

# DLTI z-Transform

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# Finding ZIR & ZSR Using Laplace Transform (1)

$$y[n] = 2x[n] - x[n-1] + 3x[n-2] + \frac{9}{20}y[n-1] - \frac{1}{20}y[n-2]$$

initial condition  $y[1]=3, y[-2]=2$

input  $x[n] = u[n]$

$y[n]$	$\leftrightarrow$	$Y[z]$
$y[n-1]$	$\leftrightarrow$	$y[-1] + z^{-1}Y[z]$ $= z^{-1}Y[z] + 3$
$y[n-2]$	$\leftrightarrow$	$y[-2] + z^{-1}y[-1] + z^{-2}Y[z]$ $= z^{-2}Y[z] + 3z^{-1} + 2$

$x[n]$	$\leftrightarrow$	$X[z] = \frac{z}{z-1}$
$x[n-1]$	$\leftrightarrow$	$\frac{1}{z-1}$
$x[n-2]$	$\leftrightarrow$	$\frac{1}{z(z-1)}$

$$Y[z] = 2\frac{z}{z-1} - \frac{1}{z-1} + \frac{3}{z(z-1)} + \frac{9}{20}(z^{-1}Y[z] + 3) - \frac{1}{20}(z^{-2}Y[z] + 3z^{-1} + 2)$$

*initial condition terms*

# Finding ZIR & ZSR Using Laplace Transform (2)

$$y[n] = 2x[n] - x[n-1] + 3x[n-2] + \frac{9}{20}y[n-1] - \frac{1}{20}y[n-2]$$

initial condition  $y[1]=3, y[-2]=2$

input  $x[n] = u[n]$

$$Y[z] = 2\frac{z}{z-1} - \frac{1}{z-1} + \frac{3}{z(z-1)} + \frac{9}{20}(z^{-1}Y[z]+3) - \frac{1}{20}(z^{-2}Y[z]+3z^{-1}+2)$$

$$\frac{Y[z]}{z} = \frac{2z^2 - z + 3}{(z-1)(z-\frac{1}{3})(z-\frac{1}{4})} + \frac{(\frac{5}{4}z - \frac{3}{20})}{(z-\frac{1}{5})(z-\frac{1}{4})}$$

*init cond terms*

*input terms*

$$\frac{Y[z]}{z} = \frac{\frac{20}{3}}{(z-1)} + \frac{72}{(z-\frac{1}{5})} + \frac{\frac{230}{3}}{(z-\frac{1}{4})} - \frac{2}{(z-\frac{1}{5})} + \frac{\frac{13}{4}}{(z-\frac{1}{4})}$$

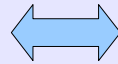
$$y[n] = \frac{30}{3} + 72\left(\frac{1}{5}\right)^n - \frac{230}{3}\left(\frac{1}{4}\right)^n - 2\left(\frac{1}{5}\right)^n + \frac{13}{4}\left(\frac{1}{4}\right)^n = \frac{20}{3} + 70\left(\frac{1}{5}\right)^n - \frac{881}{12}\left(\frac{1}{4}\right)^n$$

*Zero State Resp*

*Zero Input Resp*

# Region Of Convergence (1)

$$X(z) = \frac{1}{1-0.5z^{-1}}$$



$$X(z) = \frac{(0.5)^{-1}z}{0.5z^{-1}-1} = -\frac{(0.5)^{-1}z}{1-0.5z^{-1}}$$

$$\frac{a}{1-r}$$

Infinite Geometric Series

$$a = 1$$

$$r = 0.5z^{-1}$$

$$|(0.5)z^{-1}| < 1$$

$$1 + r + r^2 + r^3 + \dots$$

$$1 + 0.5z^{-1} + 0.5^2z^{-2} + 0.5^3z^{-3} + \dots$$

$$x[0] + x[1]z^{-1} + x[2]z^{-2} + x[3]z^{-3} + \dots$$

*Initial Term*

*Common Ratio*

*ROC*

$$\frac{a}{1-r}$$

$$a = -(0.5)^{-1}z$$

$$r = (0.5)^{-1}z$$

$$|(0.5)^{-1}z| < 1$$

$$a + ar + ar^2 + ar^3 + \dots$$

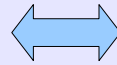
$$-0.5^{-1}z - 0.5^{-2}z^2 - 0.5^{-3}z^3 + \dots$$

$$x[-1]z^1 + x[-2]z^2 + x[-3]z^3 + \dots$$

$$-2z - 2^2z^2 - 2^3z^3 - \dots$$

# Region Of Convergence (2)

$$X(z) = \frac{1}{1-0.5z^{-1}}$$



$$X(z) = \frac{(0.5)^{-1}z}{0.5z^{-1}-1} = -\frac{(0.5)^{-1}z}{1-0.5z^{-1}}$$

$$|(0.5)z^{-1}| < 1$$

ROC

$$|(0.5)^{-1}z| < 1$$

$$1 + r + r^2 + r^3 + \dots$$

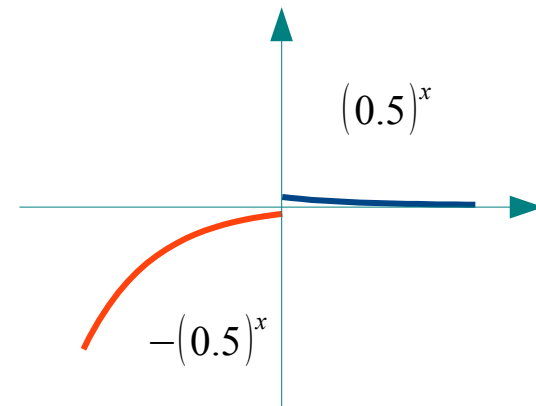
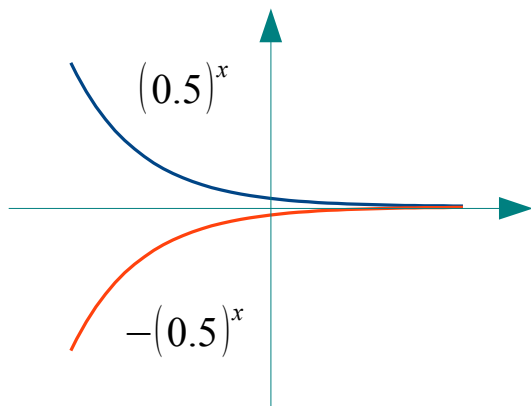
$$a + ar + ar^2 + ar^3 + \dots$$

$$1 + 0.5z^{-1} + 0.5^2z^{-2} + 0.5^3z^{-3} + \dots$$

$$-2z - 2^2z^2 - 2^3z^3 - \dots$$

$$x[0] + x[1]z^{-1} + x[2]z^{-2} + x[3]z^{-3} + \dots$$

$$x[-1]z^1 + x[-2]z^2 + x[-3]z^3 + \dots$$



## References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] B.P. Lathi, Linear Systems and Signals (2<sup>nd</sup> Ed)
- [4] D. Sundararajan, A Practical Approach to Signals and Systems