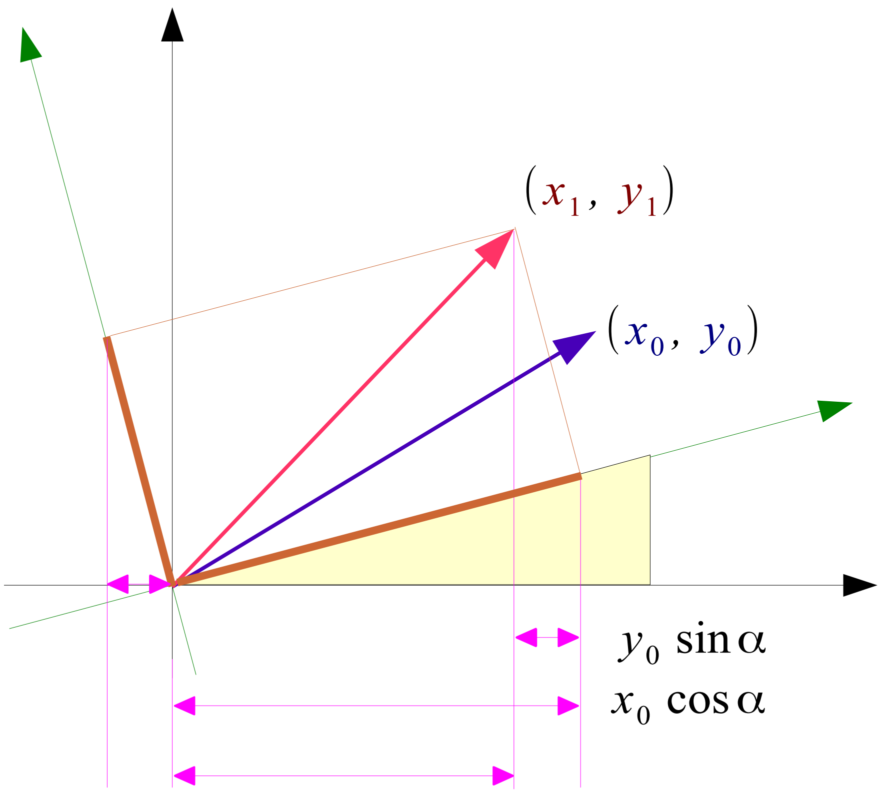
$$x_1 = x_0 \cos\alpha - y_0 \sin\alpha$$

Vector Rotation (2)

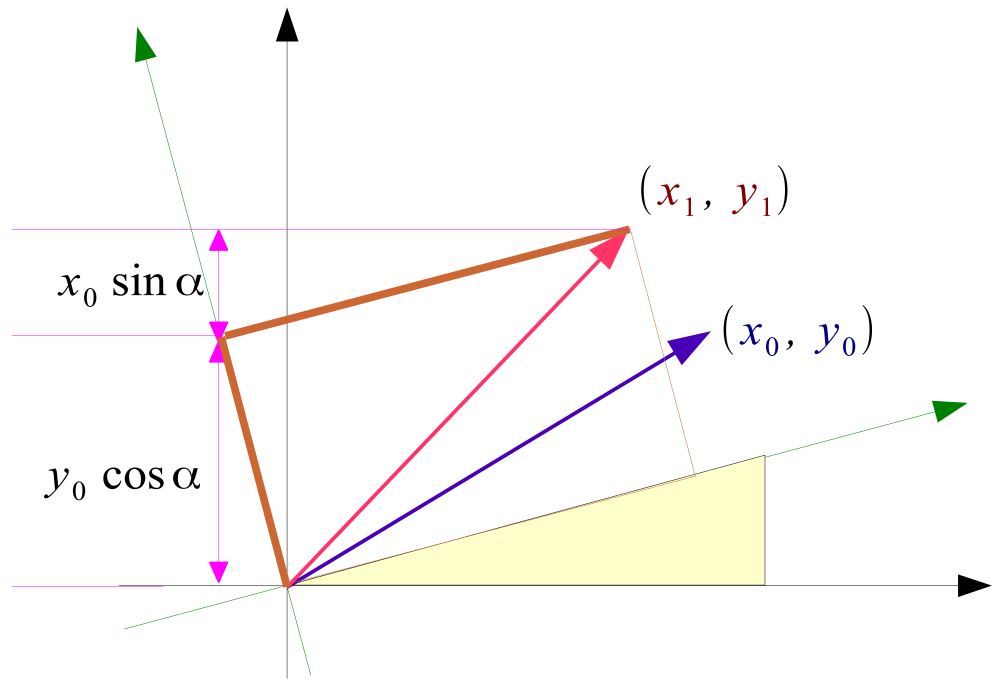




Vector Rotation (3)



$$x_1 = x_0 \cos\alpha - y_0 \sin\alpha$$



$$y_1 = x_0 \sin\alpha + y_0 \cos\alpha$$

CORDIC Iteration Equations

Given Vector
Given Angle

Rotate →

Computing
new coordinates

Given Unit Vector
Given Angle α

Rotate →

$$x = \cos\alpha$$
$$y = \sin\alpha$$

Given Vector $(x_{0,} \quad y_0)$
Given Angle α

Rotate →

$$x_n = x_0 \cos\alpha - y_0 \sin\alpha$$
$$y_n = x_0 \sin\alpha + y_0 \cos\alpha$$

$$\alpha_0 \rightarrow \cdots \rightarrow \alpha_i \rightarrow \alpha_{i+1} \rightarrow \cdots \rightarrow 0$$

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \rightarrow \cdots \rightarrow \begin{pmatrix} x_i \\ y_i \end{pmatrix} \rightarrow \begin{pmatrix} x_{i+1} \\ y_{i+1} \end{pmatrix} \rightarrow \cdots \rightarrow \begin{pmatrix} x_n \\ y_n \end{pmatrix}$$

CORDIC Iteration Equations – Pseudo-Rotation

$$\alpha_0 \rightarrow \cdots \rightarrow \alpha_i \rightarrow \alpha_{i+1} \rightarrow \cdots \rightarrow 0$$

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \rightarrow \cdots \rightarrow \begin{pmatrix} x_i \\ y_i \end{pmatrix} \rightarrow \begin{pmatrix} x_{i+1} \\ y_{i+1} \end{pmatrix} \rightarrow \cdots \rightarrow \begin{pmatrix} x_n \\ y_n \end{pmatrix}$$

$$x_{i+1} = x_i \cos\alpha_i - y_i \sin\alpha_i$$
$$y_{i+1} = x_i \sin\alpha_i + y_i \cos\alpha_i$$

Pseudo-rotation

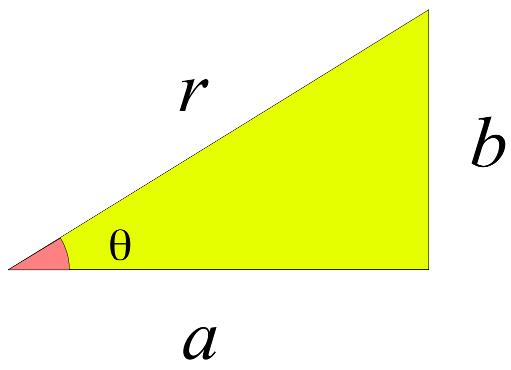
$$x'_{i+1} = (x_i - y_i \tan\alpha_i)$$
$$y'_{i+1} = (x_i \tan\alpha_i + y_i)$$

$$x_{i+1} = \cos\alpha_i (x_i - y_i \tan\alpha_i)$$
$$y_{i+1} = \cos\alpha_i (x_i \tan\alpha_i + y_i)$$

$$x_{i+1} = \frac{1}{\sqrt{1 + \tan^2\alpha_i}} (x_i - y_i \tan\alpha_i)$$

$$y_{i+1} = \frac{1}{\sqrt{1 + \tan^2\alpha_i}} (x_i \tan\alpha_i + y_i)$$

cos θ



$$r = \sqrt{a^2 + b^2}$$

$$\cos\theta = \frac{a}{r} = \frac{a}{\sqrt{a^2 + b^2}}$$

$$= \frac{1}{\sqrt{1 + (b/a)^2}}$$

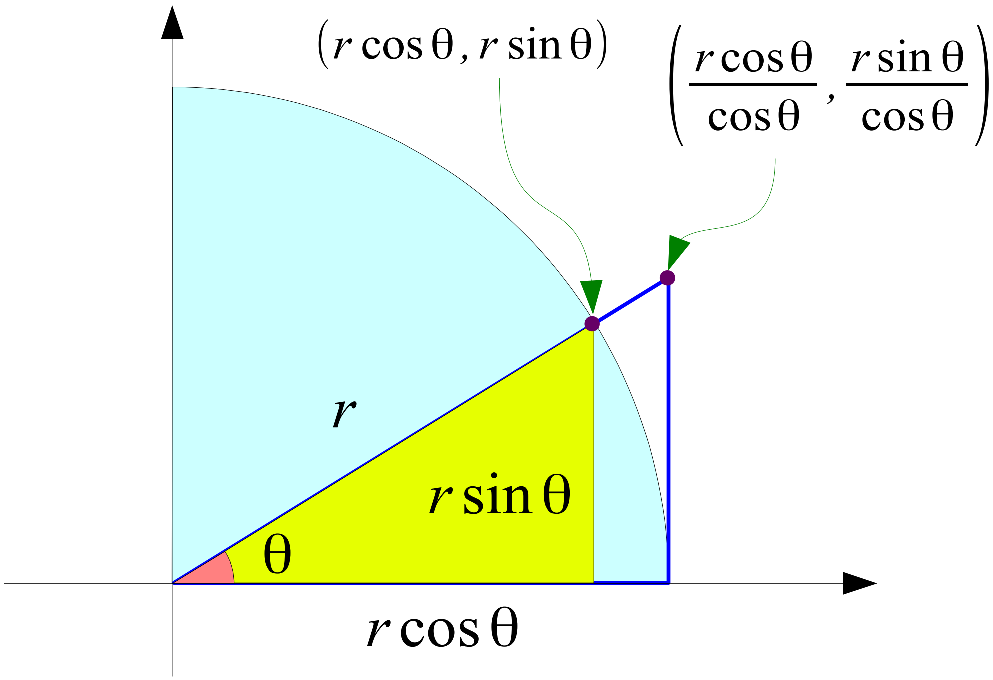
$$= \frac{1}{\sqrt{1 + \tan^2\theta}}$$

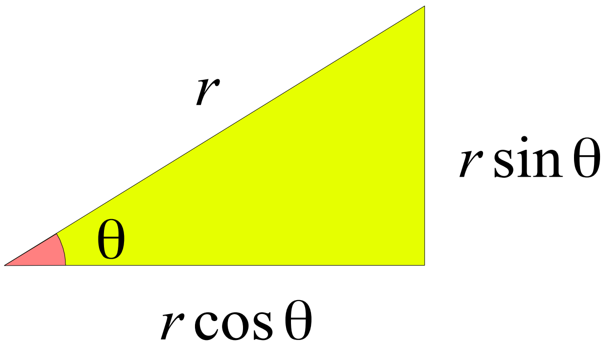
$$\cos\theta = \frac{a}{r}$$

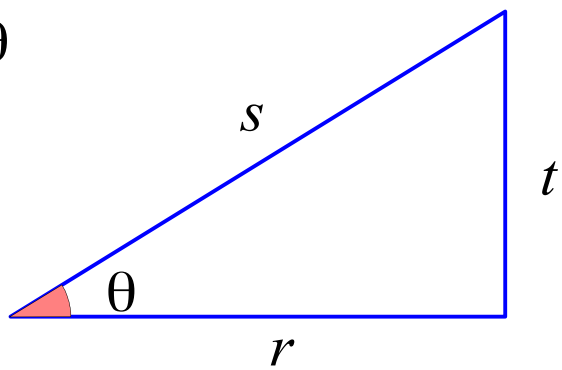
$$\sin\theta = \frac{b}{r}$$

$$\tan\theta = \frac{b}{a}$$

Pseudo-rotation



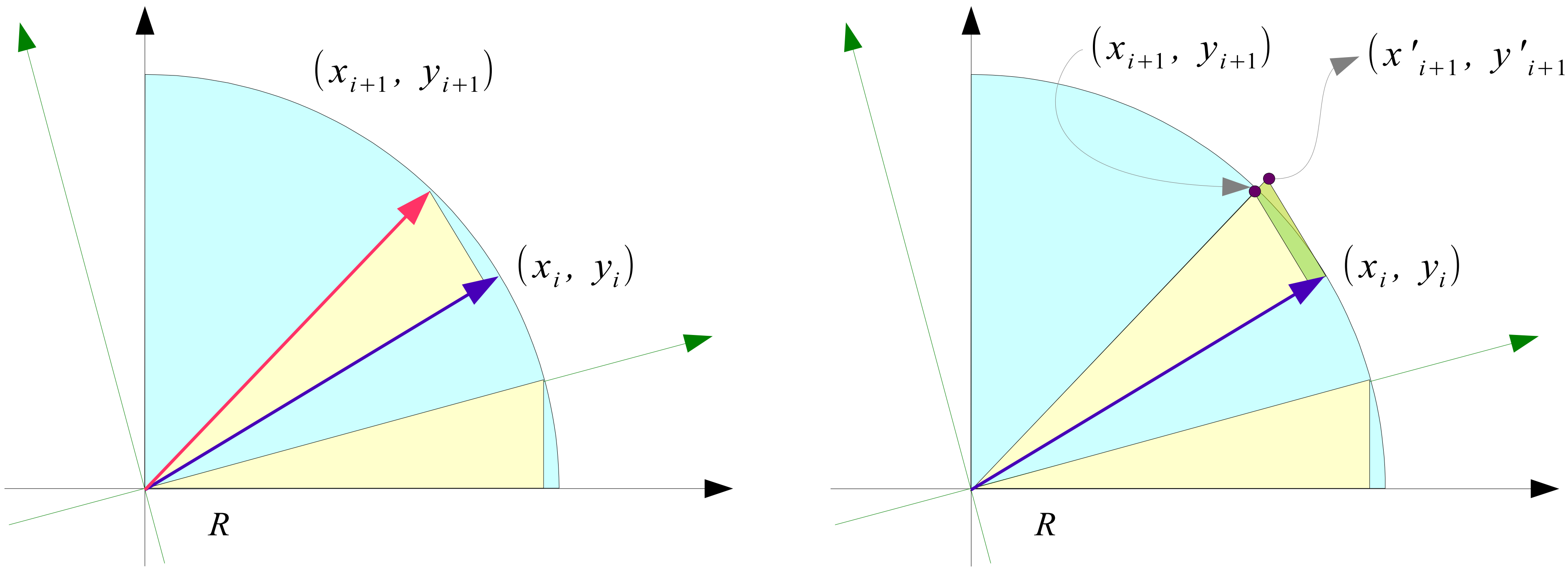




$$r : r\cos\theta = s : r \qquad r\cos\theta : r\sin\theta = r : t$$

$$s = \frac{r}{\cos\theta} \qquad s = \frac{r\sin\theta}{\cos\theta}$$

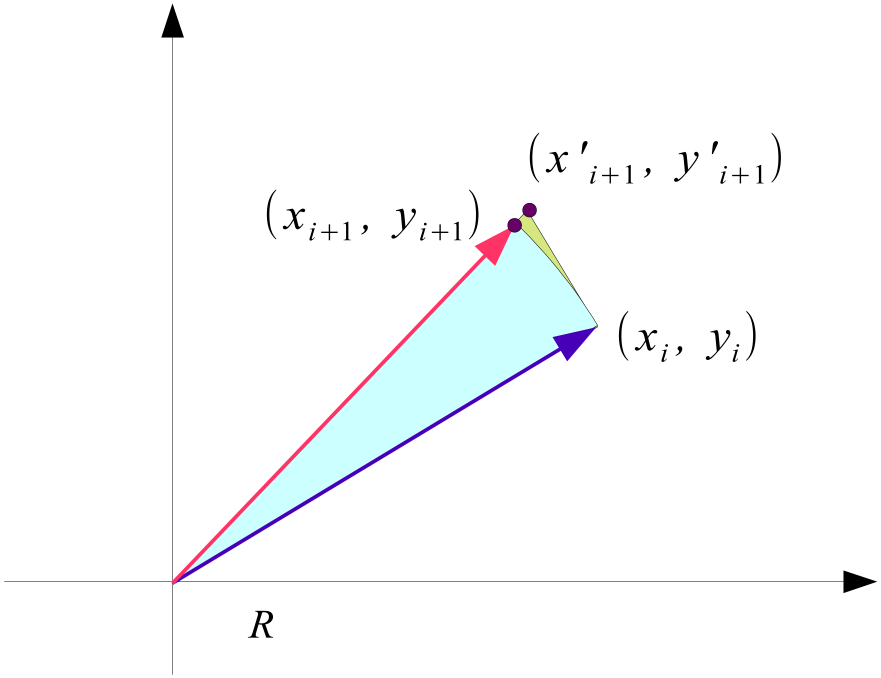
Pseudo-rotation (1)

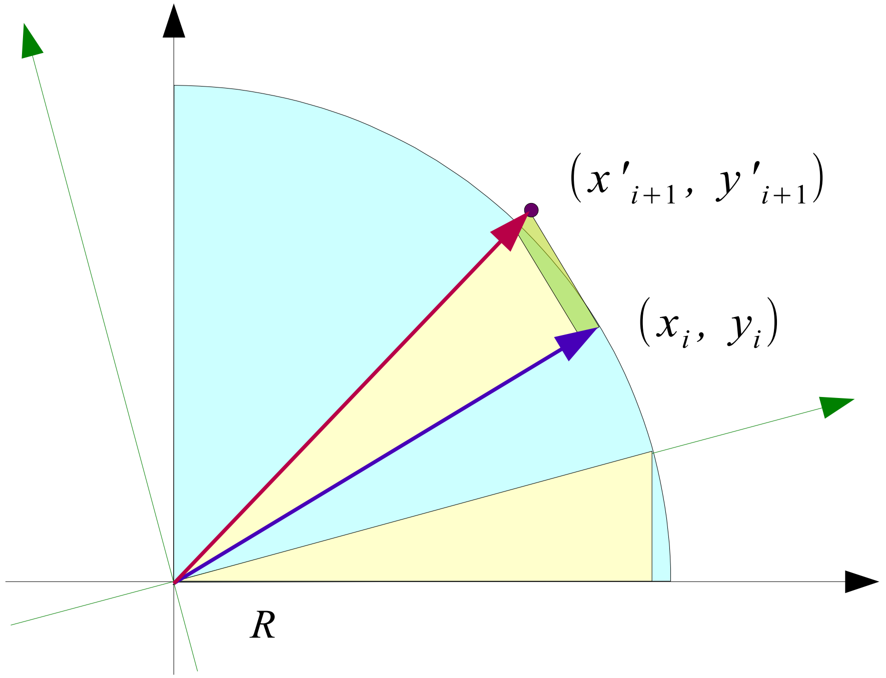


Pseudo-rotation (2)

$$x'_{i+1} = x_{i+1} / \cos\alpha_i$$
$$y'_{i+1} = y_{i+1} / \cos\alpha_i$$

$$x'_{i+1} > x_{i+1}$$
$$y'_{i+1} > y_{i+1}$$





CORDIC Iteration Equations – Pseudo-Rotation

$$\alpha_0 \rightarrow \cdots \rightarrow \alpha_i \rightarrow \alpha_{i+1} \rightarrow \cdots \rightarrow 0$$

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \rightarrow \cdots \rightarrow \begin{pmatrix} x_i \\ y_i \end{pmatrix} \rightarrow \begin{pmatrix} x_{i+1} \\ y_{i+1} \end{pmatrix} \rightarrow \cdots \rightarrow \begin{pmatrix} x_n \\ y_n \end{pmatrix}$$

$$x_{i+1} = x_i \cos\alpha_i - y_i \sin\alpha_i = \cos\alpha_i \left(x_i - y_i \tan\alpha_i\right) = \left(1/\sqrt{1+\tan^2\alpha_i}\right)\left(x_i - y_i \tan\alpha_i\right)$$

$$y_{i+1} = x_i \sin\alpha_i + y_i \cos\alpha_i = \cos\alpha_i \left(x_i \tan\alpha_i + y_i\right) = \left(1/\sqrt{1+\tan^2\alpha_i}\right)\left(x_i \tan\alpha_i + y_i\right)$$

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \rightarrow \cdots \rightarrow \begin{pmatrix} x'_i \\ y'_i \end{pmatrix} \rightarrow \begin{pmatrix} x'_{i+1} \\ y'_{i+1} \end{pmatrix} \rightarrow \cdots \rightarrow \begin{pmatrix} x'_n \\ y'_n \end{pmatrix}$$

Pseudo-rotation

$$x'_{i+1} = \left(x_i - y_i \tan\alpha_i\right)$$

$$y'_{i+1} = \left(x_i \tan\alpha_i + y_i\right)$$

References

[1] http://en.wikipedia.org/
[2] CORDIC FAQ, www.dspguru.com
[3] R. Andraka, A survey of CORDIC algorithms for FPGA based computers
[4] J. S. Walther, A Unified Algorithm for Elementary Functions
[5] J. P. Deschamps, G. A. Bioul, G.D. Sutter, Synthesis of Arithmetic Circuits