

Up-Sampling (5B)

-
-

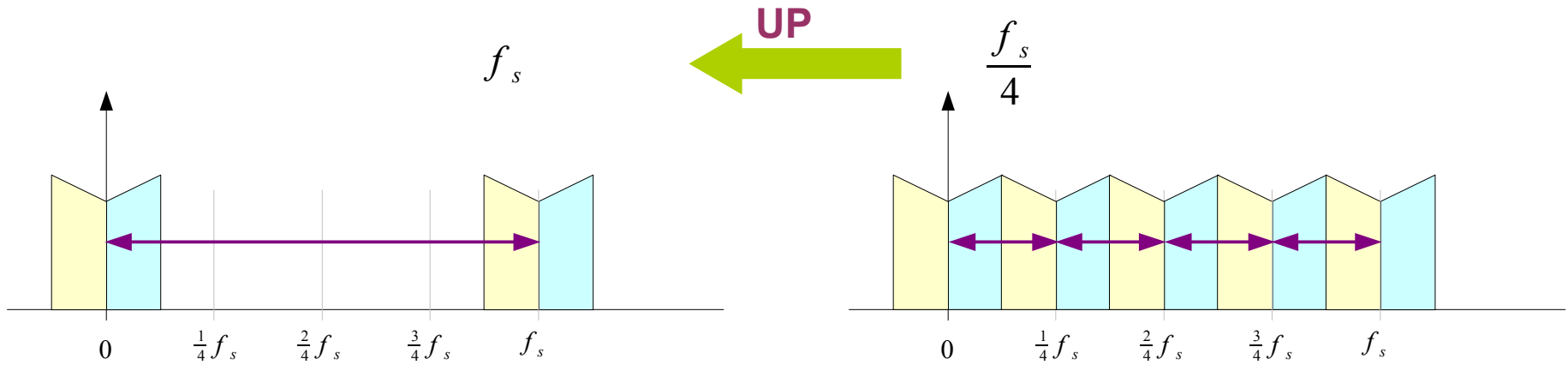
Copyright (c) 2009, 2010, 2011 Young W. Lim.

Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice and Octave.

Increasing Sampling Frequency

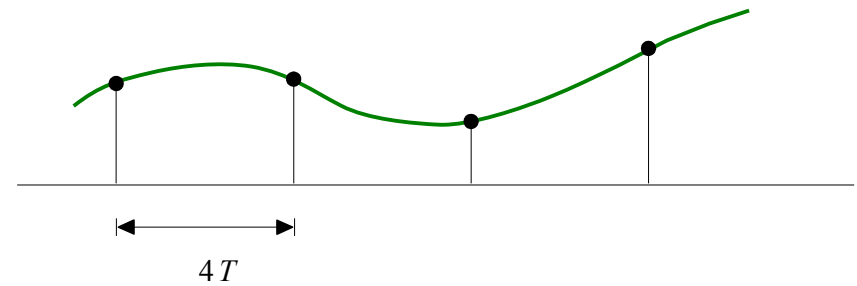
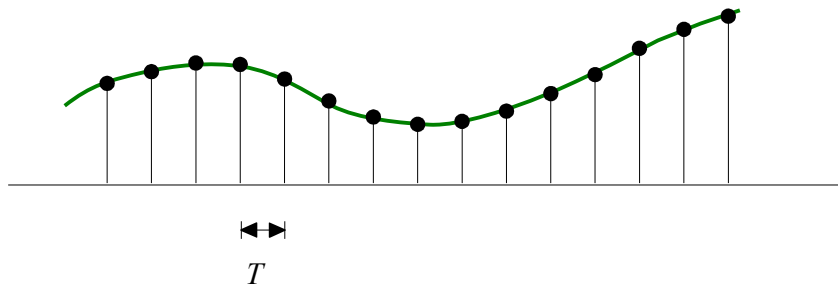


Sampling Frequency f_s

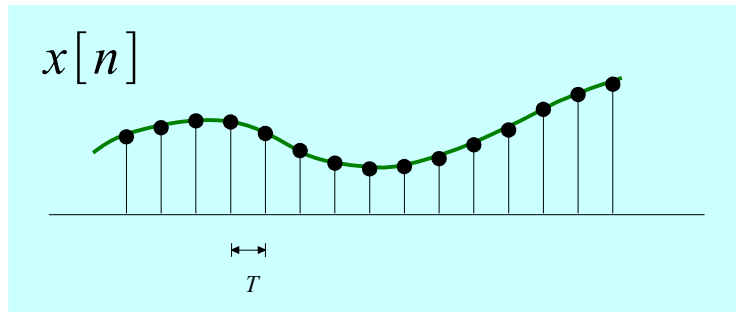
Sampling Time $T = \frac{1}{f_s}$

Sampling Frequency $f'_s = \frac{1}{4}f_s$

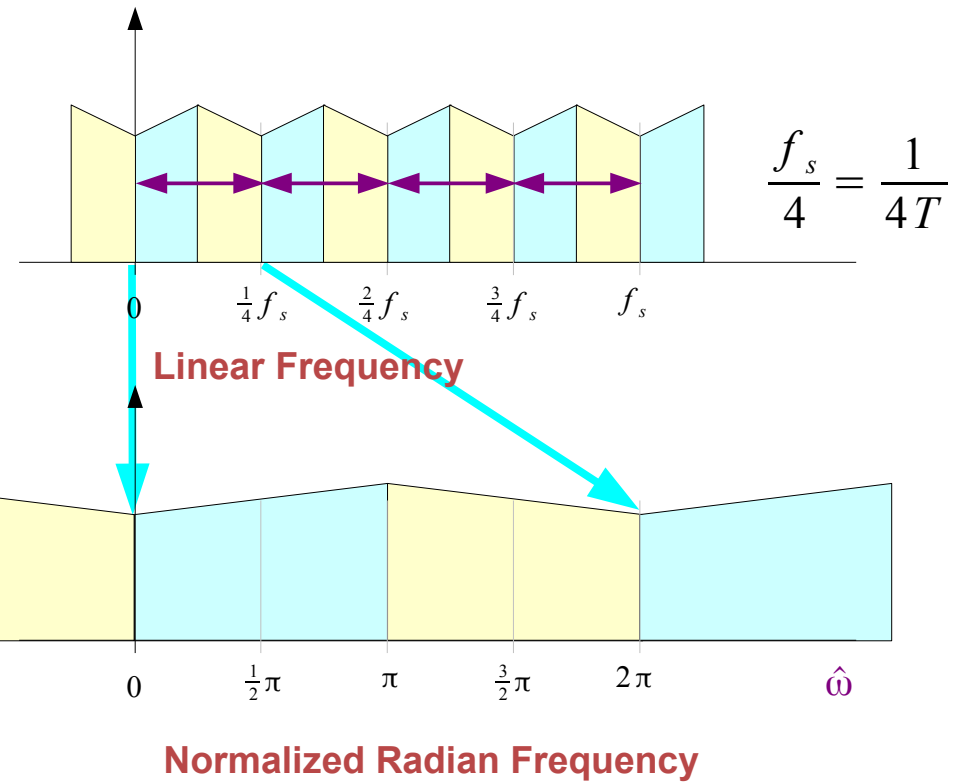
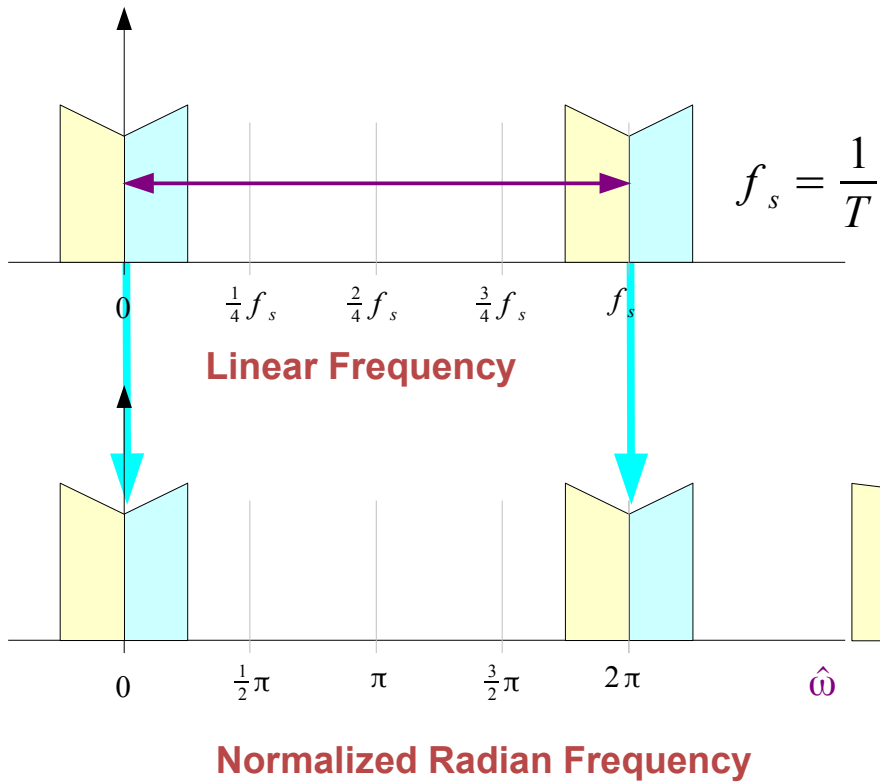
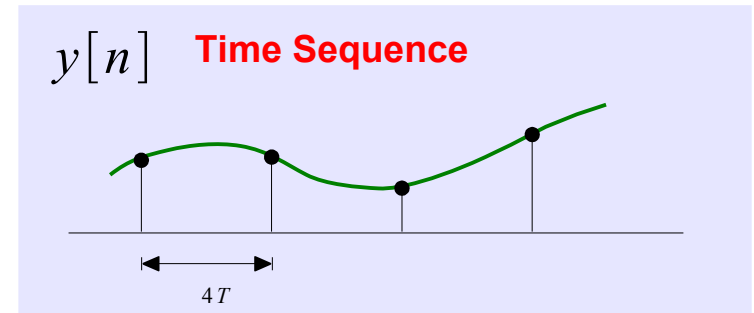
Sampling Time $T' = \frac{4}{f_s}$



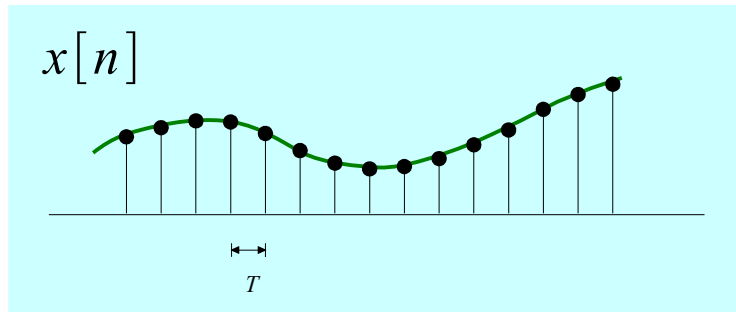
Fine Sequence & Spectrum



← UP



Normalized Radian Frequency

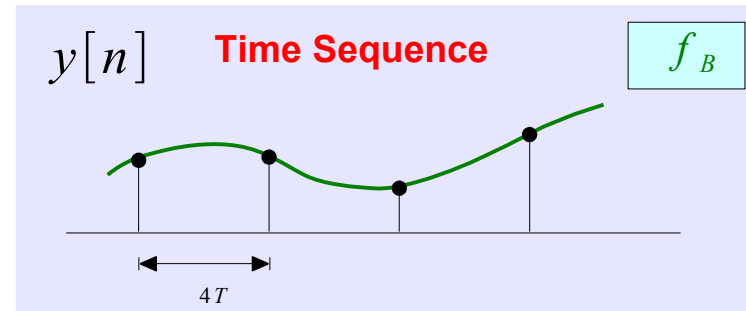


$$\hat{\omega} = \omega \cdot T_s = \frac{\omega}{1/T_s}$$

$$\hat{\omega} = \frac{\omega}{f_s} = 2\pi \frac{f}{f_s}$$

↑ ↑
Normalized to f_s

Normalized Radian Frequency

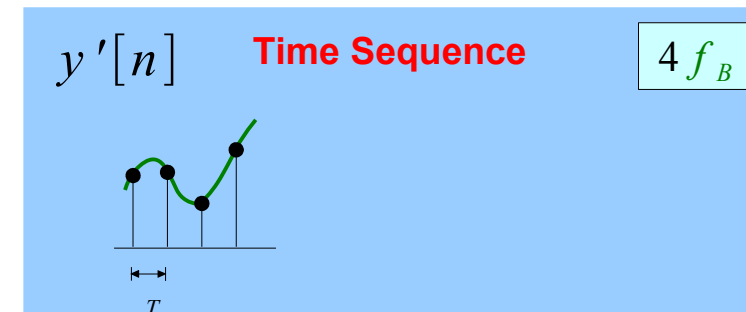


$$\frac{f}{f_s} = \frac{f_B}{1/4T} = f_B \cdot 4T$$

The Same

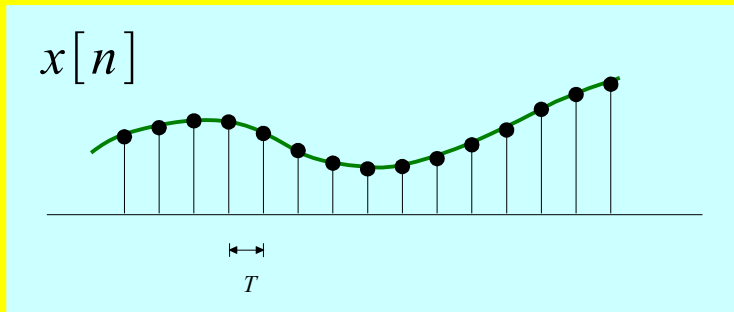
- Time Sequence
- Normalized Radian Frequency

$$\frac{f}{f_s} = \frac{4f_B}{1/T} = f_B \cdot 4T$$

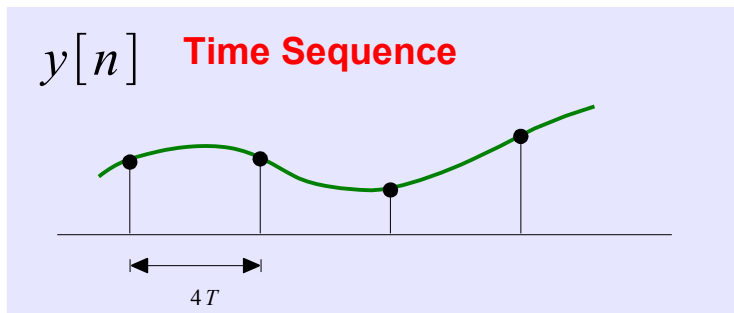


The Highest Frequency:

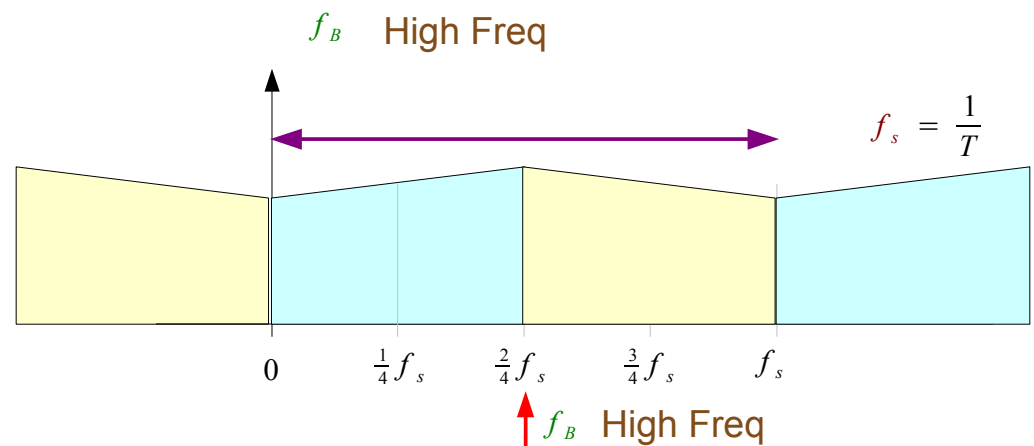
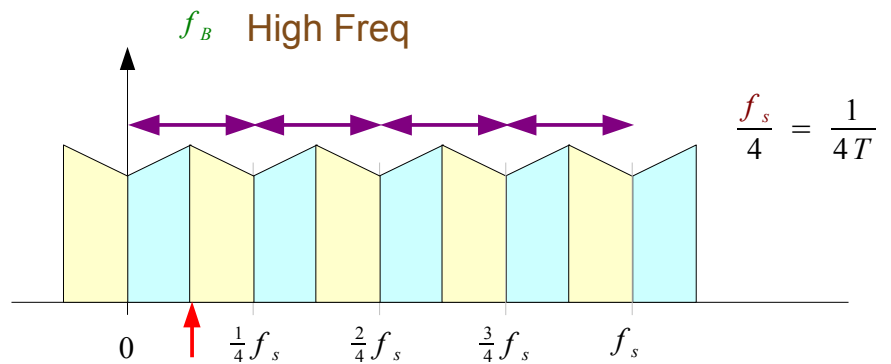
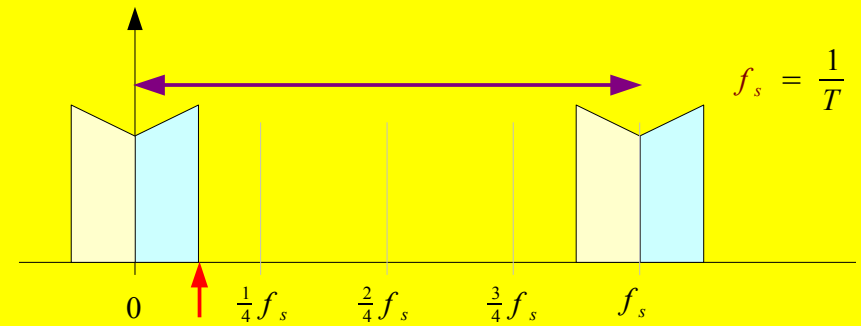
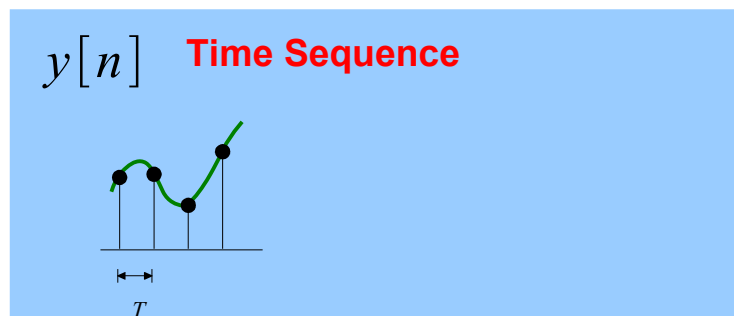
Fine Sequence Spectrum – Linear Frequency



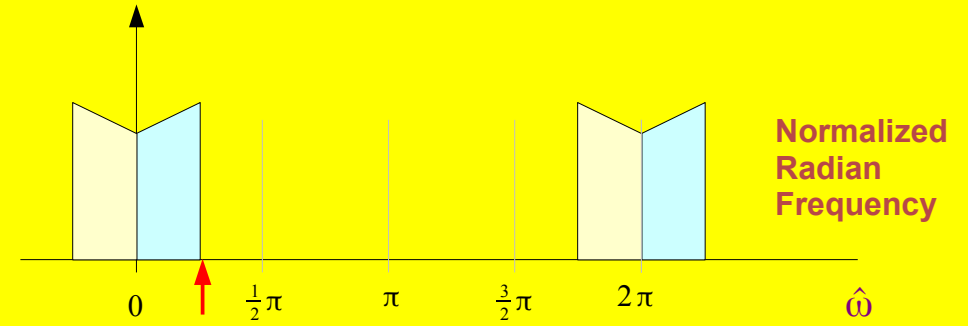
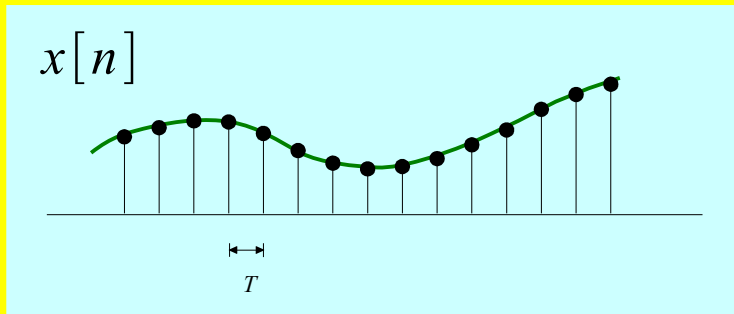
↑ UP



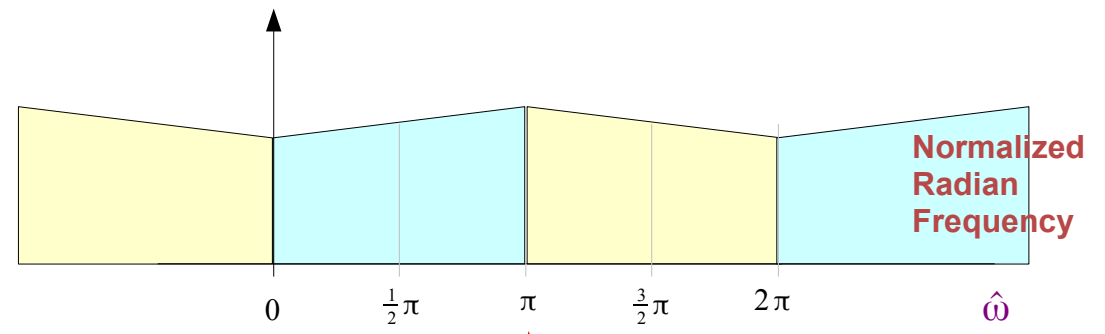
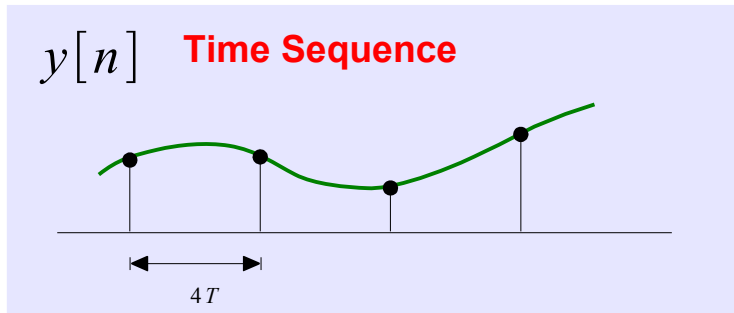
|| The Same Time Sequence



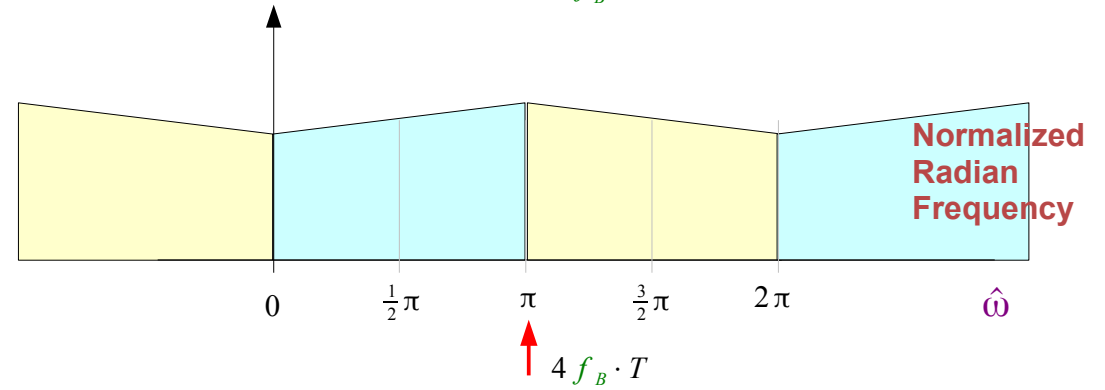
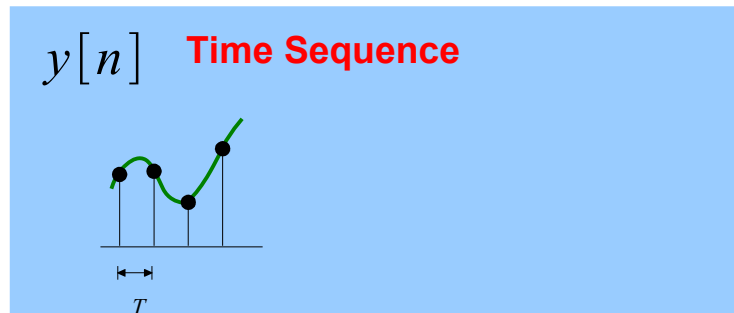
Fine Sequence Spectrum – Normalized Frequency



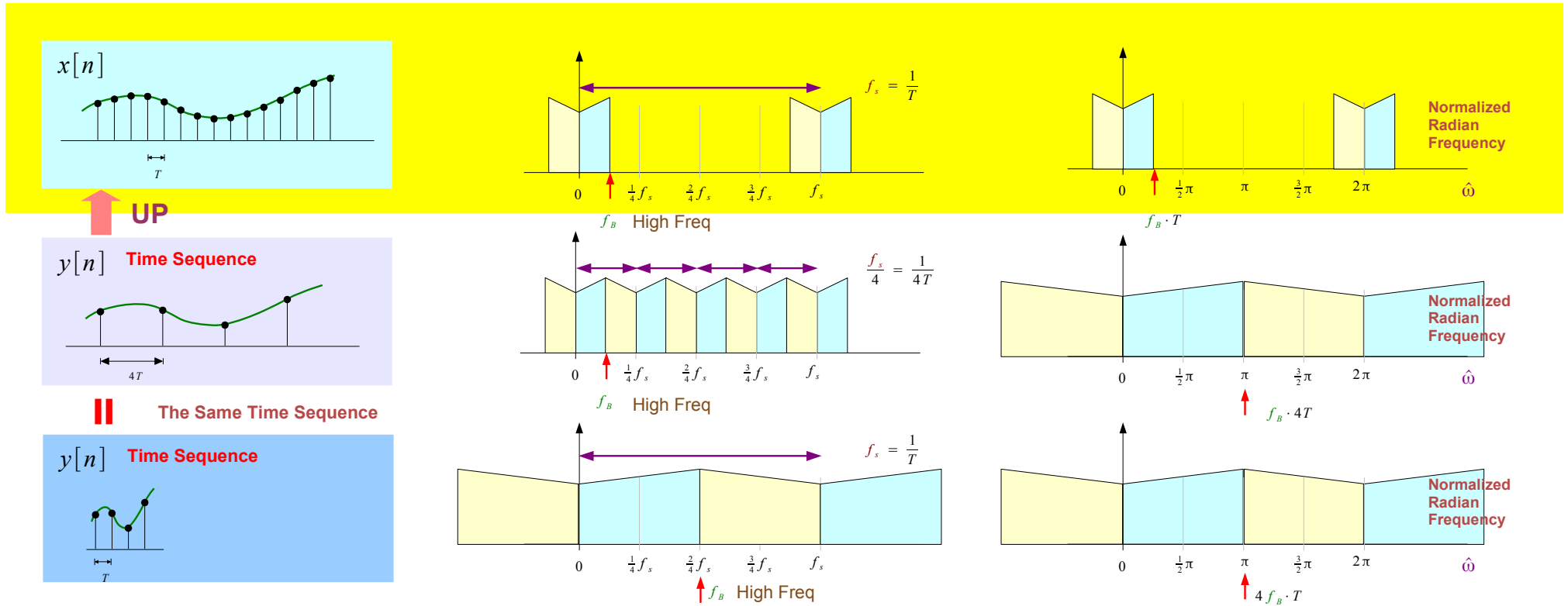
↑ UP



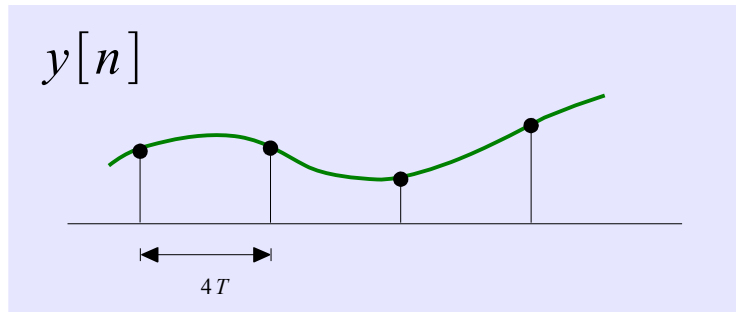
|| The Same Time Sequence



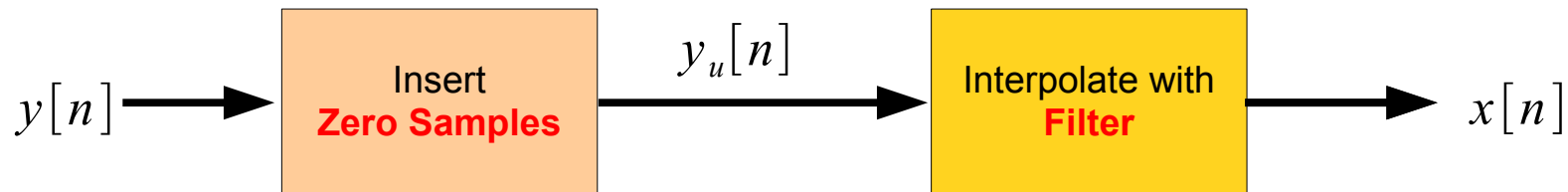
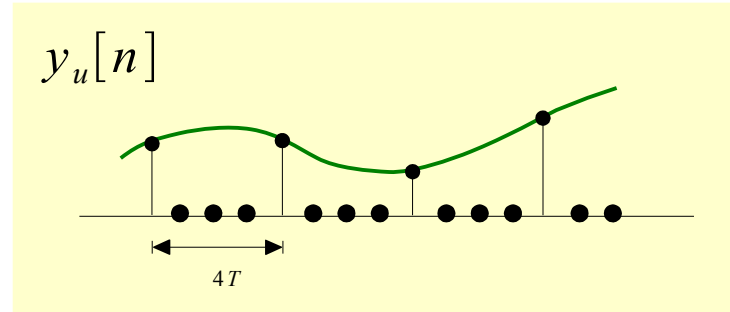
Fine Sequence Spectrum



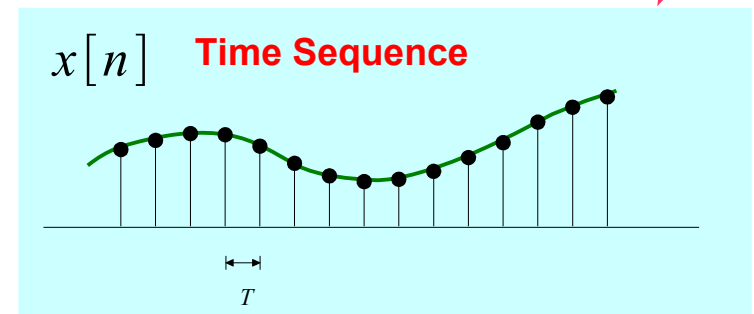
Fine Sequence Generation



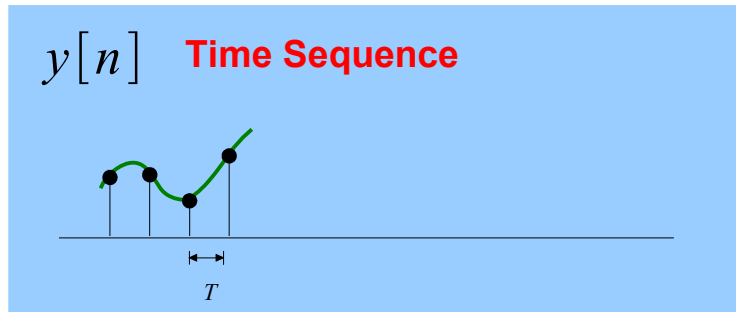
$4T$ Sampling Period



T Sampling Period

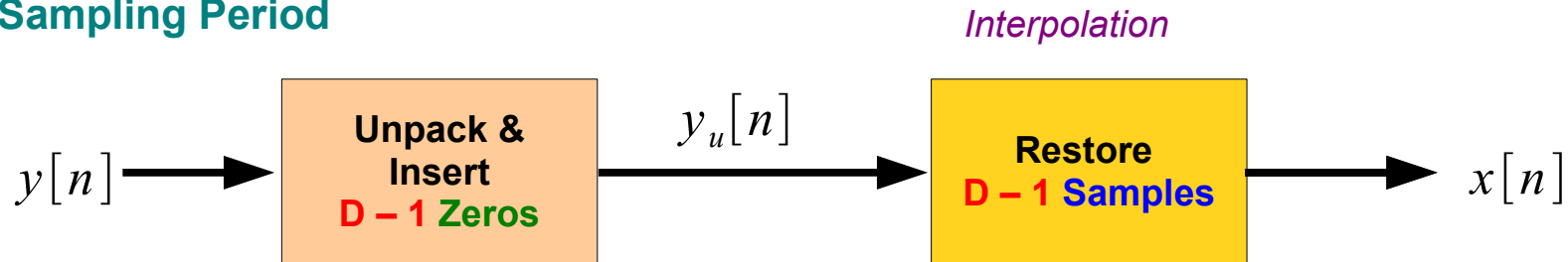
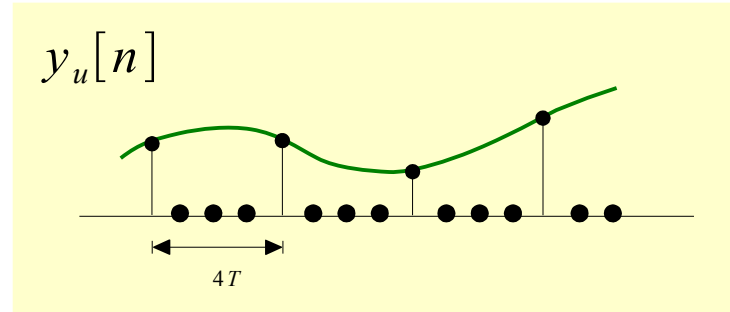


Up Sampling in Two Steps



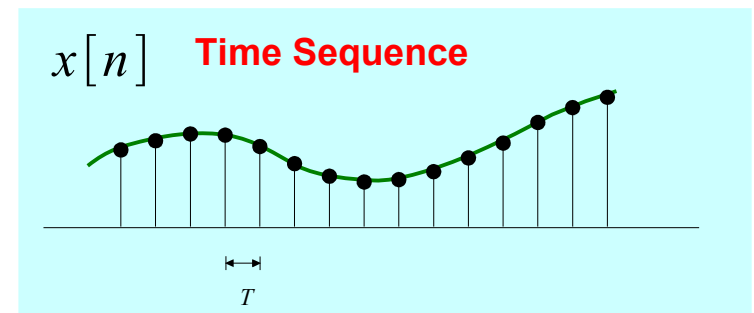
$4f_B$ Highest Frequency

T Sampling Period

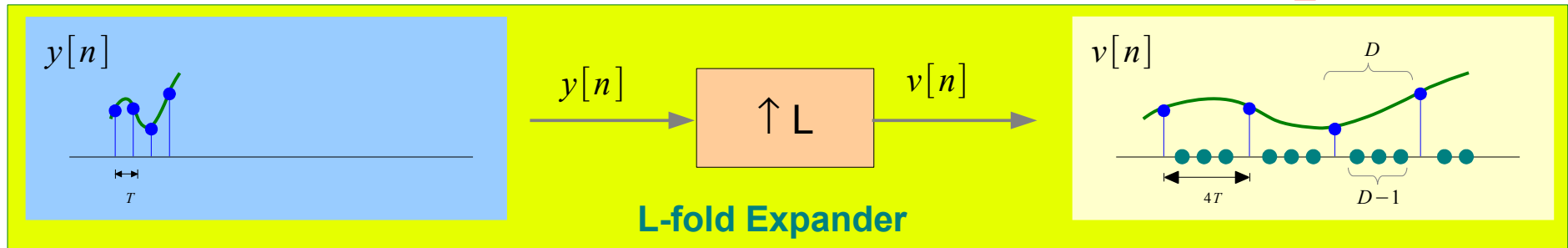
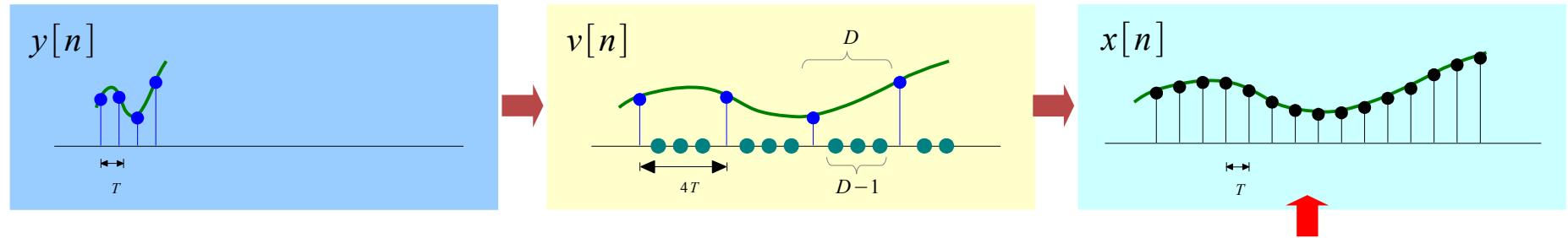


f_B Highest Frequency

T Sampling Period



Up-Sampling Operator



$$v[n] = S_L y[n] = \begin{cases} y[n/L] & \text{if } \text{mod}(n/L) = 0 \\ 0 & \text{otherwise} \end{cases}$$

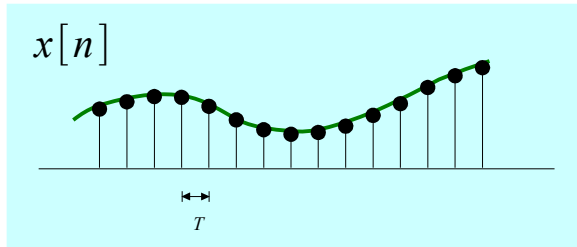
Increase sampling frequency by a factor of L

Decrease sampling period by a factor of 1/L

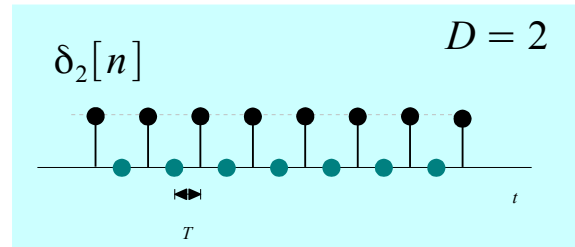
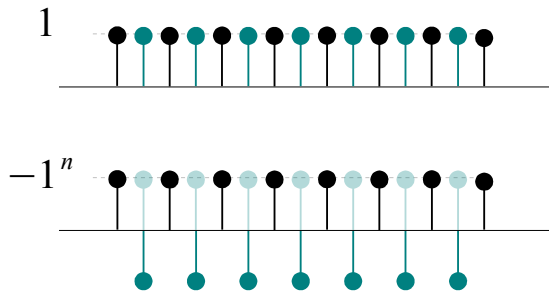
$$D = 2$$

$n=0 \cdot 2=0$	$v[0] = y[0]$	$v[1] = 0$
$n=1 \cdot 2=2$	$v[2] = y[1]$	$v[3] = 0$
$n=2 \cdot 2=4$	$v[4] = y[2]$	$v[5] = 0$
$n=3 \cdot 2=6$	$v[6] = y[3]$	$v[6] = 0$
...

Example When D=2 (1)

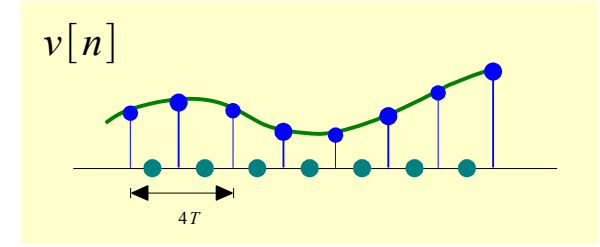


$$x[n] = e^{j\omega n}$$



$$\begin{aligned} \delta_2[n] &= \frac{1}{2}(1 + (-1)^n) \\ &= \frac{1}{2}(1 + e^{-j\pi n}) \\ &\quad (e^{-j\pi} = -1) \end{aligned}$$

$$V(z) = \frac{1}{2} \sum_{n=-\infty}^{+\infty} (x[n]z^{-n} + x[n](-z)^{-n}) = \frac{1}{2}X(z) + \frac{1}{2}X(-z)$$



$$\begin{aligned} v[n] &= \frac{1}{2}x[n] + \frac{1}{2}e^{-j\pi n}x[n] \\ &= \frac{1}{2}e^{j\omega n} + \frac{1}{2}e^{-j\pi n}e^{j\omega n} \\ &= \frac{1}{2}e^{j\omega n} + \frac{1}{2}e^{+j(\omega-\pi)n} \end{aligned}$$

$$V(e^{j\hat{\omega}}) = \frac{1}{2}X(e^{j\hat{\omega}}) + \frac{1}{2}X(e^{-j\pi}e^{j\hat{\omega}})$$

$$V(\hat{\omega}) = \frac{1}{2}X(\hat{\omega}) + \frac{1}{2}X(\hat{\omega} - \pi)$$

Z-Transform Analysis

$$\delta_D[n] = \begin{cases} 1 & \text{if } n/D \text{ is an integer} \\ 0 & \text{otherwise} \end{cases}$$

$$v[n] = \delta_D[n]x[n]$$

$$V[z] = \cdots + v[0]z^0 + v[D]z^{-D} + v[2D]z^{-2D} + \cdots \quad y[n]$$

$$V[z] = \sum_{n=-\infty}^{+\infty} v[n]z^{-n} = \sum_{m=-\infty}^{+\infty} v[mD]z^{-mD} = F(z^D)$$

T Sampling Period

Z-Transform Analysis

$$\delta_2[n] = \frac{1}{2}(1 + (-1)^n) = \frac{1}{2}(1 + e^{-j\pi n}) = \begin{cases} 1 & \text{if } n/2 \text{ is an integer (even)} \\ 0 & \text{otherwise} \end{cases}$$

$$e^{-j\pi} = -1$$

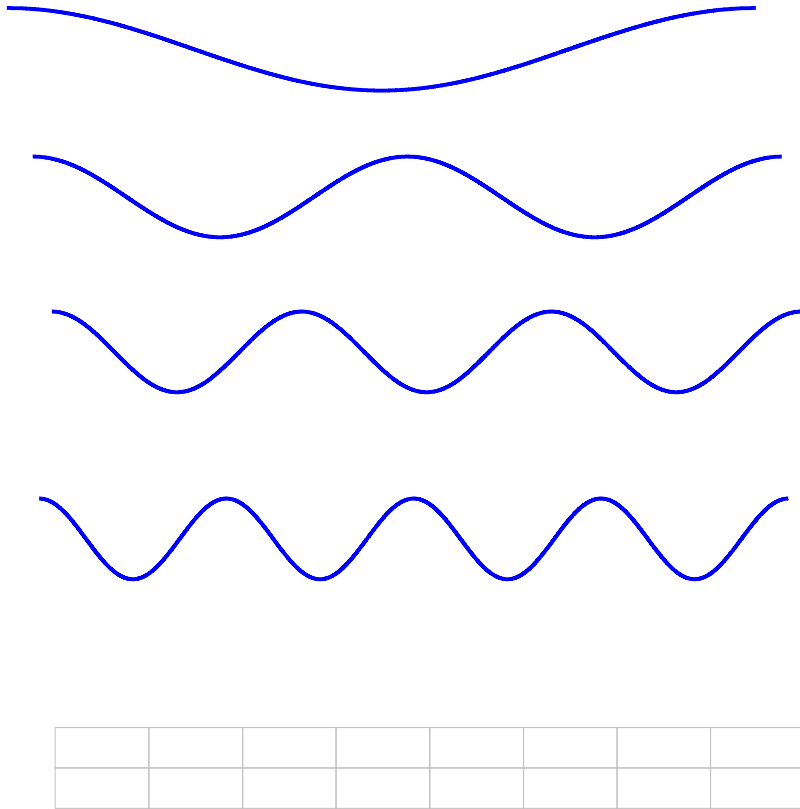
$$v[n] = \frac{1}{2}x[n] + \frac{1}{2}e^{-j\pi n}x[n] \quad x[n] = e^{j\omega n}$$

$$v[n] = \frac{1}{2}e^{j\omega n} + \frac{1}{2}e^{-j\pi n}e^{j\omega n} = \frac{1}{2}e^{j\omega n} + \frac{1}{2}e^{+j(\omega-\pi)n}$$

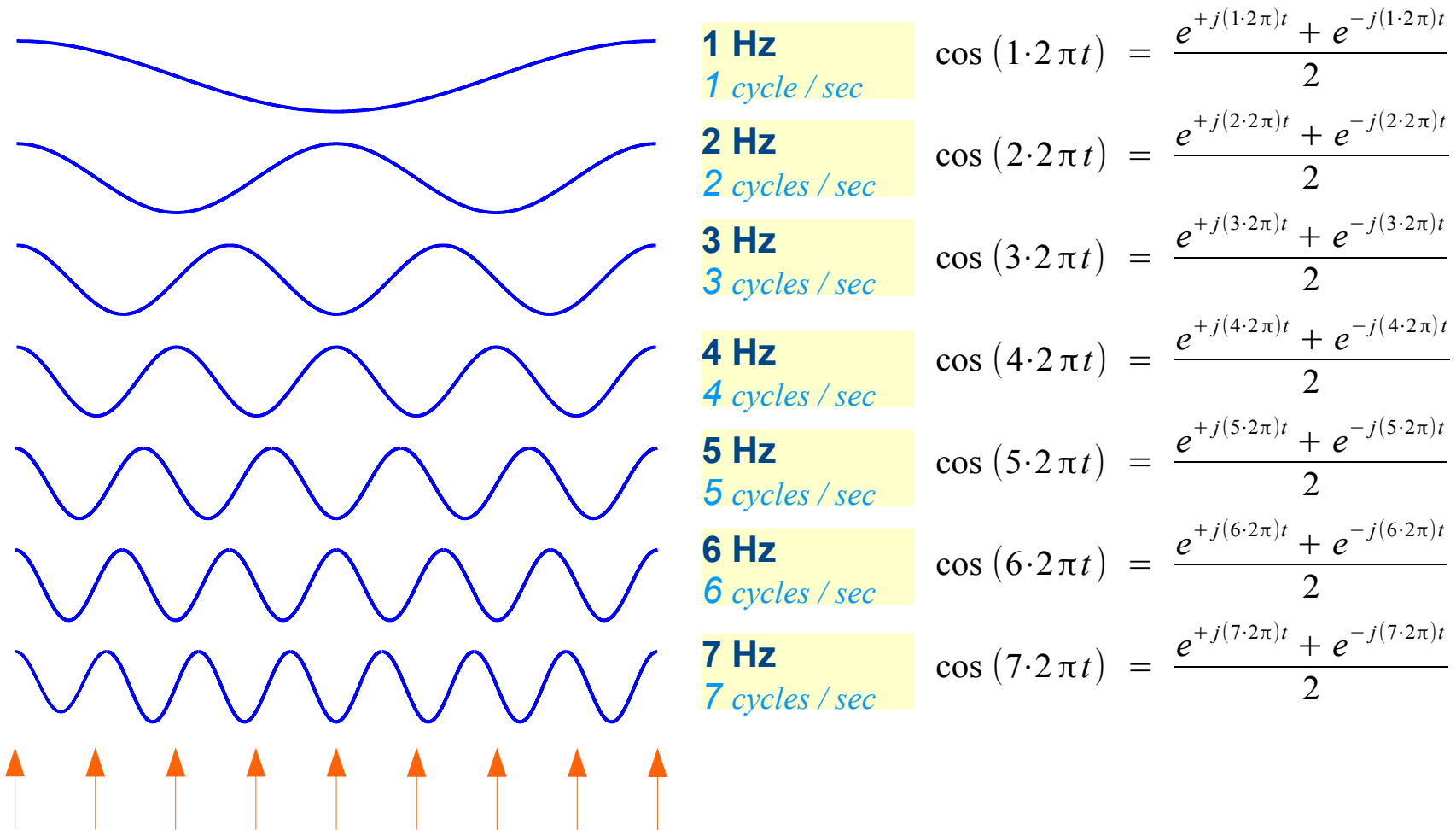
$$V(z) = \frac{1}{2} \sum_{n=-\infty}^{+\infty} (x[n]z^{-n} + x[n](-z)^{-n}) = \frac{1}{2}X(z) + \frac{1}{2}X(-z)$$

$$V(\omega) = V(e^{j\omega}) = \frac{1}{2}X(e^{j\omega}) + \frac{1}{2}X(e^{-j\pi}e^{j\omega}) = \frac{1}{2}X(\omega) + \frac{1}{2}X(\omega - \pi)$$

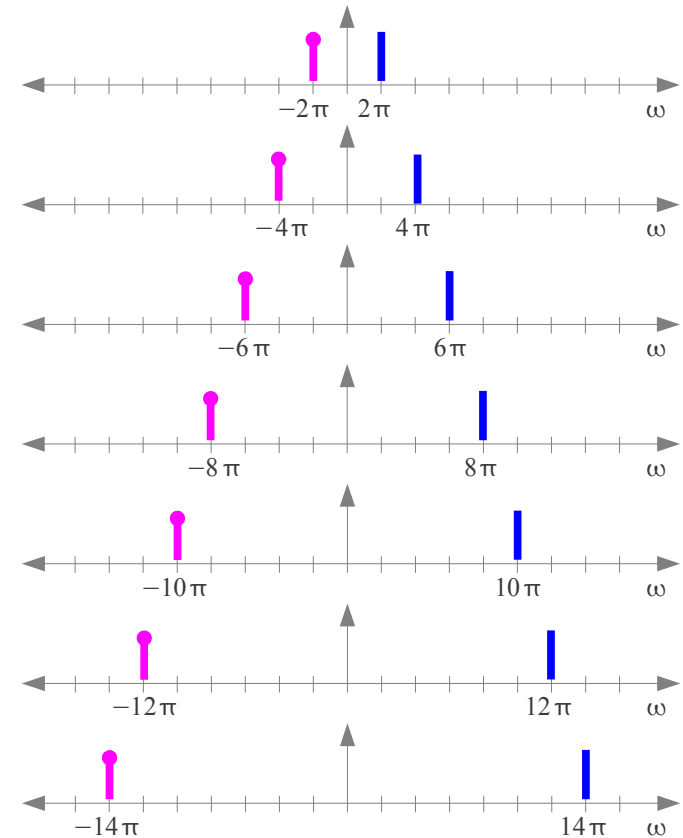
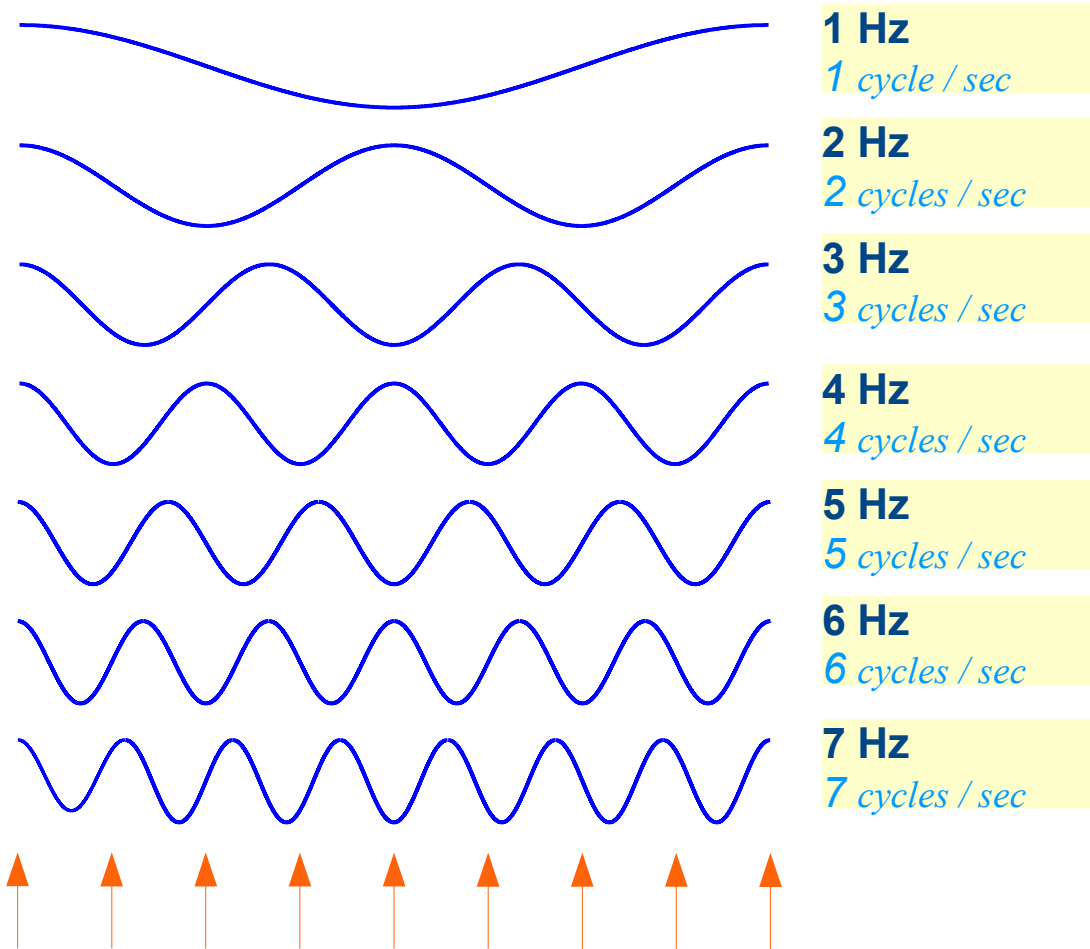
Measuring Rotation Rate



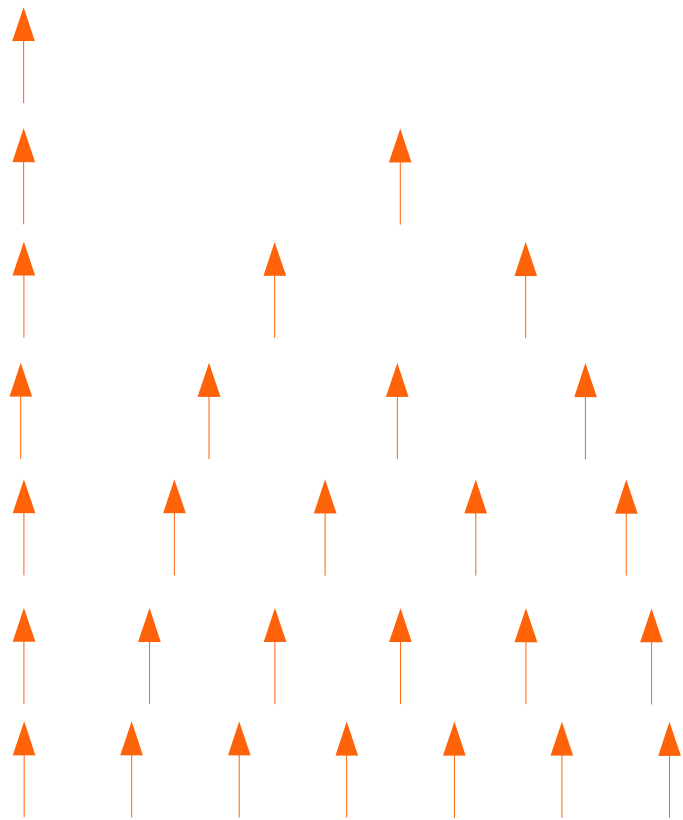
Signals with Harmonic Frequencies (1)



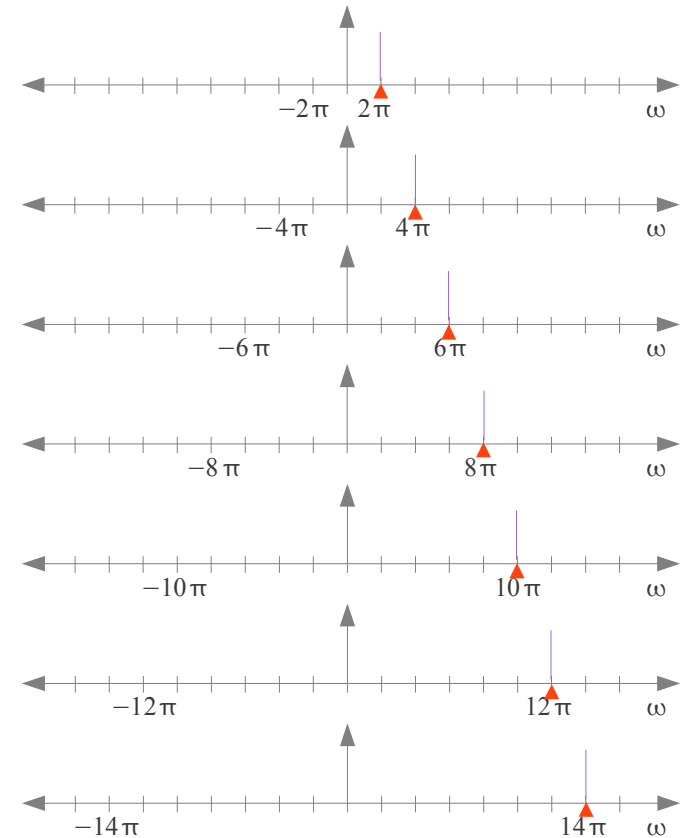
Signals with Harmonic Frequencies (2)



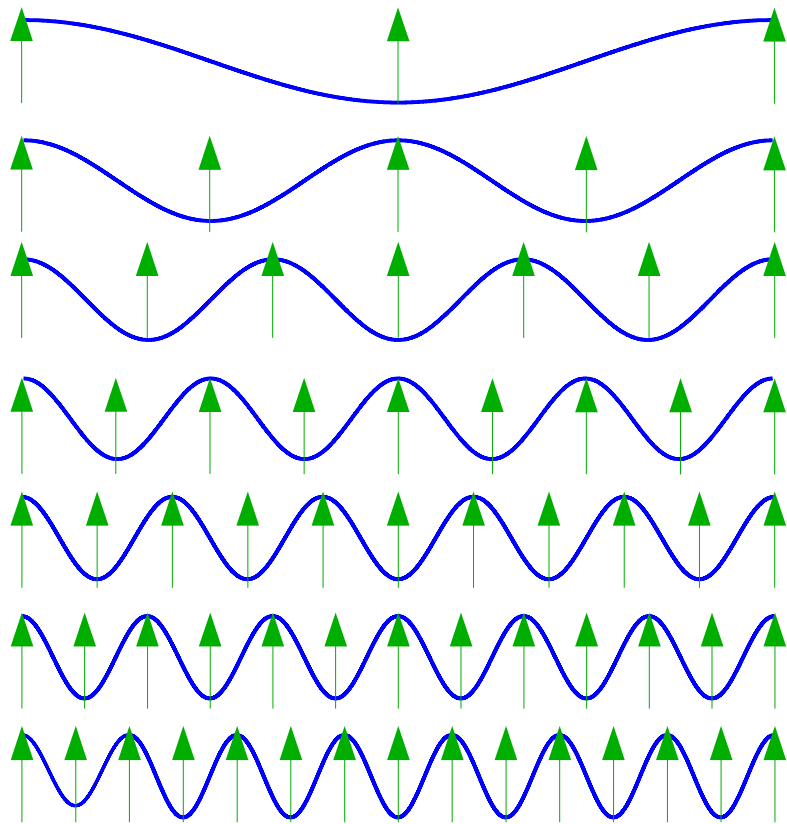
Sampling Frequency



- 1 Hz
1 sample / sec
- 2 Hz
2 samples / sec
- 3 Hz
3 samples / sec
- 4 Hz
4 samples / sec
- 5 Hz
5 samples / sec
- 6 Hz
6 samples / sec
- 7 Hz
7 samples / sec



Nyquist Frequency



1 Hz
1 cycle / sec

2x1 sample / sec

2 Hz
2 cycles / sec

2x2 samples / sec

3 Hz
3 cycles / sec

2x3 samples / sec

4 Hz
4 cycles / sec

2x4 samples / sec

5 Hz
5 cycles / sec

2x5 samples / sec

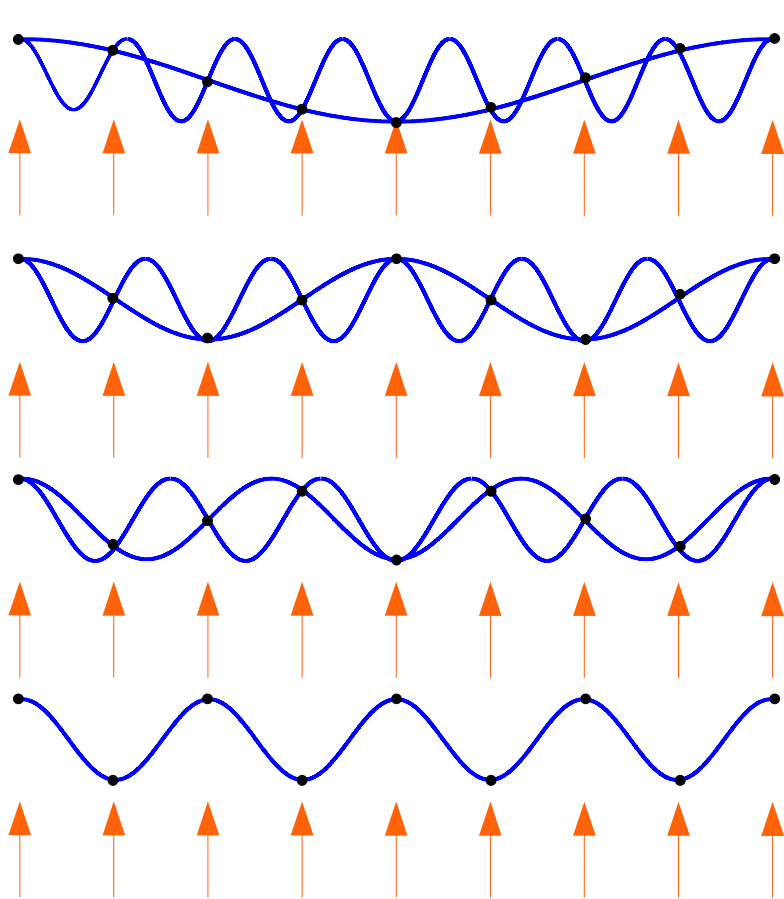
6 Hz
6 cycles / sec

2x6 samples / sec

7 Hz
7 cycles / sec

2x7 samples / sec

Aliasing



1 Hz
7 Hz

2x4 samples / sec

2 Hz
6 Hz

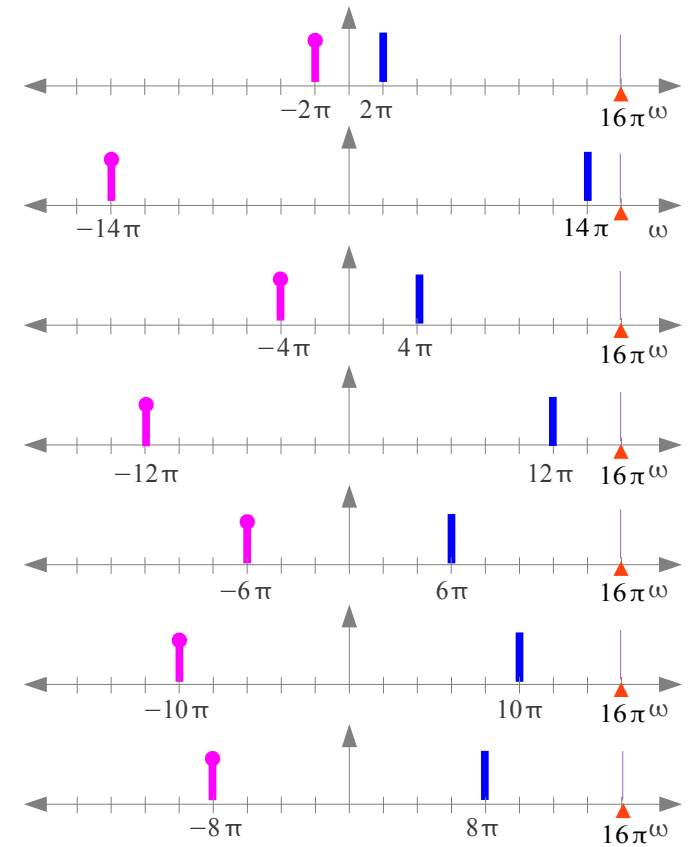
2x4 samples / sec

3 Hz
5 Hz

2x4 samples / sec

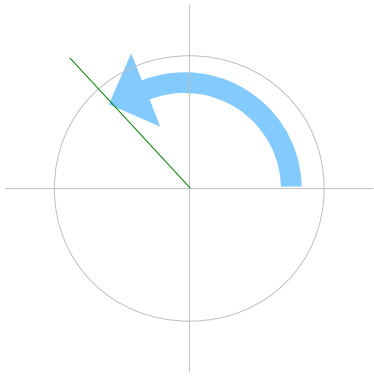
4 Hz

2x4 samples / sec



Sampling

$$\omega_s = 2\pi f_s \text{ (rad/sec)}$$



$$\omega_1 = 2\pi f_1$$

$$\omega_1 = \frac{\omega_s}{2} \text{ (rad/sec)}$$

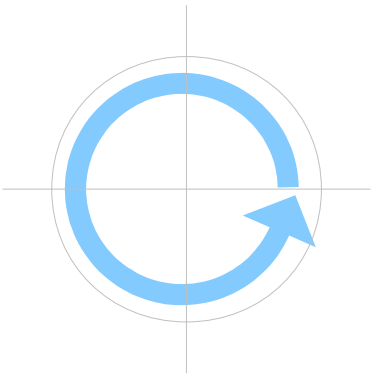
$$f_1 = \frac{f_s}{2} \text{ (rad/sec)}$$

$$\omega_2 = 2\pi f_2$$

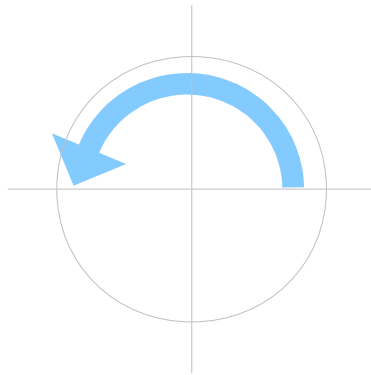
$$\omega_2 = -\frac{\omega_s}{2} \text{ (rad/sec)}$$

$$f_2 = -\frac{f_s}{2} \text{ (rad/sec)}$$

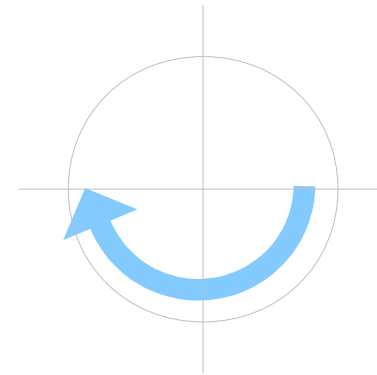
$$2\pi \text{ (rad)} / T_s \text{ (sec)}$$



$$\pi \text{ (rad)} / T_s \text{ (sec)}$$

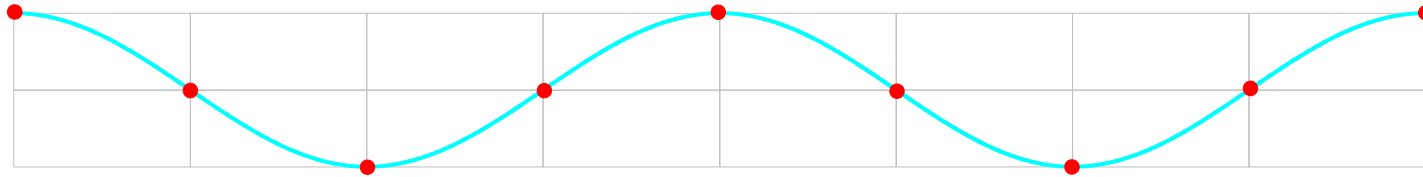


$$-\pi \text{ (rad)} / T_s \text{ (sec)}$$

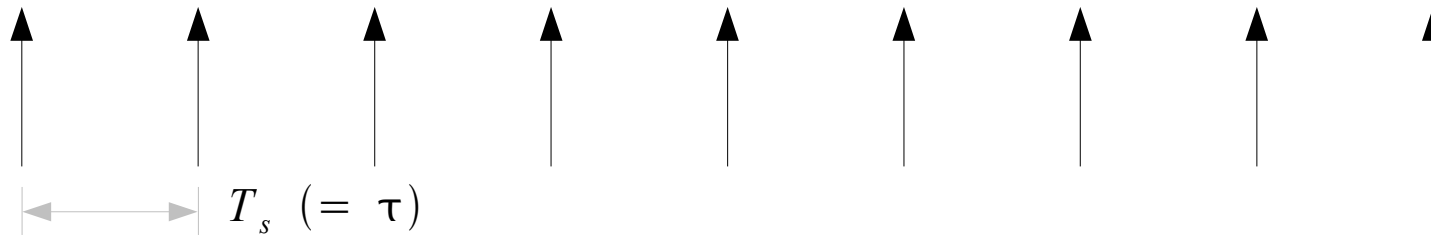


Sampling

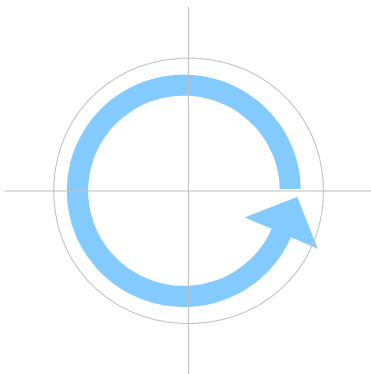
$$\omega_1 = 2\pi f_1 \text{ (rad/sec)}$$



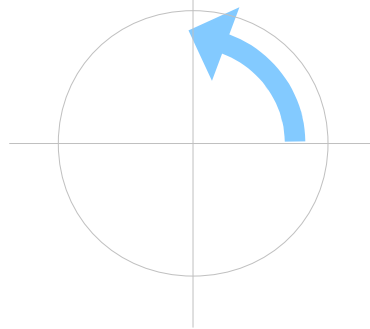
$$\omega_s = 2\pi f_s \text{ (rad/sec)}$$



$$2\pi \text{ (rad)} / T_s \text{ (sec)}$$



$$\frac{\pi}{2} \text{ (rad)} / T_s \text{ (sec)}$$



For the period of T_s
Angular displacement $\frac{\pi}{2}$ (rad)

$$\begin{aligned} \hat{\omega} &= \omega \cdot T_s \text{ (rad)} \\ &= 2\pi f_1 \cdot T_s \text{ (rad)} \\ &= 2\pi \frac{f_s}{4} \cdot T_s \text{ (rad)} \\ &= \frac{\pi}{2} \text{ (rad)} \end{aligned}$$

Angular Frequencies in Sampling

continuous-time signals

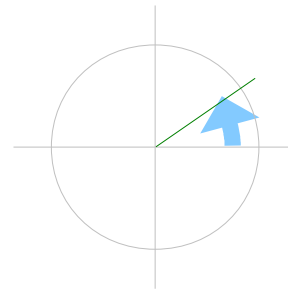
Signal Frequency

$$f_0 = \frac{1}{T_0}$$

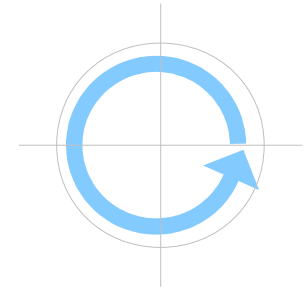
Signal Angular Frequency

$$\omega_0 = 2\pi f_0 \text{ (rad/sec)}$$

For 1 second
 $2\pi f_0 \text{ (rad/sec)}$



For 1 revolution
 $2\pi \text{ (rad)}$
 $T_0 \text{ (sec)}$



sampling sequence

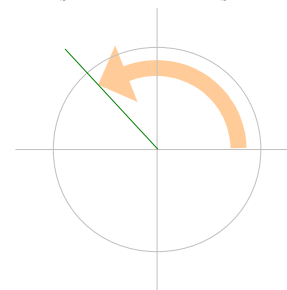
Sampling Frequency

$$f_s = \frac{1}{T_s}$$

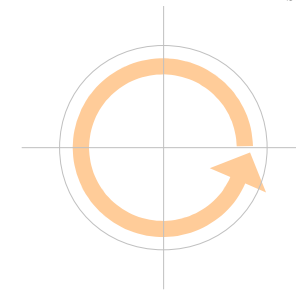
Sampling Angular Frequency

$$\omega_s = 2\pi f_s \text{ (rad/sec)}$$

For 1 second
 $2\pi f_s \text{ (rad/sec)}$



For 1 revolution
 $2\pi \text{ (rad)}$
 $T_s \text{ (sec)}$



References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] A "graphical interpretation" of the DFT and FFT, by Steve Mann
- [4] R. Cristi, "Modern Digital Signal Processing"