Trigonometry (3A)

- Quadrant Angle Trigonometry
- Negative Angle Trigonometry
- Reference Angle Trigonometry
- Sinusoidal Waves

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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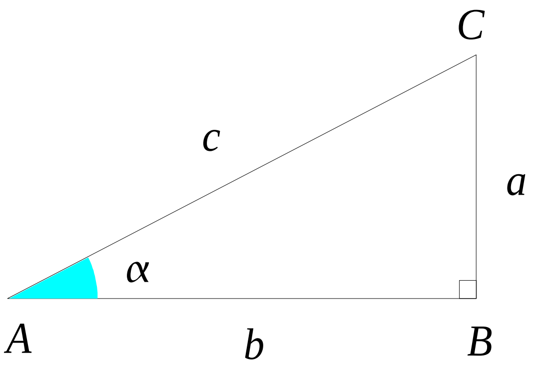
Triangle Trigonometry

Right Triangle

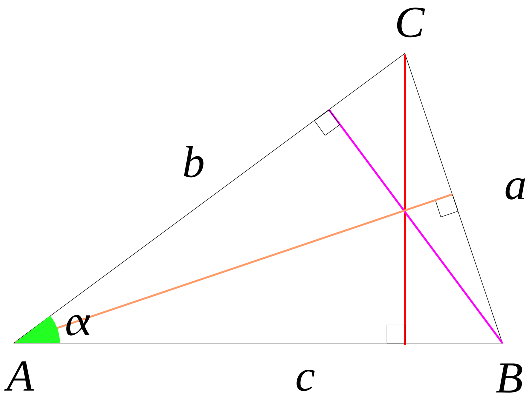


$$0^\circ < \alpha < 90^\circ$$

$$\sin\alpha = a / c$$
$$\cos\alpha = b / c$$
$$\tan\alpha = a / b$$

Oblique Triangle

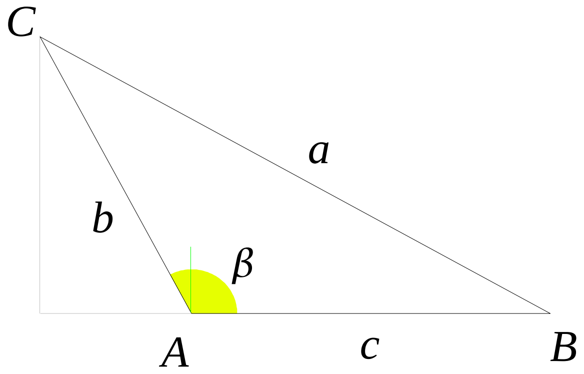
All Acute Angles



$$0^\circ < \alpha < 90^\circ$$

$$\sin\alpha = ?$$
$$\cos\alpha = ?$$
$$\tan\alpha = ?$$

One Obtuse Angle

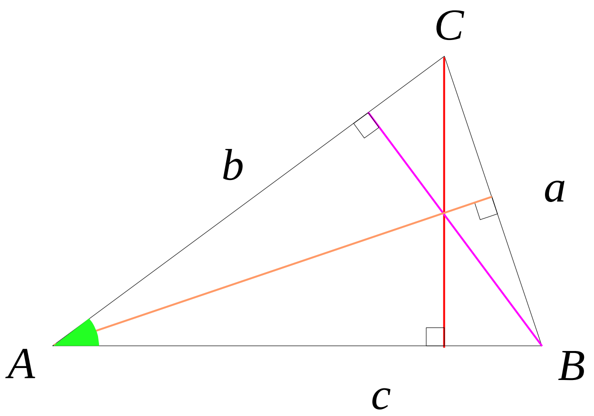


$$90^\circ < \beta < 180^\circ$$

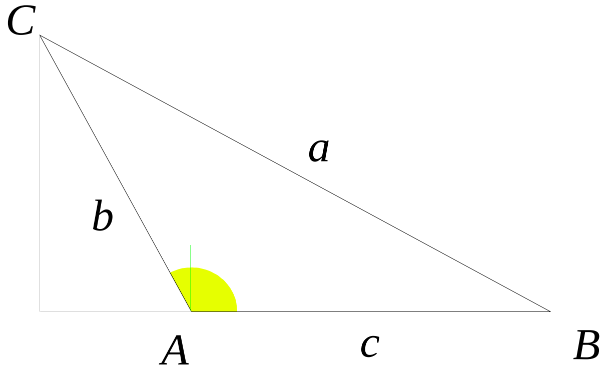
$$\sin\beta = ?$$
$$\cos\beta = ?$$
$$\tan\beta = ?$$

Oblique Triangles Trigonometry

All Acute Angles



One Obtuse Angle



Assuming

$$\sin\beta = \sin(180^\circ - \alpha) = +\sin\alpha$$
$$\cos\beta = \cos(180^\circ - \alpha) = -\cos\alpha$$
$$\tan\beta = \tan(180^\circ - \alpha) = -\tan\alpha$$

$$0^\circ < \alpha < 90^\circ, \qquad 0^\circ < \beta < 180^\circ$$

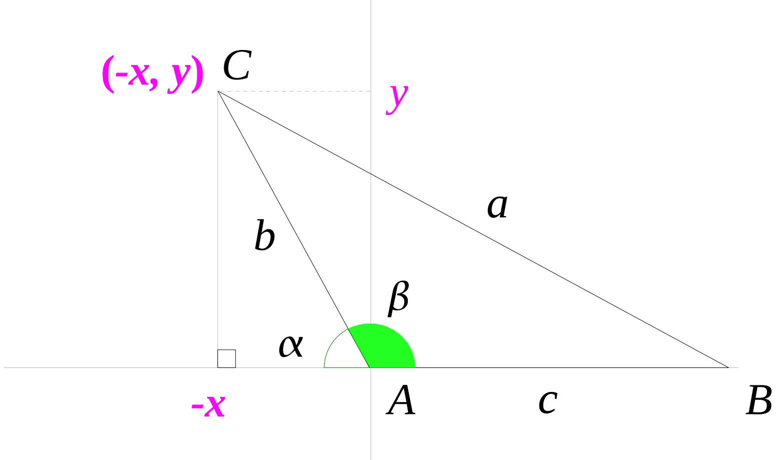
The Law of Sines

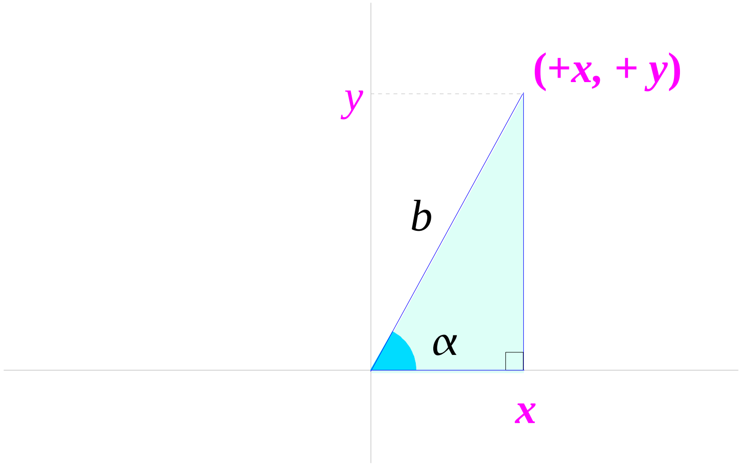
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = c^2 + a^2 - 2ca\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Trigonometry in Quadrant Angles (1)



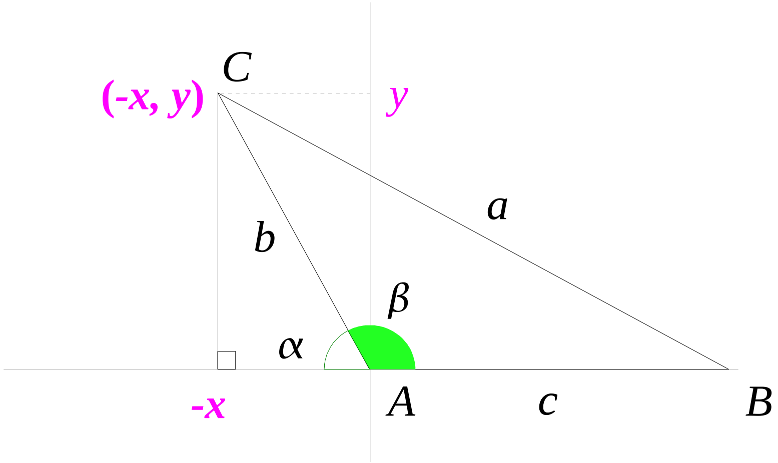


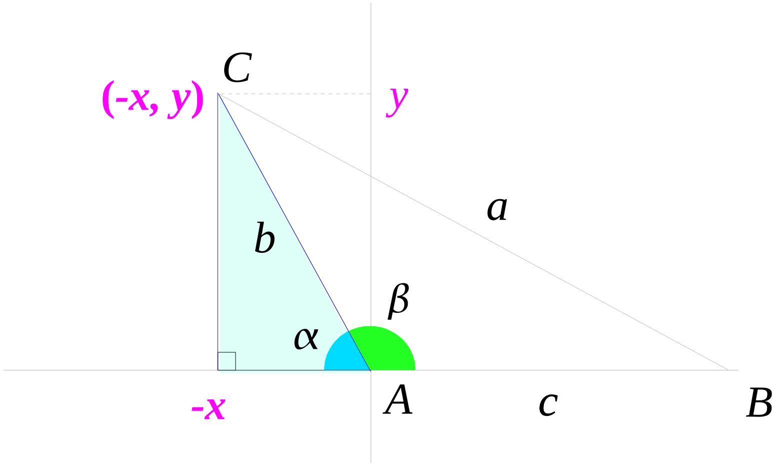
$$\sin\beta = \sin(180^\circ - \alpha) = +\sin\alpha \qquad +(\ \sin\alpha = y\,/\,b\ )$$

$$\cos\beta = \cos(180^\circ - \alpha) = -\cos\alpha \qquad -(\ \cos\alpha = x\,/\,b\ )$$

$$\tan\beta = \tan(180^\circ - \alpha) = -\tan\alpha \qquad -(\ \tan\alpha = y\,/\,x\ )$$

Trigonometry in Quadrant Angles (2)



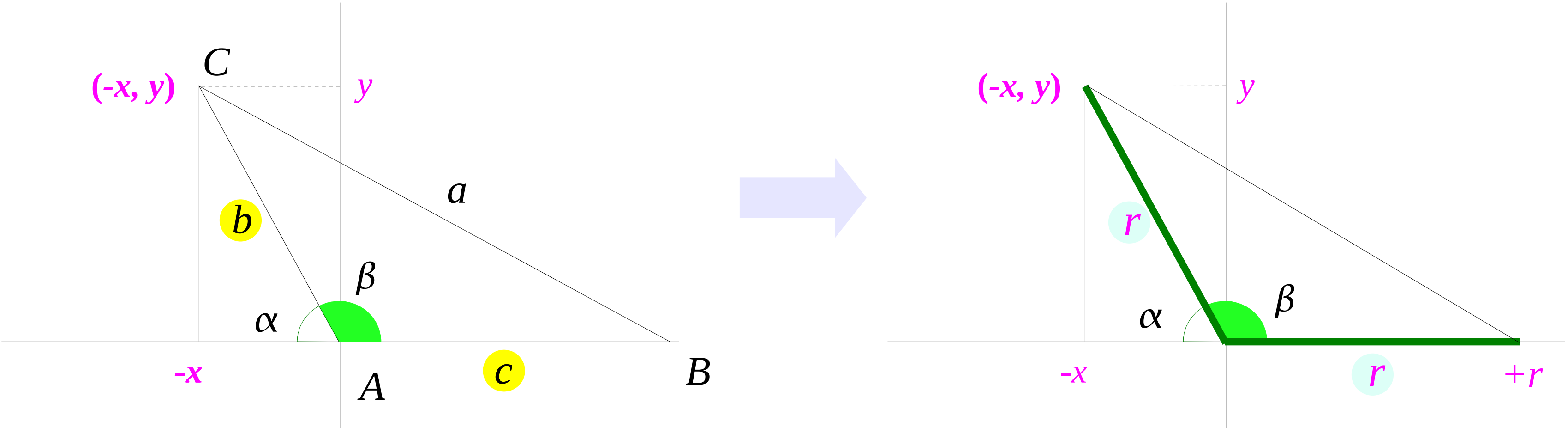


$$\sin\beta = \sin(180^\circ - \alpha) = +\sin\alpha$$
$$\cos\beta = \cos(180^\circ - \alpha) = -\cos\alpha$$
$$\tan\beta = \tan(180^\circ - \alpha) = -\tan\alpha$$

$$\sin\beta = +y\,/\,b$$
$$\cos\beta = -x\,/\,b$$
$$\tan\beta = -\,y/x$$

Trigonometry in Quadrant Angles (3)

Isosceles Triangle



$$r = \sqrt{x^2 + y^2}$$

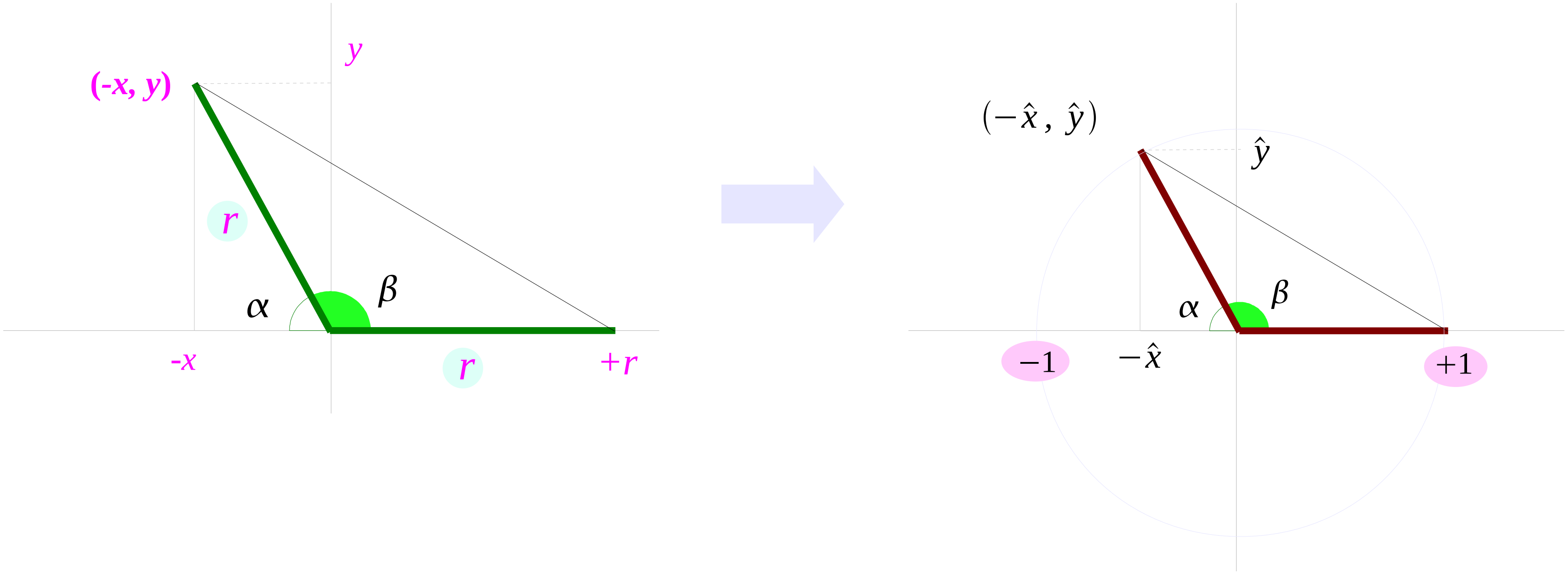
$$\sin\beta = +y / b \qquad \sin\beta = +y / r$$

$$\cos\beta = -x / b \qquad \cos\beta = -x / r$$

$$\tan\beta = -\ y/x \qquad \tan\beta = -y / x$$

Trigonometry in Quadrant Angles (4)

Unit Circle



$$r = \sqrt{x^2 + y^2}$$

$$\sin\beta = +y \,/\, r$$

$$\cos\beta = -x \,/\, r$$

$$\tan\beta = -y \,/\, x$$

$$1 = \sqrt{\hat{x}^2 + \hat{y}^2}$$

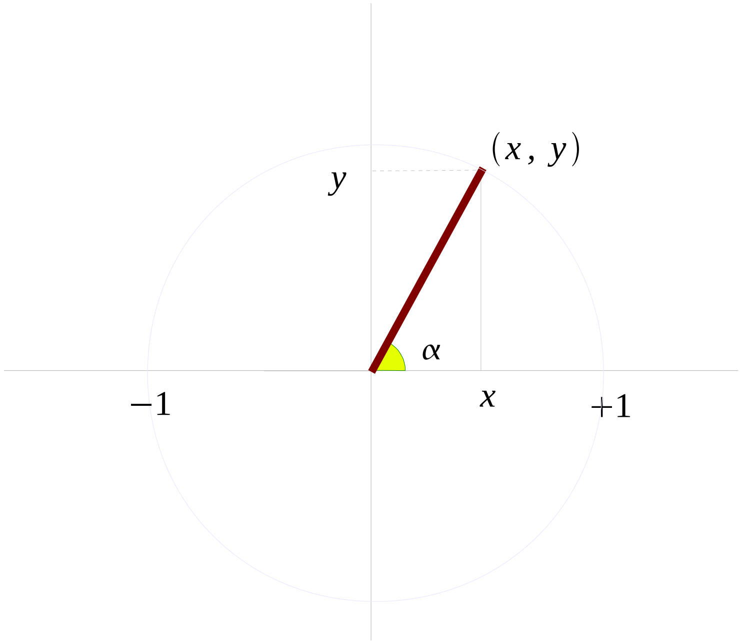
$$\sin\beta = +\hat{y}$$

$$\cos\beta = -\hat{x}$$

$$\tan\beta = -\hat{y}/\hat{x}$$

Negative Angle Trigonometry (1)

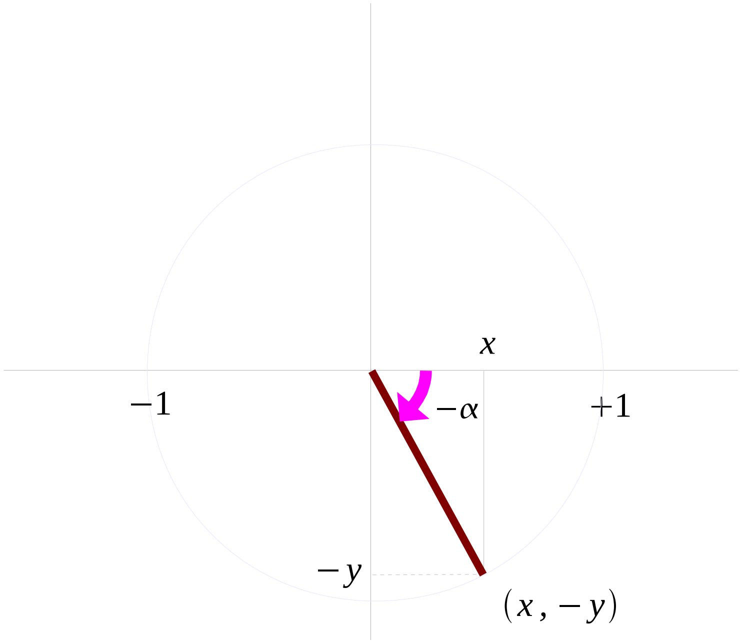
1st Quadrant Angle



$0^\circ < \alpha < 90^\circ$

$$\sin\alpha = y$$
$$\cos\alpha = x$$
$$\tan\alpha = y/x$$

4th Quadrant Angle

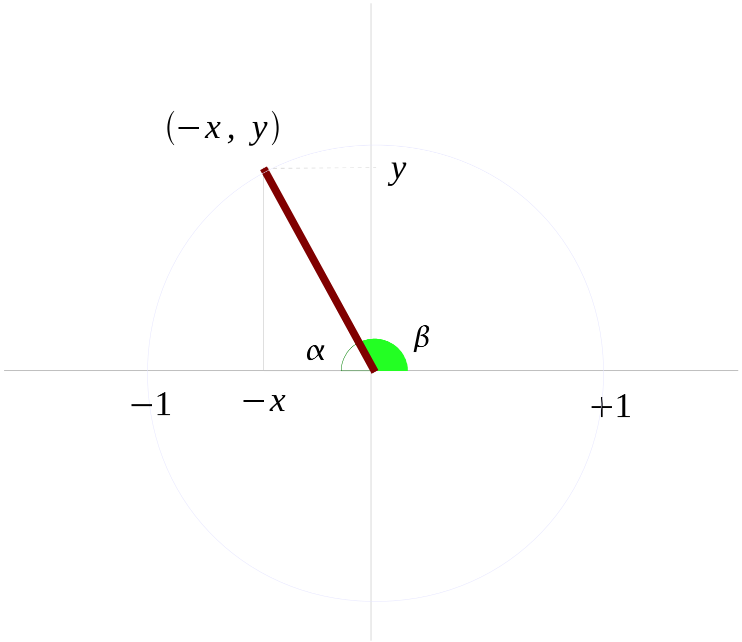


$-90^\circ < -\alpha < 0^\circ$

$$\sin(-\alpha) = -\sin\alpha = -y$$
$$\cos(-\alpha) = +\cos\alpha = +x$$
$$\tan(-\alpha) = -\tan\alpha = -y/x$$

Negative Angle Trigonometry (2)

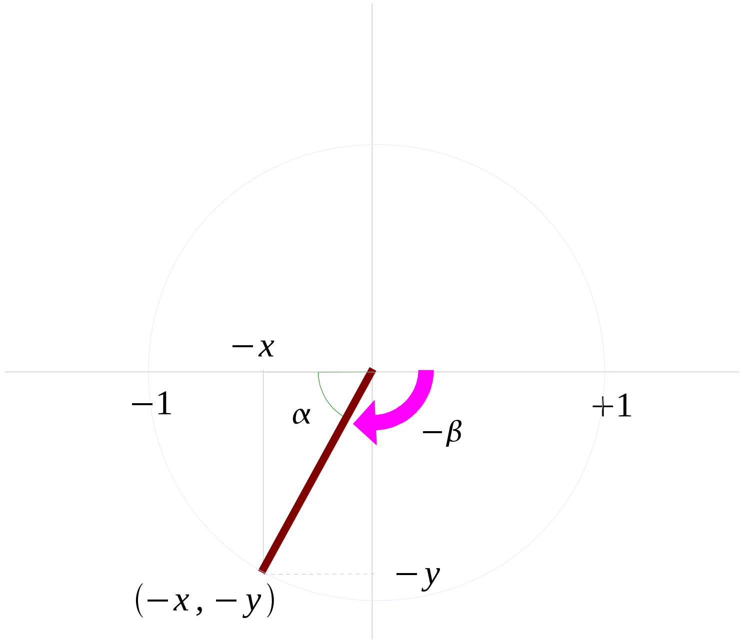
2nd Quadrant Angle



$90^\circ < \beta < 180^\circ$

$$\begin{aligned} \sin\beta &= +\sin\alpha = +y \\ \cos\beta &= -\cos\alpha = -x \\ \tan\beta &= -\tan\alpha = -y/x \end{aligned}$$

3rd Quadrant Angle

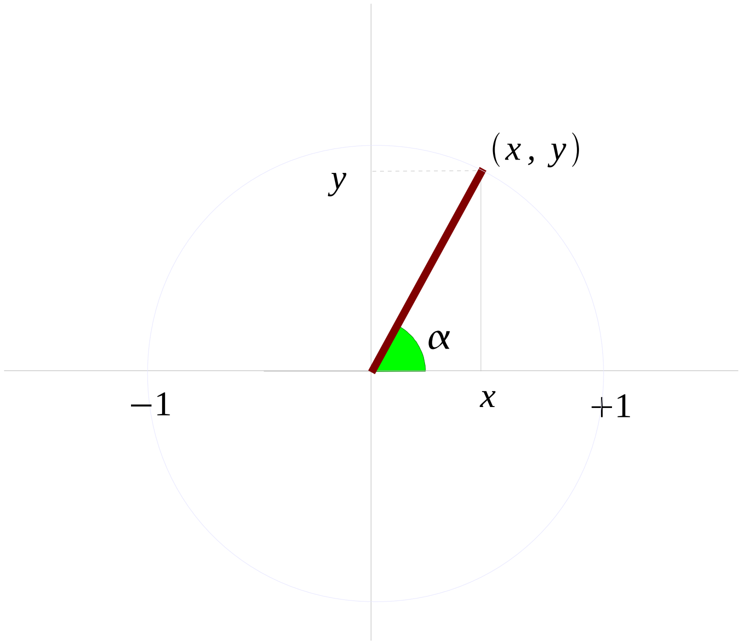


$-180^\circ < -\beta < -90^\circ$

$$\begin{aligned} \sin(-\beta) &= -\sin\alpha = -y \\ \cos(-\beta) &= -\cos\alpha = -x \\ \tan(-\beta) &= +\tan\alpha = +y/x \end{aligned}$$

Reference Angle (1)

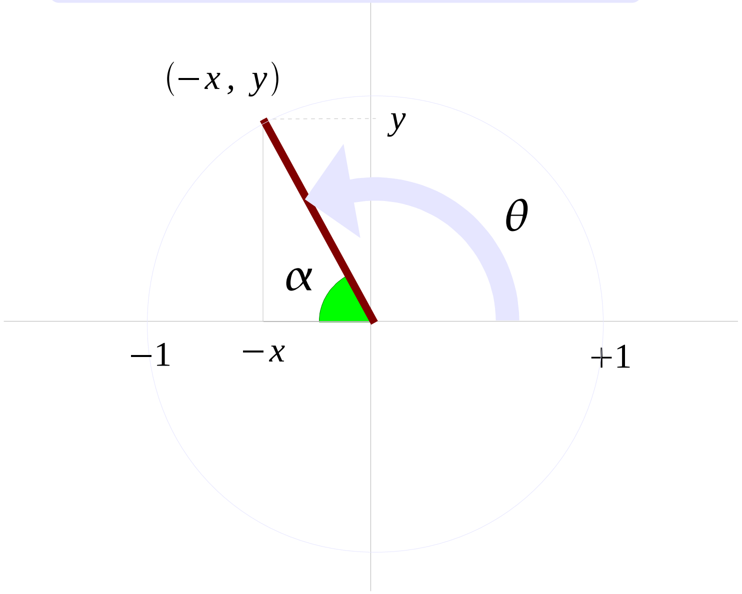
1^{st} Quadrant Angle θ



$$\sin\alpha = y$$
$$\cos\alpha = x$$
$$\tan\alpha = y/x$$

2^{nd} Quadrant Angle θ

$$90^\circ < \theta < 180^\circ$$



Reference Angle α

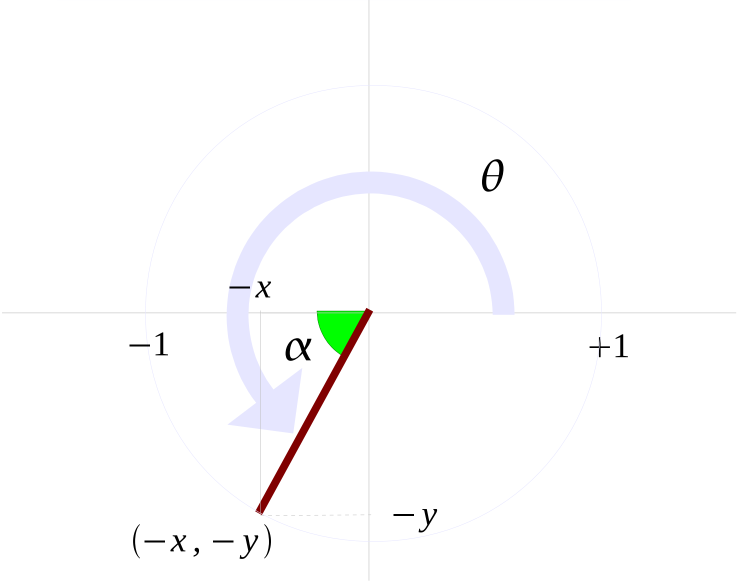
$$\alpha = 180^\circ - \theta$$

$$\sin\theta = +\sin\alpha = +y$$
$$\cos\theta = -\cos\alpha = -x$$
$$\tan\theta = -\tan\alpha = -y/x$$

Reference Angle (2)

3^{rd} Quadrant Angle θ

$180^\circ < \theta < 270^\circ$



Reference Angle α

$\alpha = \theta - 180^\circ$

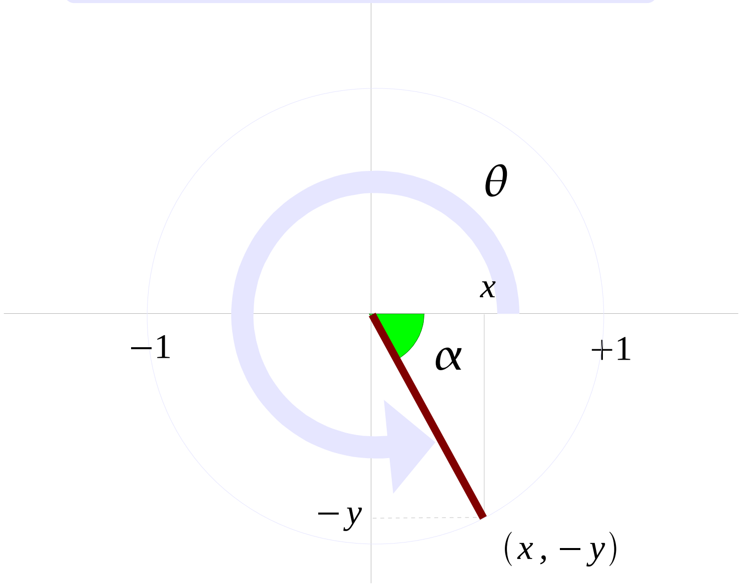
$\sin\theta = - \sin\alpha = -y$

$\cos\theta = - \cos\alpha = -x$

$\tan\theta = + \tan\alpha = + y/x$

4^{th} Quadrant Angle θ

$270^\circ < \theta < 360^\circ$



Reference Angle α

$\alpha = 360^\circ - \theta$

$\sin\theta = - \sin\alpha = -y$

$\cos\theta = + \cos\alpha = +x$

$\tan\theta = - \tan\alpha = - y/x$

Reference Angle (3)

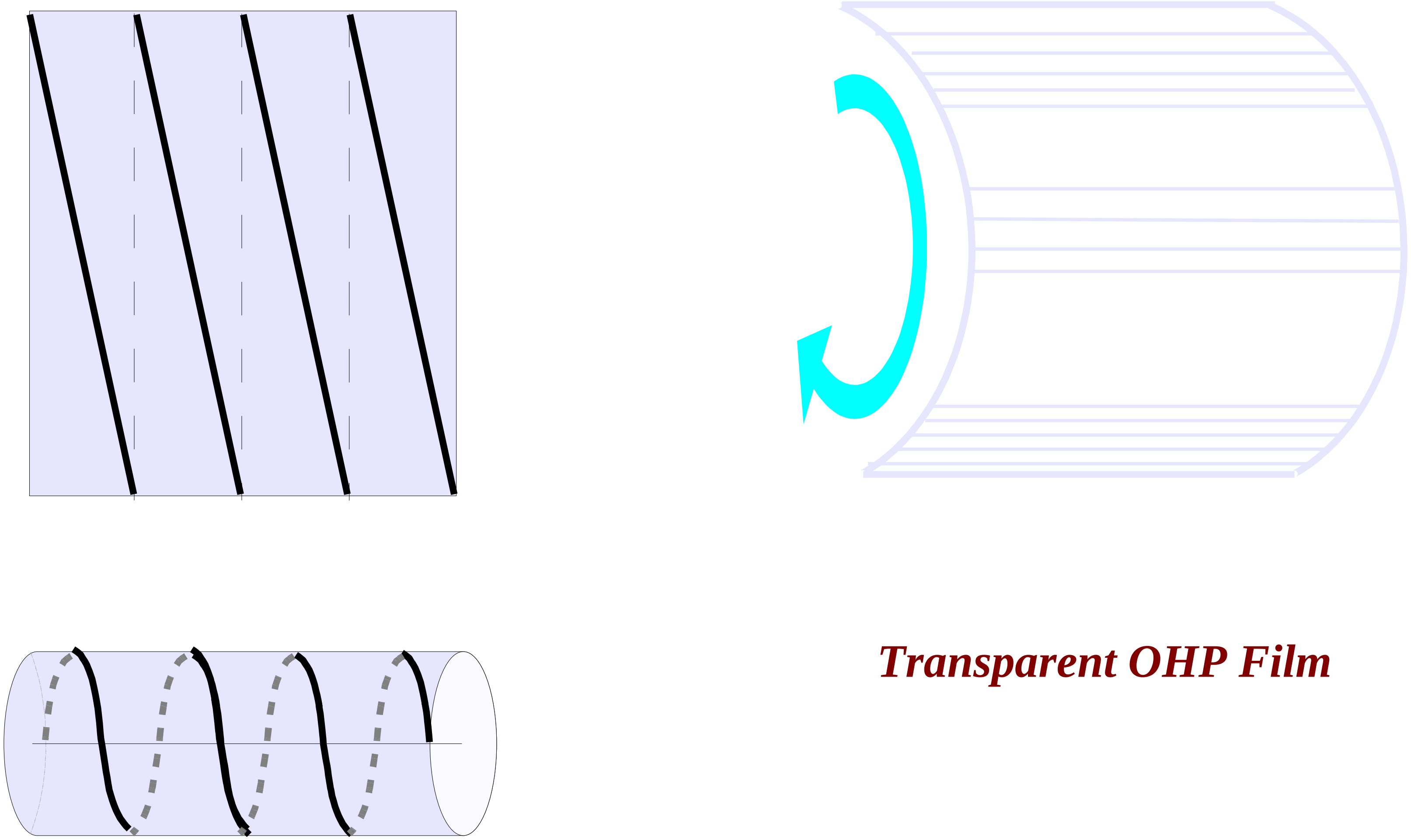
A Quadrant Angle θ

Reference Angle α

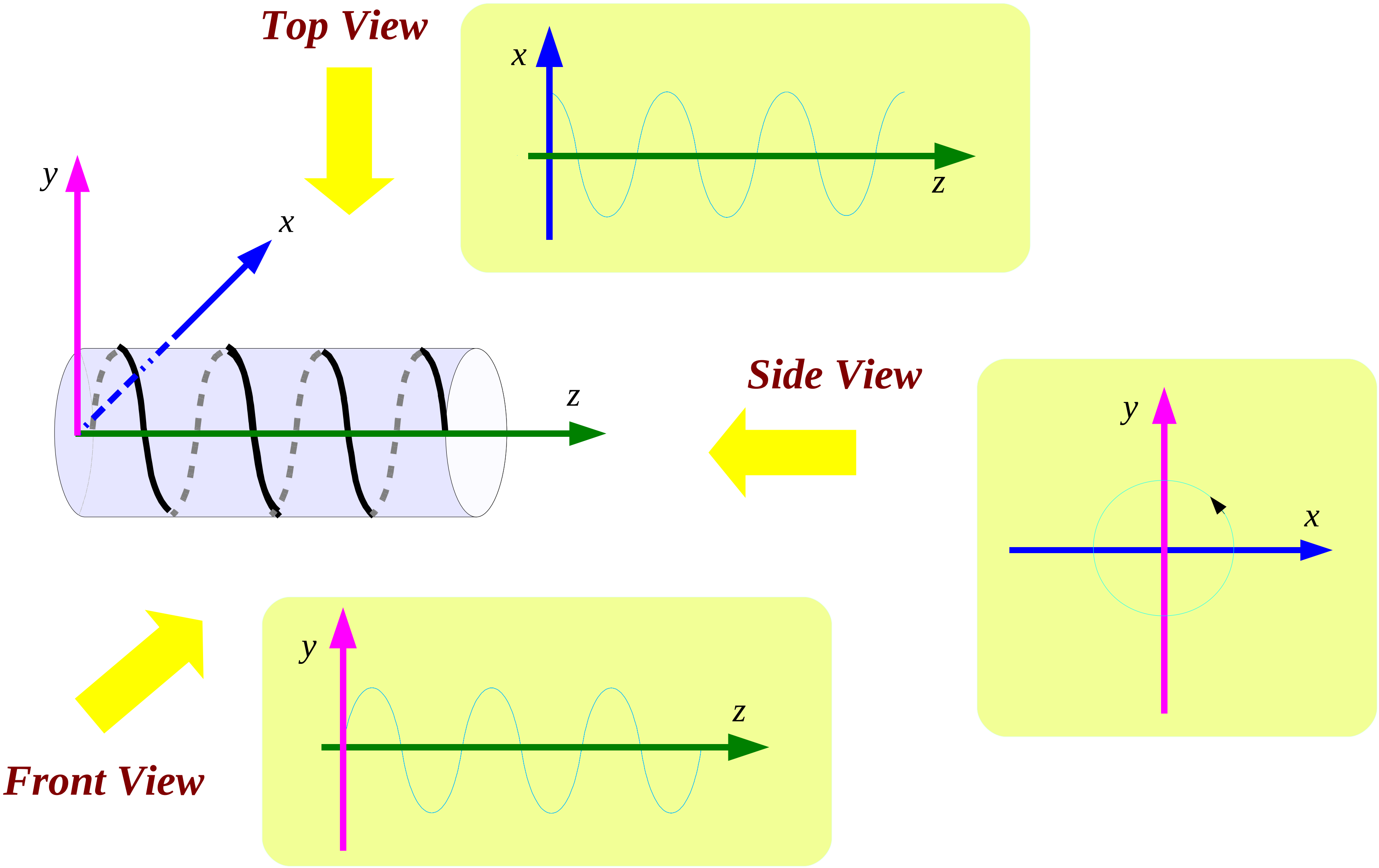
$\alpha = \pi - \theta$ $\sin\theta = +\sin\alpha$ $\cos\theta = -\cos\alpha$ $\tan\theta = -\tan\alpha$	$\alpha = \theta$ $\sin\theta = \sin\alpha$ $\cos\theta = \cos\alpha$ $\tan\theta = \tan\alpha$
$\sin\theta = -\sin\alpha$ $\cos\theta = -\cos\alpha$ $\tan\theta = +\tan\alpha$ $\alpha = \theta - \pi$	$\sin\theta = -\sin\alpha$ $\cos\theta = +\cos\alpha$ $\tan\theta = -\tan\alpha$ $\alpha = 2\pi - \theta$

only ***sin*** +	***All*** +
only ***tan*** +	*only* ***cos*** +

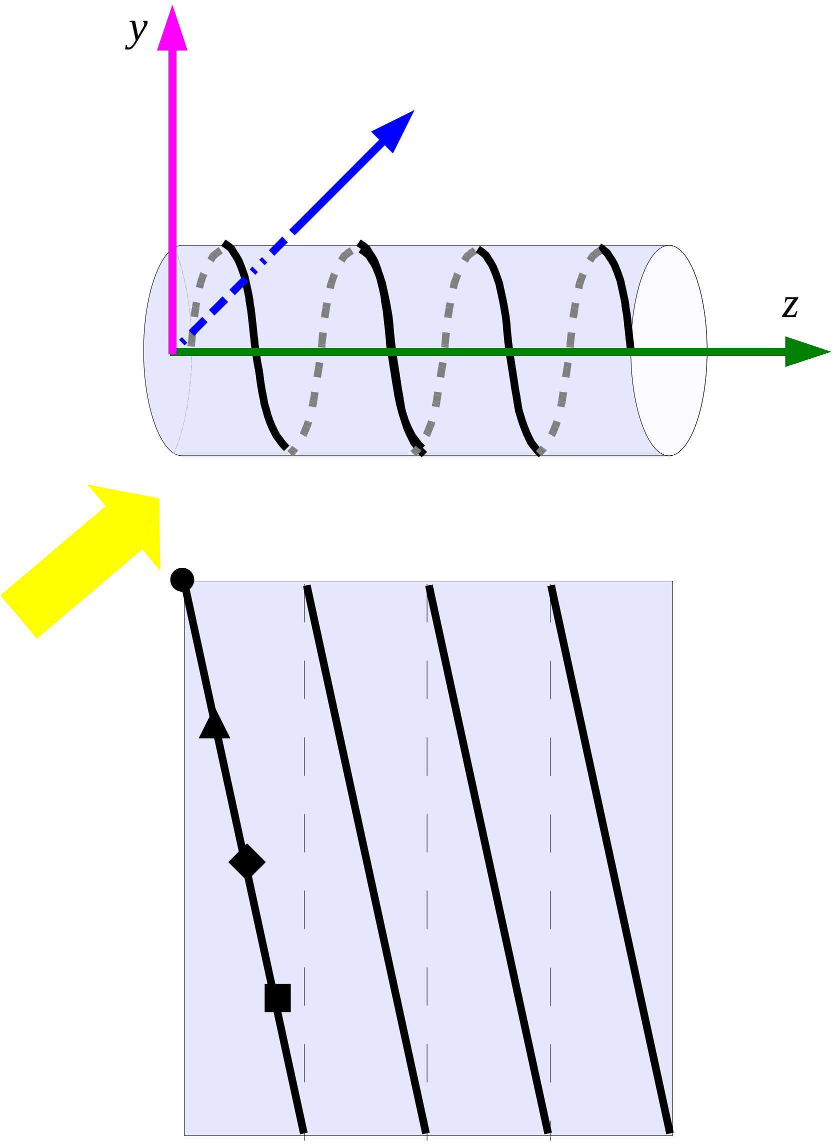
Making a Helix



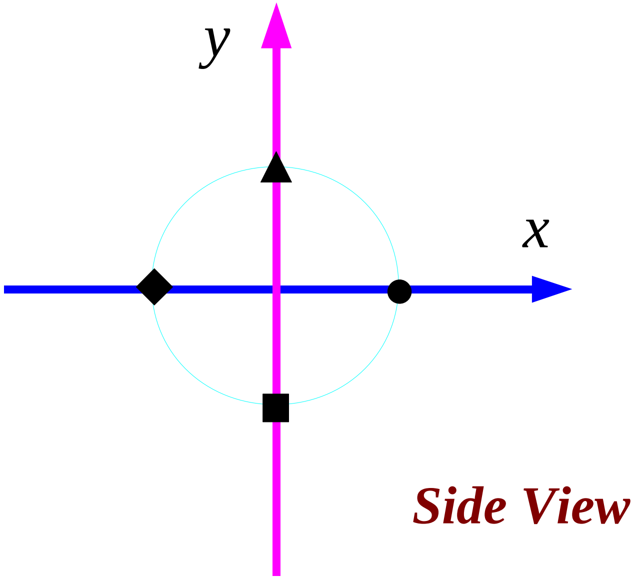
A Helix and Viewpoints



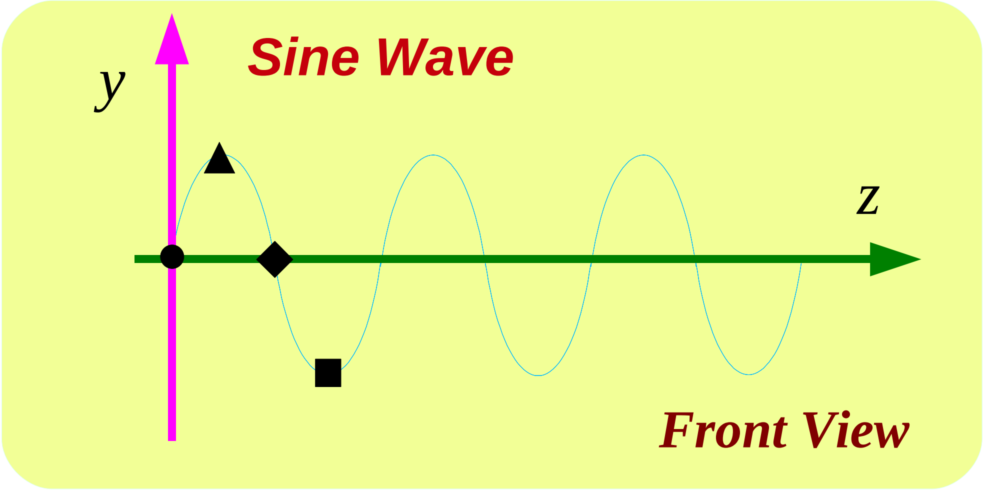
Sine Wave



●	0
▲	$\frac{\pi}{2}$
◆	π
■	$\frac{3}{2}\pi$



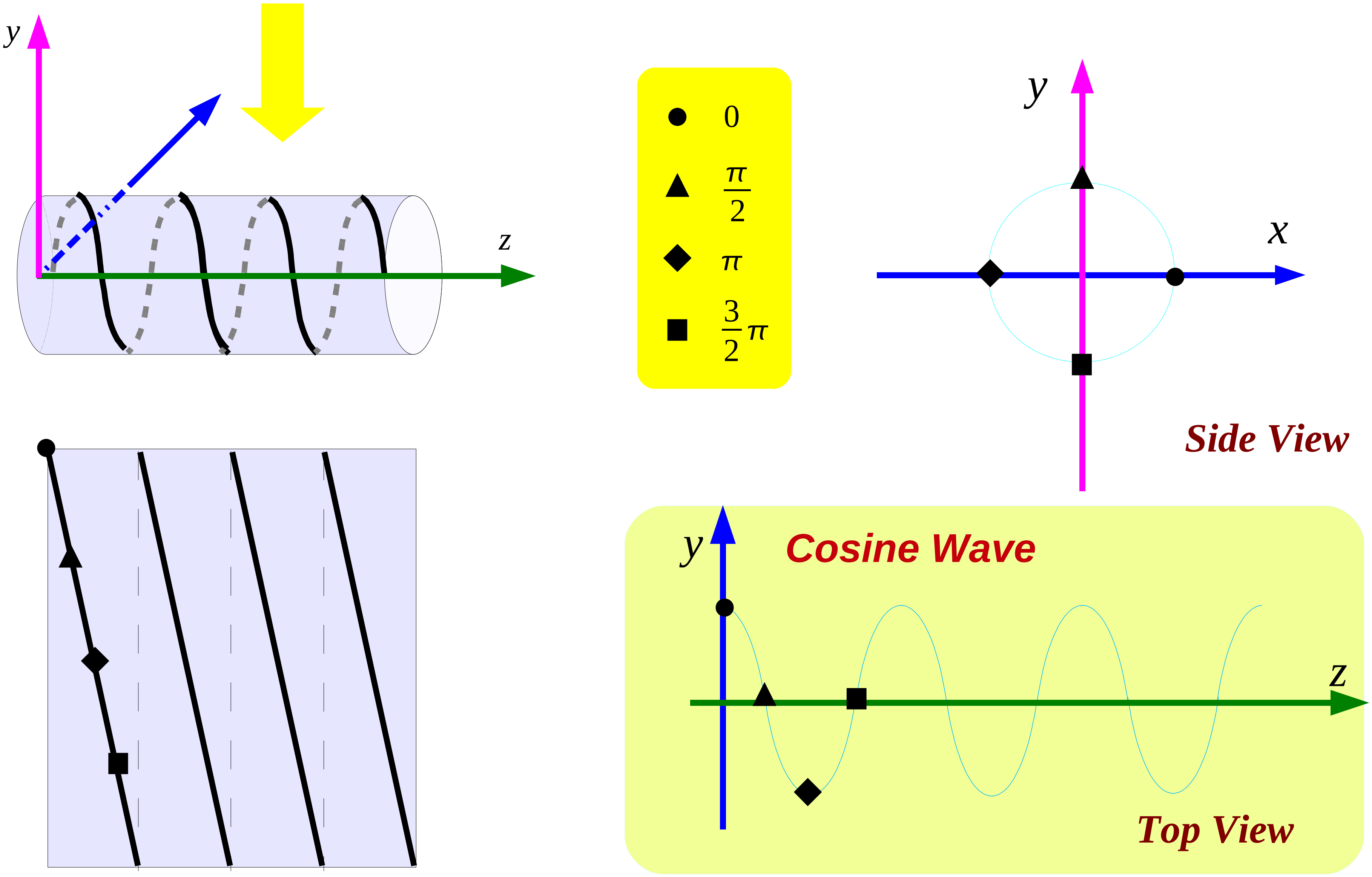
Side View



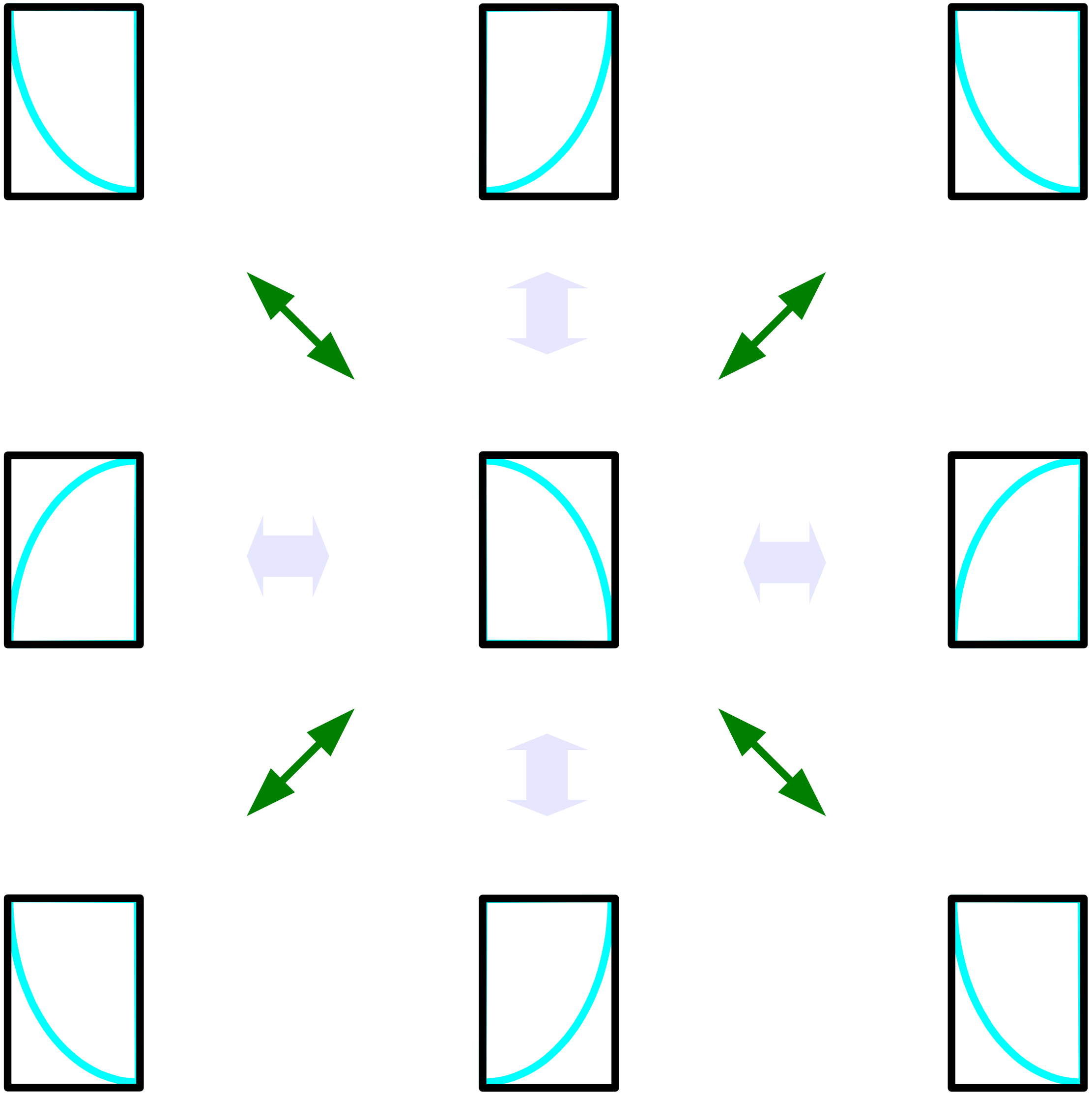
Sine Wave

Front View

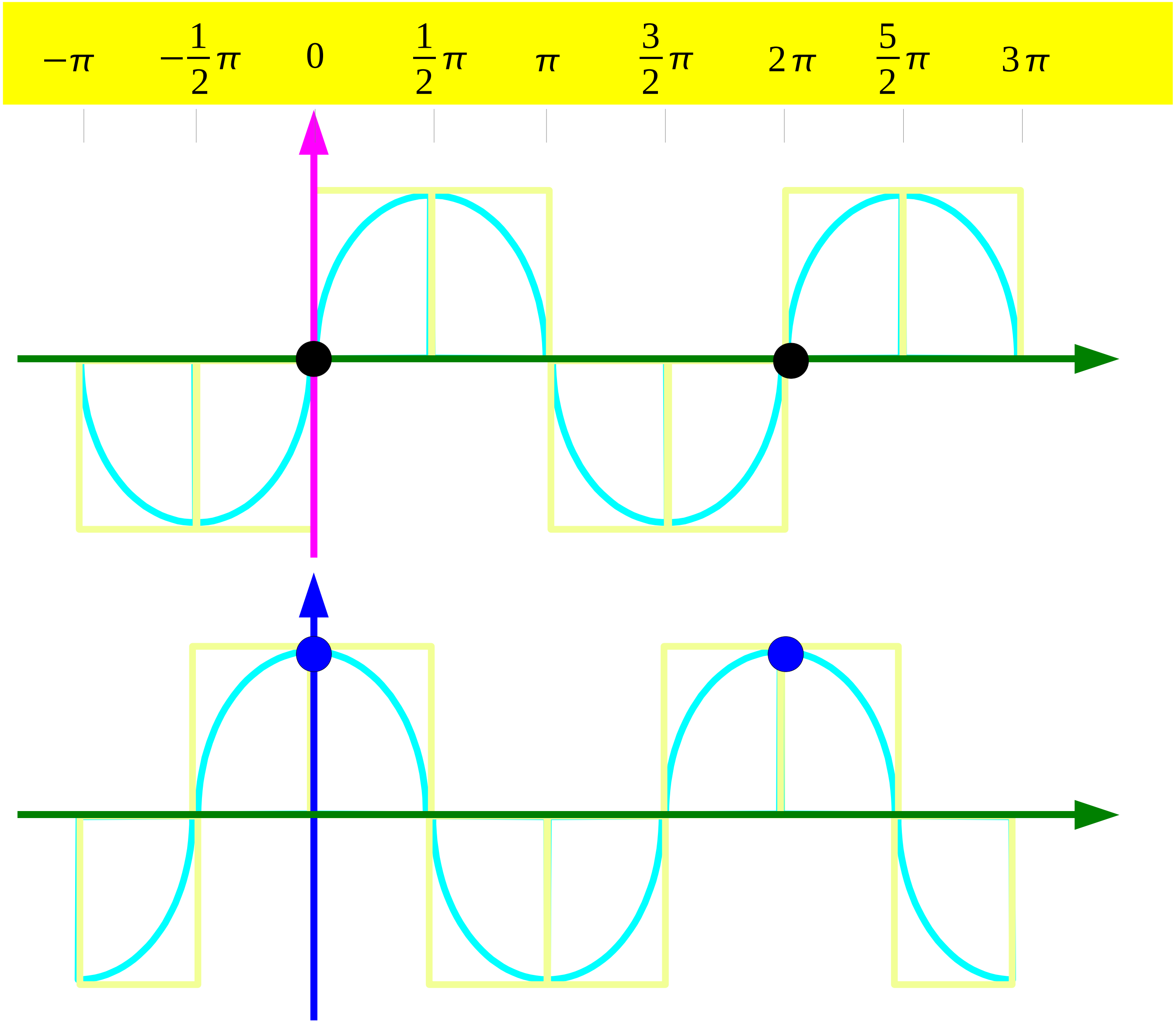
Cosine Wave



Symmetry in Sinusoid



Sine and Cosine Waves

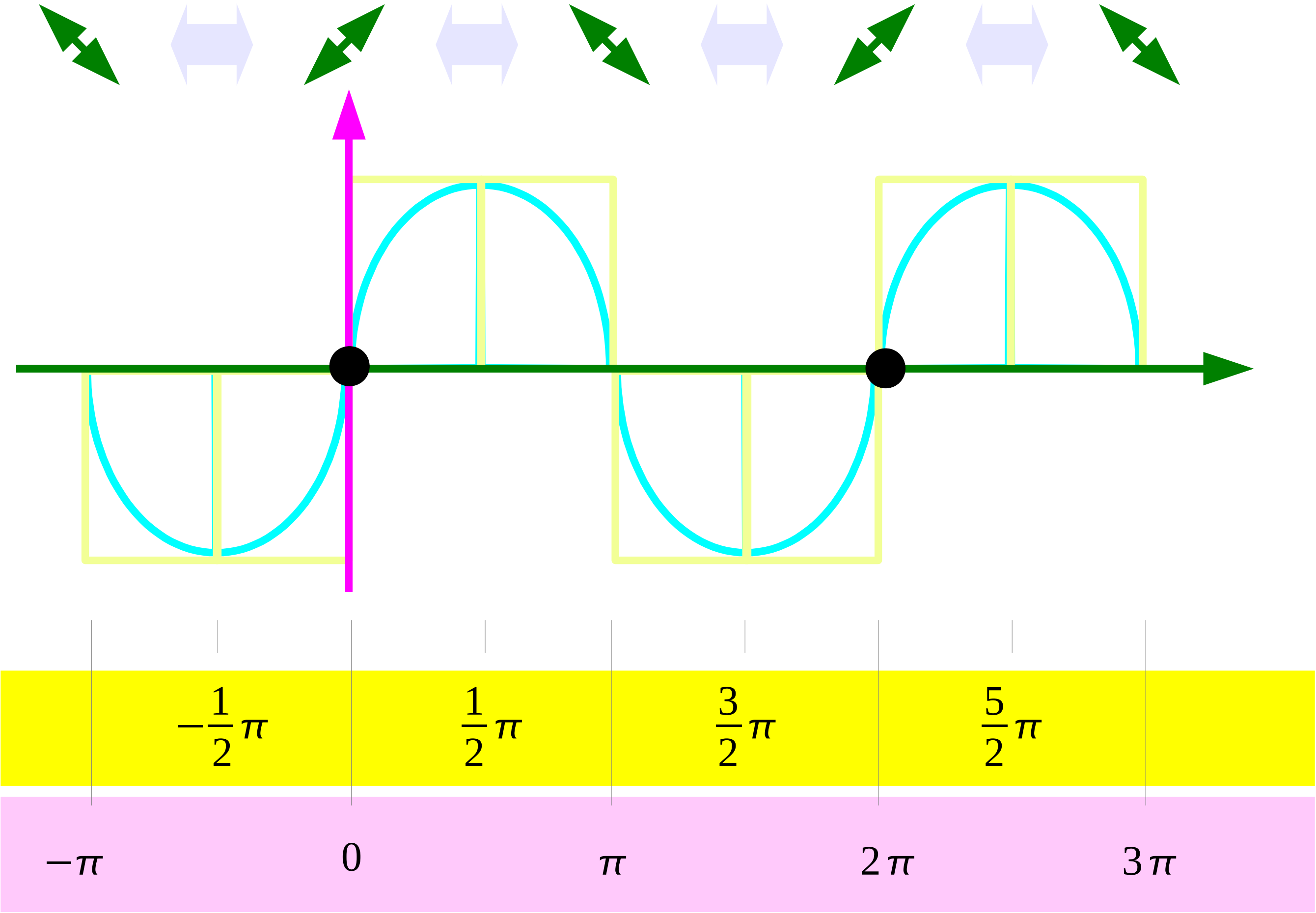


Sine Wave

Cosine Wave

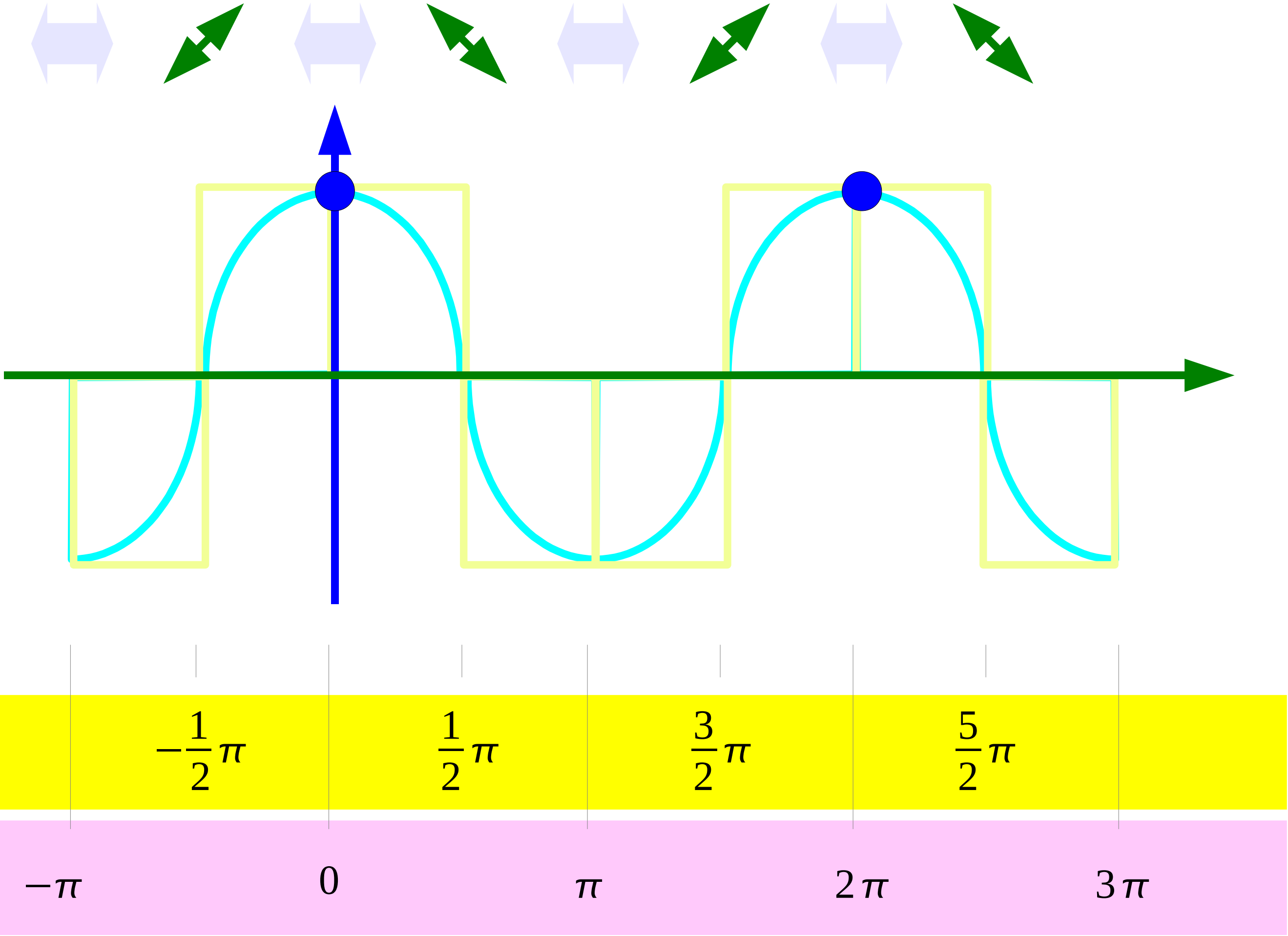
Sine Wave Symmetry

Sine Wave



Cosine Wave Symmetry

Cosine Wave



References

[1] http://en.wikipedia.org/
[2] http://planetmath.org/
[3] Blitzer, R. “Algebra & Trigonometry.” 3rd ed, Prentice Hall
[4] Smith, R. T., Minton, R. B. “Calculus: Concepts & Connections,” Mc Graw Hill
[5] 홍성대, “기본/실력 수학의 정석,”성지출판