DFT Matrix (7A)

- DFT Matrix
- Symmetric Matrix
- Unitary Matrix

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4/19/10

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Discrete Fourier **Transform**

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn} \quad \Longleftrightarrow \quad x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{+j(2\pi/N)kn}$$

$$W_N \triangleq e^{-j(2\pi/N)}$$

$$W_N^{nk} \triangleq e^{-j(2\pi/N)nk}$$

$$X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn} \quad \Longleftrightarrow \quad x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, W_N^{-kn}$$

N=8 DFT

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad\qquad W_8^{kn} = e^{-j(\frac{2\pi}{8})kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} = \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\ W_8^0 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\ W_8^0 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\ W_8^0 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\ W_8^0 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\ W_8^0 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49} \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix}$$

N=8 DFT Matrix (1)

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad W_8^{kn} = e^{-j(\frac{2\pi}{8})kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} = \begin{bmatrix}
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 7} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 5} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 3} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 1}
\end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix}$$

N=8 DFT Matrix (2)

$$W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn} = \cos\left(\frac{\pi}{4}\cdot k\cdot n\right) - j\sin\left(\frac{\pi}{4}\cdot k\cdot n\right)$$

$\cos(\pi/4)\cdot 0 - j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0 - j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0 - j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0 - j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0 - j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0 - j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0 - j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0 - j\sin(\pi/4)\cdot 0$
$\cos(\pi/4)\cdot 0 - j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 1 - j\sin(\pi/4)\cdot 1$	$\cos(\pi/4)\cdot 2 - j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 3 - j\sin(\pi/4)\cdot 3$	$\cos(\pi/4)\cdot 4 - j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 5 - j\sin(\pi/4)\cdot 5$	$\cos(\pi/4)\cdot 6 - j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 7 - j\sin(\pi/4)\cdot 7$
$\cos(\pi/4)\cdot 0 - j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 2 - j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 4 - j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 6 - j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 0 - j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 2 - j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 4 - j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 6 - j\sin(\pi/4)\cdot 6$
$\cos(\pi/4)\cdot 0 - j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 3 - j\sin(\pi/4)\cdot 3$	$\cos(\pi/4)\cdot 6 - j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 1 - j\sin(\pi/4)\cdot 1$	$\cos(\pi/4)\cdot 4 - j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 7 - j\sin(\pi/4)\cdot 7$	$\cos(\pi/4)\cdot 2 - j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 5 - j\sin(\pi/4)\cdot 5$
$\cos(\pi/4)\cdot 0 - j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 4 - j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 0 - j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 4 - j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 0 - j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 4 - j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 0 - j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 4 - j\sin(\pi/4)\cdot 4$
$\cos(\pi/4)\cdot 0 - j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 5 - j\sin(\pi/4)\cdot 5$	$\cos(\pi/4)\cdot 2 - j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 7 - j\sin(\pi/4)\cdot 7$	$\cos(\pi/4)\cdot 4 - j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 1 - j\sin(\pi/4)\cdot 1$	$\cos(\pi/4)\cdot 6 - j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 3 - j\sin(\pi/4)\cdot 3$
$\cos(\pi/4)\cdot 0 - j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 6 - j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 4 - j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 2 - j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 0 - j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 6 - j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 4 - j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 2 - j\sin(\pi/4)\cdot 2$
$\cos(\pi/4)\cdot 0 - j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 7 - j\sin(\pi/4)\cdot 7$	$\cos(\pi/4)\cdot 6 - j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 5 - j\sin(\pi/4)\cdot 5$	$\cos(\pi/4)\cdot 4 - j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 3 - j\sin(\pi/4)\cdot 3$	$\cos(\pi/4)\cdot 2 - j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 1 - j\sin(\pi/4)\cdot 1$

N=8 DFT Matrix (3)

$$W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn} = \cos\left(\frac{\pi}{4}\cdot k\cdot n\right) - j\,\sin\left(\frac{\pi}{4}\cdot k\cdot n\right)$$

$$\begin{matrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & +\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}} & -j & -\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}} & -1 & -\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}} & +j & +\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}} \\
0 & -j & -1 & +j & 0 & -j & -1 & +j \\
0 & -\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}} & +j & +\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}} & -1 & +\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}} & -j & -\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}} \\
0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 \\
0 & -\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}} & -j & +\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}} & -1 & +\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}} & +j & -\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}} \\
0 & +j & -1 & -j & 0 & +j & -1 & -j \\
0 & +\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}} & +j & -\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}} & -1 & -\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}} & -j & +\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}
\end{matrix}$$

N=8 IDFT

$$x[n] = \frac{1}{N} \sum_{k=0}^{7} W_8^{-kn} X[k] \qquad W_8^{-kn} = e^{+j(\frac{2\pi}{8})kn}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \frac{1}{N} \begin{bmatrix} W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} \\ W_8^{0} & W_8^{-1} & W_8^{-2} & W_8^{-3} & W_8^{-4} & W_8^{-5} & W_8^{-6} & W_8^{-7} \\ W_8^{0} & W_8^{-2} & W_8^{-4} & W_8^{-6} & W_8^{-8} & W_8^{-10} & W_8^{-12} & W_8^{-14} \\ W_8^{0} & W_8^{-3} & W_8^{-6} & W_8^{-9} & W_8^{-12} & W_8^{-15} & W_8^{-18} & W_8^{-21} \\ W_8^{0} & W_8^{-4} & W_8^{-8} & W_8^{-12} & W_8^{-16} & W_8^{-20} & W_8^{-24} & W_8^{-28} \\ W_8^{0} & W_8^{-5} & W_8^{-10} & W_8^{-15} & W_8^{-20} & W_8^{-25} & W_8^{-30} & W_8^{-35} \\ W_8^{0} & W_8^{-6} & W_8^{-12} & W_8^{-18} & W_8^{-24} & W_8^{-30} & W_8^{-36} & W_8^{-42} \\ W_8^{0} & W_8^{-7} & W_8^{-14} & W_8^{-21} & W_8^{-28} & W_8^{-35} & W_8^{-42} & W_8^{-49} \end{bmatrix} \begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix}$$

N=8 IDFT Matrix (3)

$$x[n] = \frac{1}{N} \sum_{k=0}^{7} W_8^{-kn} \, X[k] \qquad W_8^{-kn} = e^{+j(\frac{2\pi}{8})kn}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \begin{bmatrix}
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 7} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 6} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 7} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 5} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 7} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 3} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 2} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 7} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 1}
\end{bmatrix} \begin{bmatrix} \frac{X[0]}{N} \\ \frac{X[1]}{N} \\ \frac{X[2]}{N} \\ \frac{X[3]}{N} \\ \frac{X[4]}{N} \\ \frac{X[5]}{N} \\ \frac{X[6]}{N} \\ \frac{X[7]}{N} \end{bmatrix}$$

N=8 IDFT Matrix (4)

$$W_8^{-kn} = e^{+j\left(\frac{2\pi}{8}\right)kn} = \cos\left(\frac{\pi}{4}\cdot k\cdot n\right) + j\sin\left(\frac{\pi}{4}\cdot k\cdot n\right)$$

$\cos(\pi/4)\cdot 0 + j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0 + j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0 + j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0 + j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0 + j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0 + j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0 + j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0 + j\sin(\pi/4)\cdot 0$
$\cos(\pi/4)\cdot 0 + j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 1 + j\sin(\pi/4)\cdot 1$	$\cos(\pi/4)\cdot 2 + j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 3 + j\sin(\pi/4)\cdot 3$	$\cos(\pi/4)\cdot 4 + j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 5 + j\sin(\pi/4)\cdot 5$	$\cos(\pi/4)\cdot 6 + j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 7 + j\sin(\pi/4)\cdot 7$
$\cos(\pi/4)\cdot 0 + j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 2 + j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 4 + j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 6 + j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 0 + j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 2 + j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 4 + j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 6 + j\sin(\pi/4)\cdot 6$
$\cos(\pi/4)\cdot 0 + j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 3 + j\sin(\pi/4)\cdot 3$	$\cos(\pi/4)\cdot 6 + j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 1 + j\sin(\pi/4)\cdot 1$	$\cos(\pi/4)\cdot 4 + j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 7 + j\sin(\pi/4)\cdot 7$	$\cos(\pi/4)\cdot 2 + j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 5 + j\sin(\pi/4)\cdot 5$
$\cos(\pi/4)\cdot 0 + j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 4 + j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 0 + j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 4 + j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 0 + j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 4 + j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 0 + j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 4 + j\sin(\pi/4)\cdot 4$
$\cos(\pi/4)\cdot 0 + j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 5 + j\sin(\pi/4)\cdot 5$	$\cos(\pi/4)\cdot 2 + j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 7 + j\sin(\pi/4)\cdot 7$	$\cos(\pi/4)\cdot 4 + j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 1 + j\sin(\pi/4)\cdot 1$	$\cos(\pi/4)\cdot 6 + j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 3 + j\sin(\pi/4)\cdot 3$
$\cos(\pi/4)\cdot 0 + j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 6 + j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 4 + j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 2 + j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 0 + j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 6 + j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 4 + j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 2 + j\sin(\pi/4)\cdot 2$
$\cos(\pi/4)\cdot 0 + j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 7 + j\sin(\pi/4)\cdot 7$	$\cos(\pi/4)\cdot 6 + j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 5 + j\sin(\pi/4)\cdot 5$	$\cos(\pi/4)\cdot 4 + j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 3 + j\sin(\pi/4)\cdot 3$	$\cos(\pi/4)\cdot 2 + j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 1 + j\sin(\pi/4)\cdot 1$

N=8 IDFT Matrix (5)

$$W_8^{-kn} = e^{+j(\frac{2\pi}{8})kn} = \cos\left(\frac{\pi}{4}\cdot k\cdot n\right) + j\sin\left(\frac{\pi}{4}\cdot k\cdot n\right)$$

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & +\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}} & +j & -\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}} & -1 & -\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}} & -j & +\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}} \\
0 & +j & -1 & -j & 0 & +j & -1 & -j \\
0 & -\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}} & -j & +\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}} & -1 & +\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}} & +j & -\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}} \\
0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 \\
0 & -\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}} & +j & +\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}} & -1 & +\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}} & -j & -\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}} \\
0 & -j & -1 & +j & 0 & -j & -1 & +j \\
0 & +\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}} & -j & -\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}} & -1 & -\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}} & +j & +\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}
\end{bmatrix}$$

N=8 DFT & IDFT Matrix (1)

	0	1	2	3	4	5	6	7	
0	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	DFT
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	IDFT
1	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 1}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 3}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 5}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 7}$	DFT
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 1}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 3}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 5}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 7}$	IDFT
2	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	DFT
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	IDFT
3	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 3}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 1}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 7}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 5}$	DFT
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 3}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 1}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 7}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 5}$	IDFT

N=8 DFT & IDFT Matrix (2)

	0	1	2	3	4	5	6	7	
4	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	DFT
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	IDFT
5	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 5}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 7}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 1}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 3}$	DFT
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 5}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 7}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 1}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 3}$	IDFT
6	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	DFT
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	IDFT
7	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 7}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 5}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 3}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 1}$	DFT
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 7}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 5}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 3}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 1}$	IDFT

N=8 DFT Matrix (1)

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad\qquad W_8^{kn} = e^{-j(\frac{2\pi}{8})kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} = \begin{bmatrix}
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 7} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 5} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 3} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 1}
\end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix}$$

A

N=8 IDFT Matrix (1)

$$x[n] = \frac{1}{N}\sum_{k=0}^{7} W_8^{-kn}\, X[k] \qquad W_8^{-kn} = e^{+j(\frac{2\pi}{8})kn}$$

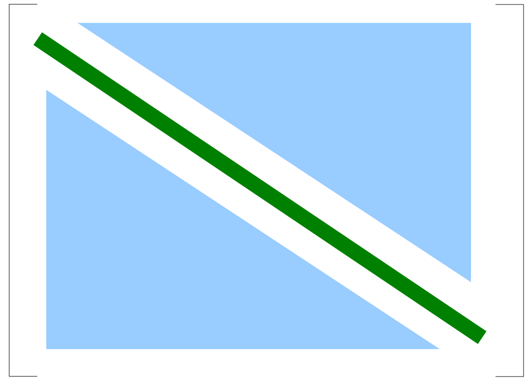
B

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \begin{bmatrix}
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 7} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 6} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 7} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 5} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 7} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 3} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 2} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 7} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 1}
\end{bmatrix} \begin{bmatrix} \frac{X[0]}{N} \\ \frac{X[1]}{N} \\ \frac{X[2]}{N} \\ \frac{X[3]}{N} \\ \frac{X[4]}{N} \\ \frac{X[5]}{N} \\ \frac{X[6]}{N} \\ \frac{X[7]}{N} \end{bmatrix}$$

Symmetric Matrices

DFT

$$A =$$



$$A = A^T$$

ID
F

$$B =$$

$$B = B^T$$

Conjugate Transpose Matrices

$$A = B^*$$

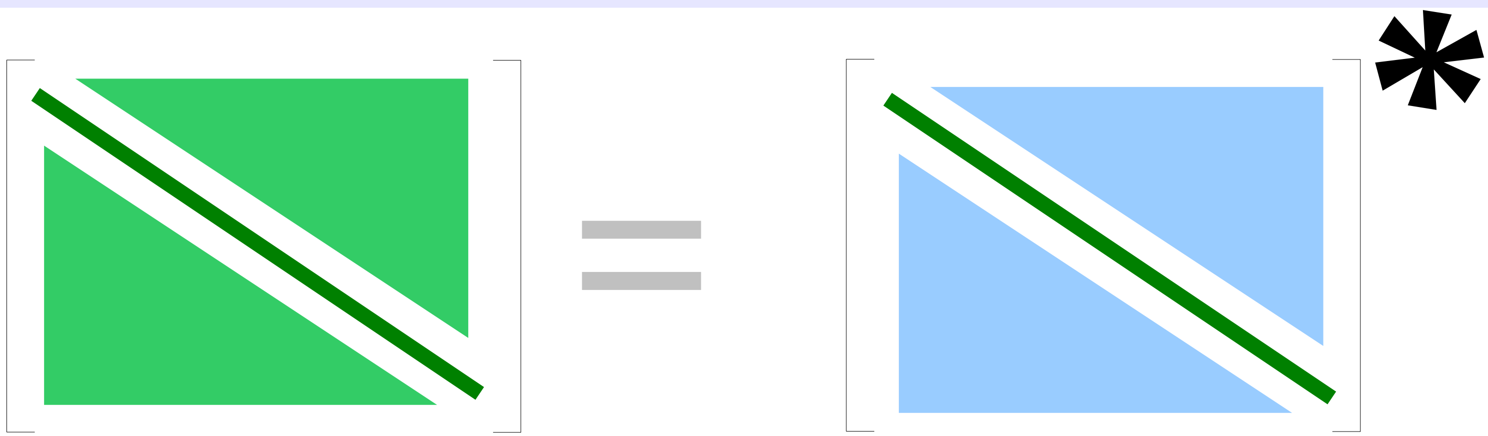
DFT

$A =$

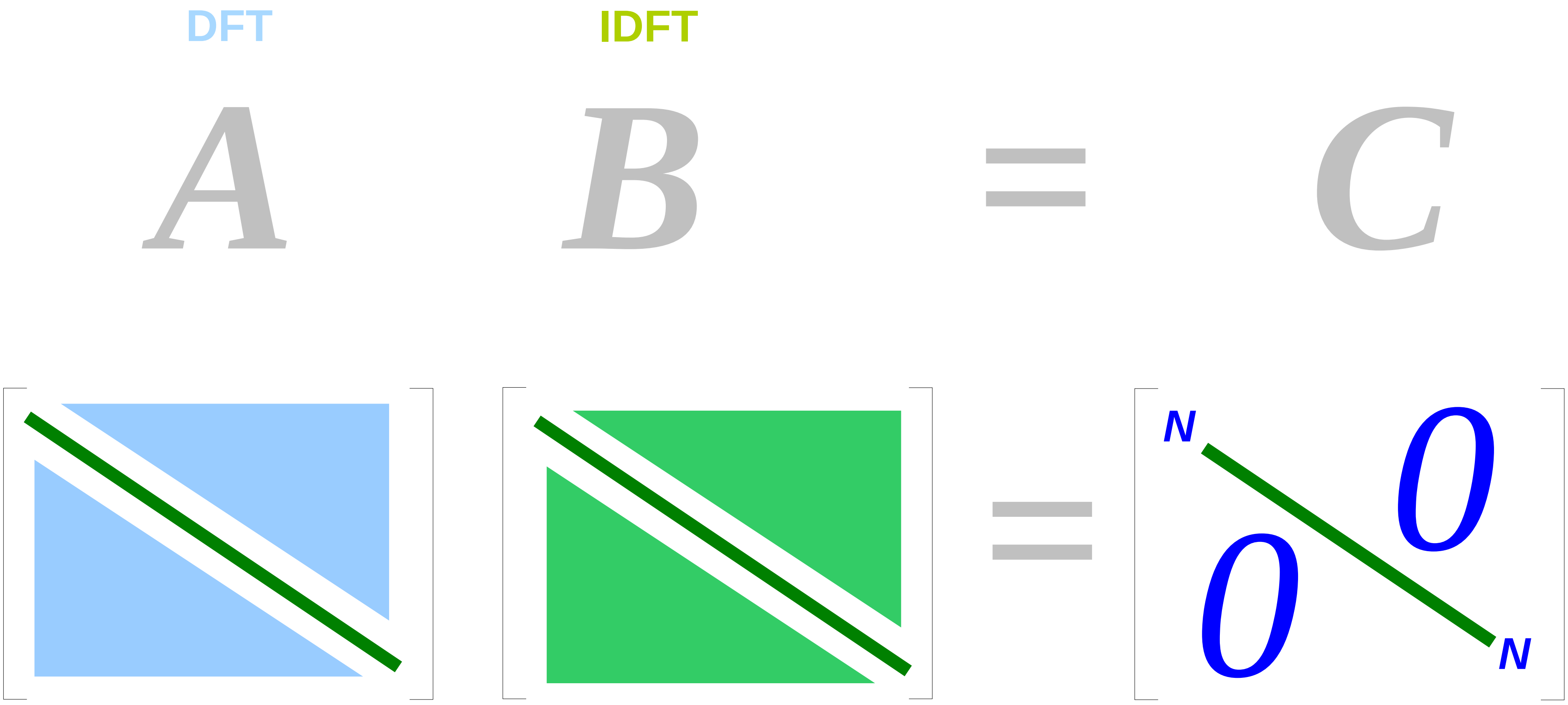
$$B = A^*$$

IDFT

$B =$



Product AB



Product AB – Diagonal Elements

$C = A \cdot B$

$$[\boldsymbol{C}]_{(i,j)} = [\boldsymbol{A}]_{(row\ i)} \cdot [\boldsymbol{B}]_{(col\ j)}$$

$$C_{(i,i)} = N$$

$C_{(1,1)}$

$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 1}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 3}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 5}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 7}$	
$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 1}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 3}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 5}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 7}$	
+1	+1	+1	+1	+1	+1	+1	+1	= N

Product AB – Off-Diagonal Elements

$C = A \cdot B$

$$[\boldsymbol{C}]_{(i,j)} = [\boldsymbol{A}]_{(row\ \ i)} \cdot [\boldsymbol{B}]_{(col\ \ j)}$$

$C_{(1,1)}$

$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 1}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 3}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 5}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 7}$
$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 1}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 3}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 5}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 7}$

1 +1 +1 +1 +1 +1 +1 +1 = N

$C_{(1,2)}$

$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 1}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 3}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 5}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 7}$
$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$

$$e^{+j\cdot\frac{\pi}{4}\cdot 0} + e^{+j\cdot\frac{\pi}{4}\cdot 1} + e^{+j\cdot\frac{\pi}{4}\cdot 2} + e^{+j\cdot\frac{\pi}{4}\cdot 3} + e^{+j\cdot\frac{\pi}{4}\cdot 4} + e^{+j\cdot\frac{\pi}{4}\cdot 5} + e^{+j\cdot\frac{\pi}{4}\cdot 6} + e^{+j\cdot\frac{\pi}{4}\cdot 7} = 0$$

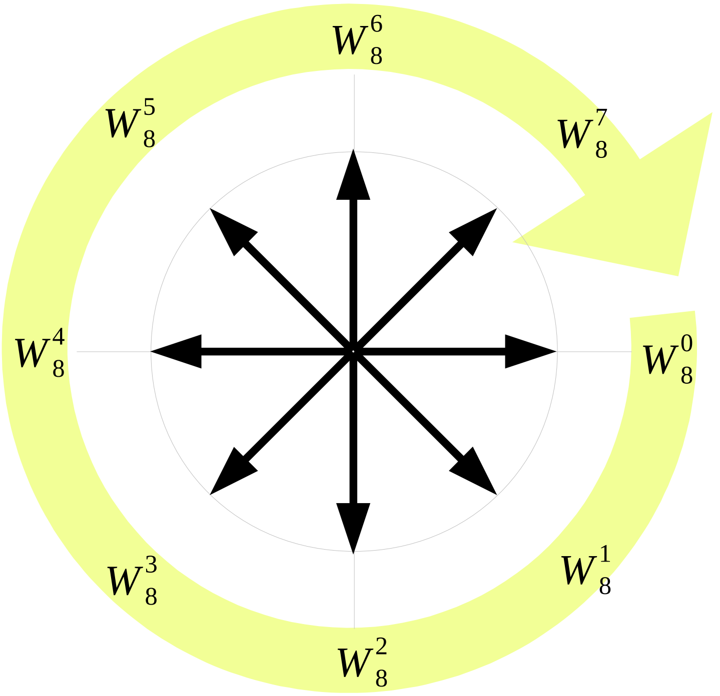
Root of Unity

$$\sum_{k=0}^{N-1} W_N^k = \sum_{k=0}^{N-1} e^{-j\left(\frac{2\pi}{N}\right)k} = 0$$

$$z \equiv e^{-j\left(\frac{2\pi}{N}\right)}$$

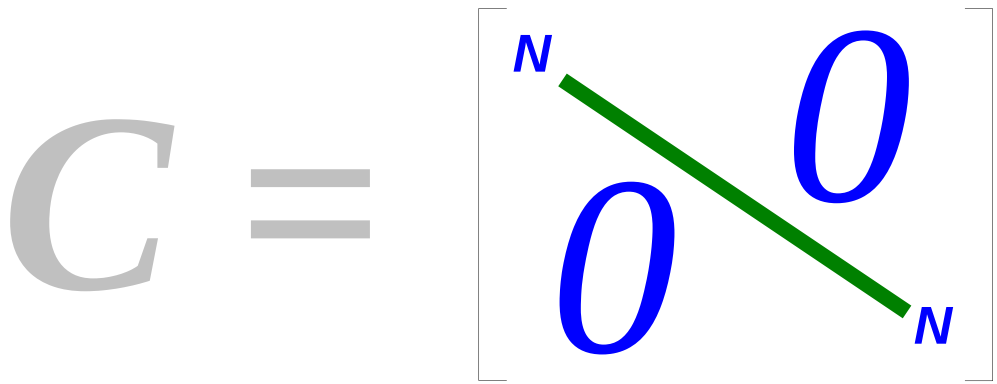
$$z^N = e^{-j\left(\frac{2\pi}{N}\right)N} = 1$$

$$\sum_{k=0}^{N-1} e^{-j\left(\frac{2\pi}{N}\right)k} = \frac{z^N - 1}{z - 1} = 0$$



$$W_8^0 + W_8^1 + W_8^2 + W_8^3 + W_8^4 + W_8^5 + W_8^6 + W_8^7 = 0$$

Unitary Matrix



$$C = A \cdot B = A \cdot A^{*} = N\ I$$

$$U \cdot U^{*} = I$$ ***Unitary Matrix***

Unitary Matrix

$$C = \begin{bmatrix} N & & 0 \\ & \ddots & \\ 0 & & N \end{bmatrix}$$

$$\boldsymbol{C} = \boldsymbol{A} \cdot \boldsymbol{B} = \boldsymbol{A} \cdot \boldsymbol{A}^{*} = N\ \boldsymbol{I}$$

$$\boldsymbol{U} \cdot \boldsymbol{U}^{*} = \boldsymbol{I}$$

Unitary Matrix

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003