DFT Sampling (5B)

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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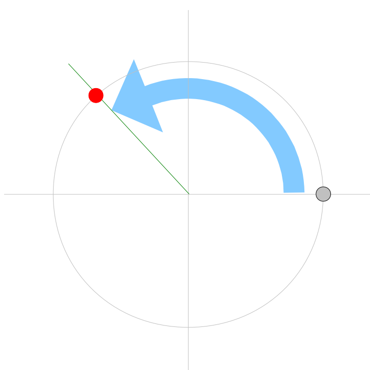
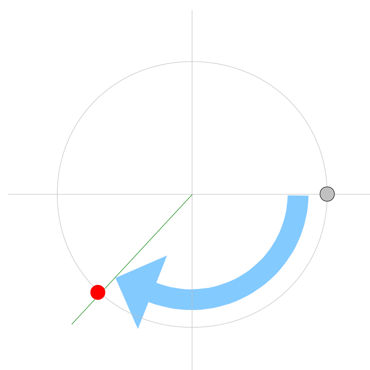
Angular Speed*: *Rotation Rate

$$\omega = \frac{2\pi}{T} = 2\pi f$$



$+\omega_0 \; rad \,/\, 1 \; sec$



$-\omega_0 \; rad \,/\, 1 \; sec$

$+\omega_0 \; (rad/sec)$

$-\omega_0 \; (rad/sec)$

rpm*: *Rotation Rate

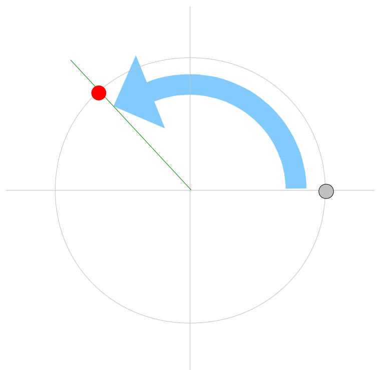
$$rpm = revolutions \,/\, minute$$

$$\begin{aligned} 1 \; rpm &= 2\pi \; rad \,/\, 1 \; min \\ &= 2\pi \; rad \,/\, 60 \; sec \\ &= \frac{\pi}{30} \; rad/sec \end{aligned}$$

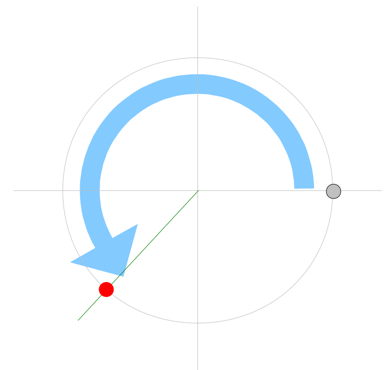
- **Negative Angles**

Angular Speed (2)

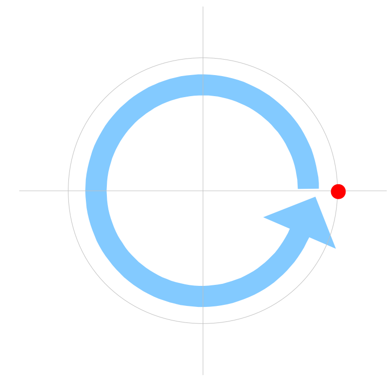
$+\omega_0 \quad (rad/sec)$



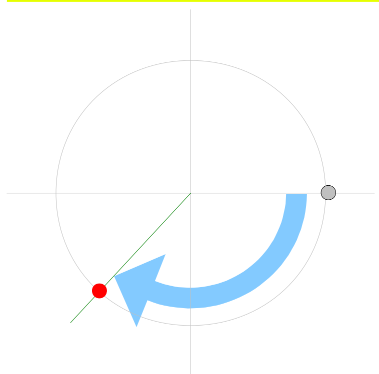
$+2\omega_0 \quad (rad/sec)$



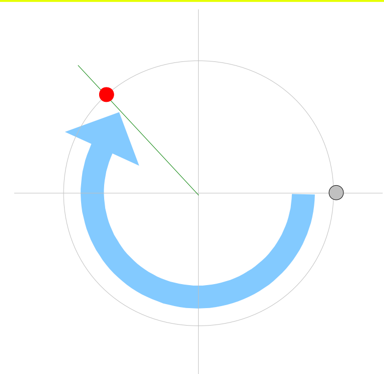
$+3\omega_0 \quad (rad/sec)$



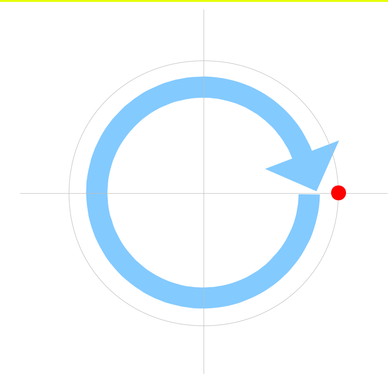
$-\omega_0 \quad (rad/sec)$



$-2\omega_0 \quad (rad/sec)$

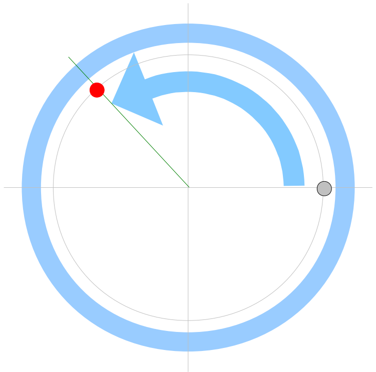


$-3\omega_0 \quad (rad/sec)$

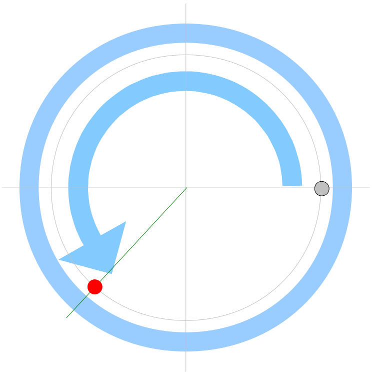


Angular Speed (3)

$+4\,\omega_0 \quad (rad\,/\,sec)$

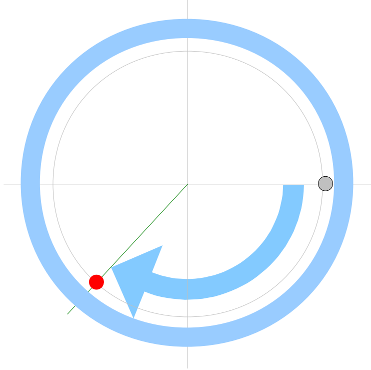


$+5\,\omega_0 \quad (rad\,/\,sec)$

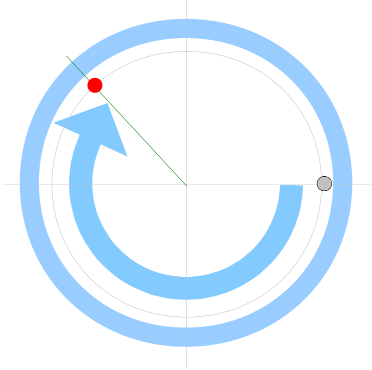


- **Co-terminal Angles**

$-4\,\omega_0 \quad (rad\,/\,sec)$



$-5\,\omega_0 \quad (rad\,/\,sec)$



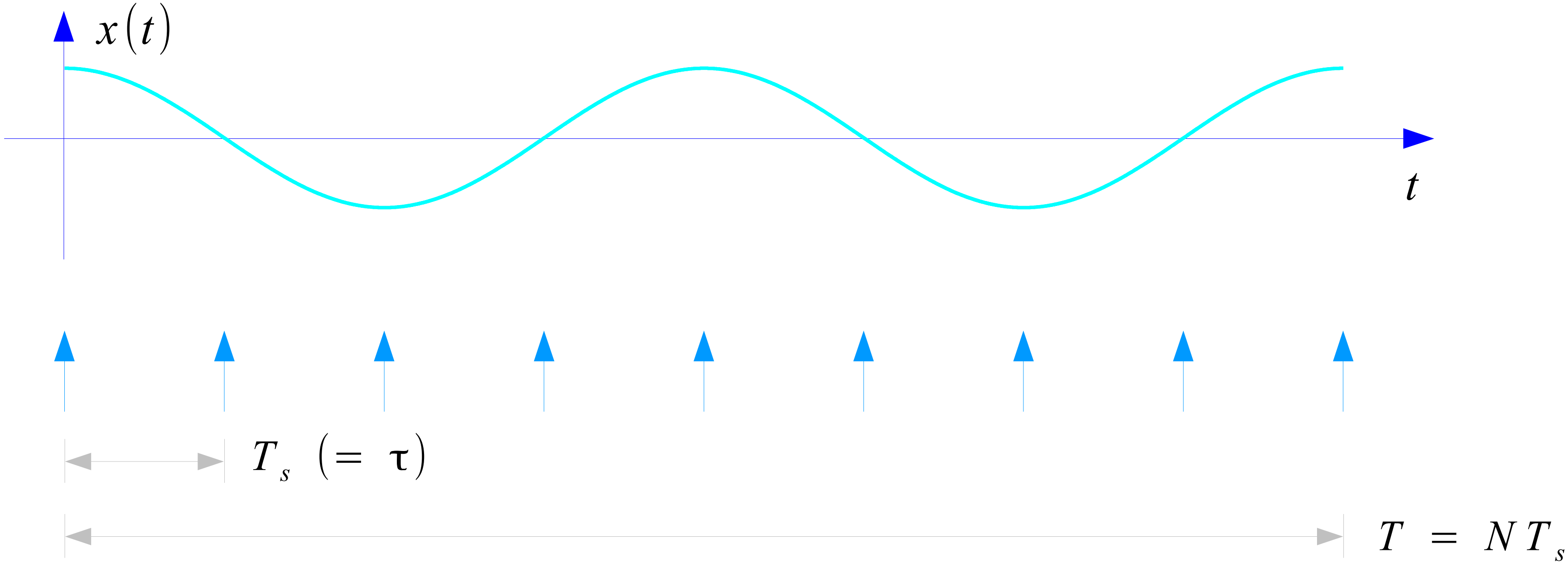
- **Co-terminal Angles**

Angular Speed (4)

$$\omega = \frac{2\pi}{T} = 2\pi f$$

T (sec)	0.01 sec	0.1 sec	1 sec	10 sec	100 sec
F (Hz)	100 Hz	10 Hz	1 Hz	0.1 Hz	0.01 Hz
ω (rad/sec)	200π (rad/sec)	20π (rad/sec)	2π (rad/sec)	0.2π (rad/sec)	0.02π (rad/sec)
	$= 628$	$= 62.8$	$= 6.28$	$= 0.628$	$=0.0628$

Sampling (1)

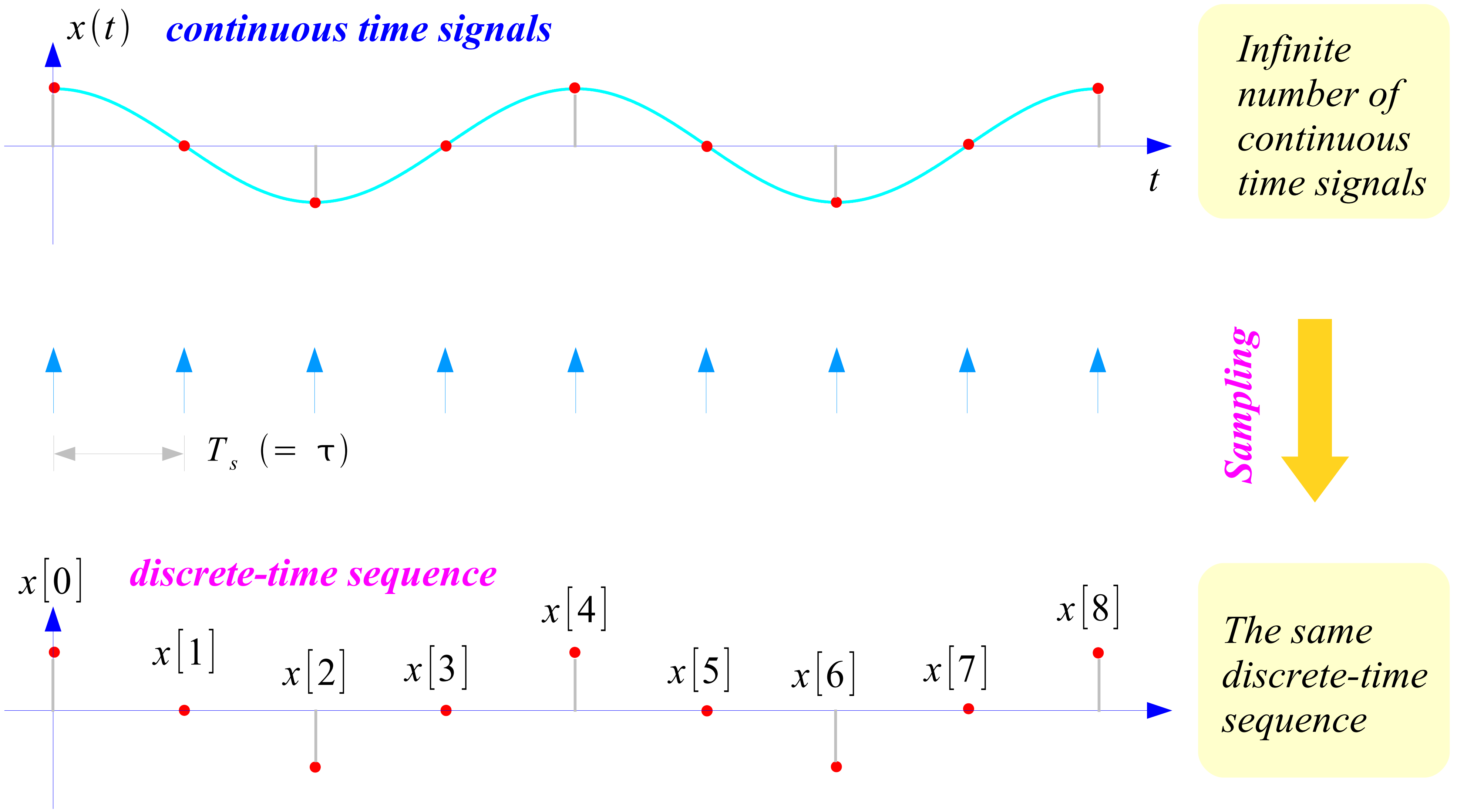


Sampling Time T_s

Sequence Time Length $T = N T_s$

Sampling Frequency $f_s = \frac{1}{\%T_s}$ *(samples per second)*

Sampling (2)

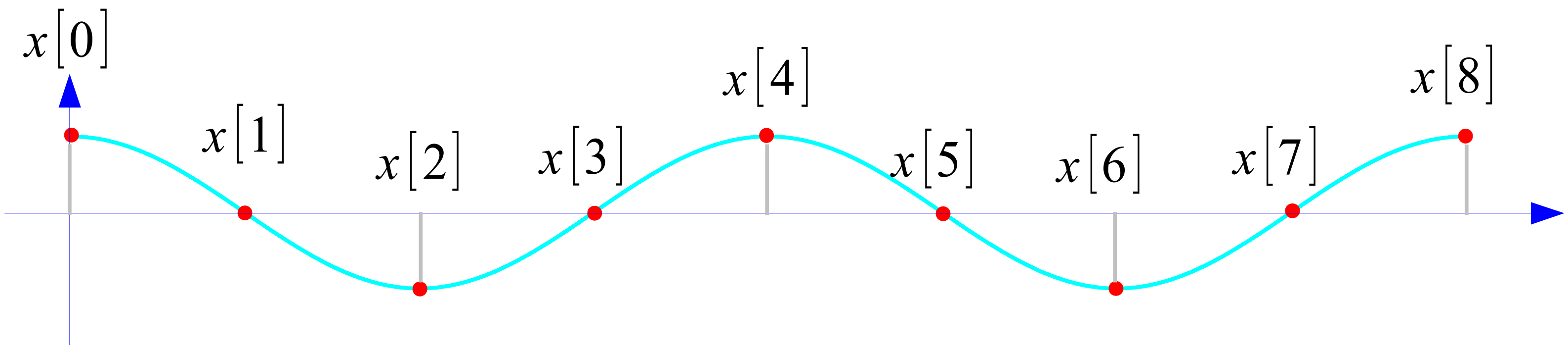


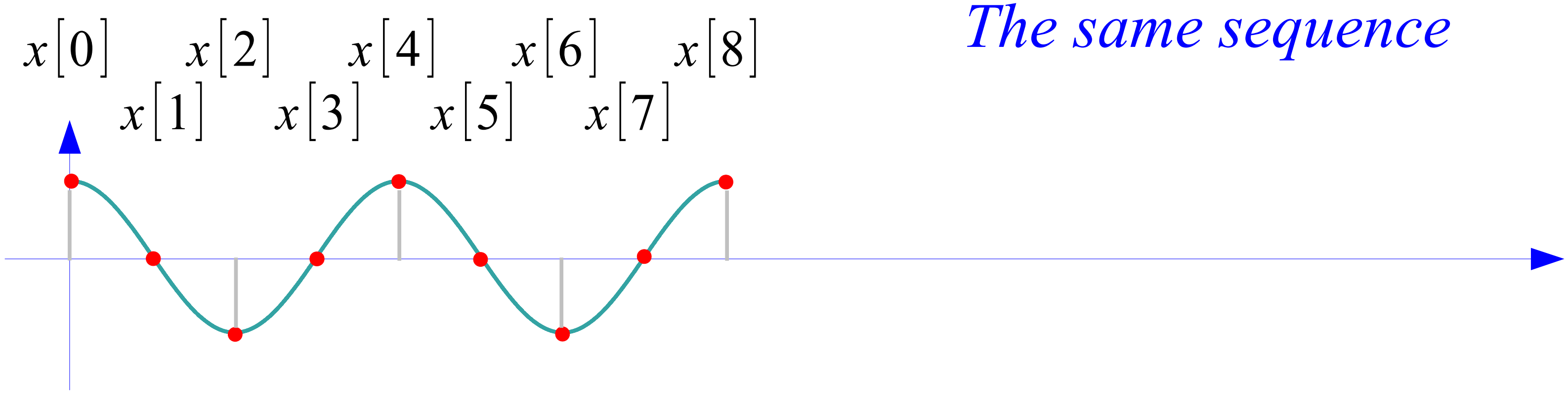
$x(t)$
continuous time signals
t
Infinite number of continuous time signals
$T_s \; (= \tau)$
Sampling
$x[0]$
discrete-time sequence
$x[1]$
$x[2]$
$x[3]$
$x[4]$
$x[5]$
$x[6]$
$x[7]$
$x[8]$
The same discrete-time sequence

Sampling (3)

Dimensionless sequence

$x[n] \Rightarrow \cdots, x[0], x[1], x[2], x[3], x[4], x[5], x[6], x[7], x[8], \cdots$





The same sequence

Sampling (4)

$$x(t) = A\cos(\omega t + \phi)$$

$$t \rightarrow n\,T_s$$

$$x[n] = x(n\,T_s)$$

$$= A\cos(\omega \cdot n\,T_s + \phi)$$

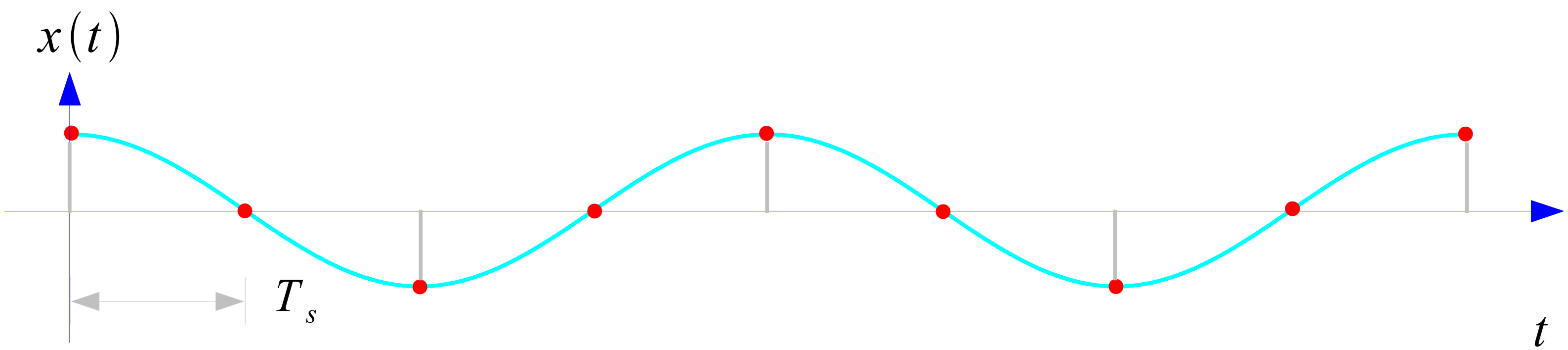
$$= A\cos(\omega \cdot T_s\, n + \phi)$$

$$= A\cos(\hat{\omega} \cdot n + \phi)$$

$$\hat{\omega} = \omega \cdot T_s = \frac{\omega}{1/T_s}$$

$$\hat{\omega} = \frac{\omega}{f_s}$$

- **Normalized to f_s**



Angular Speed (Angular Frequency, Radian Frequency)

$$\omega \quad (rad/sec)$$

$$x(t)$$

Sampling $\quad t \rightarrow n\,T_s$

$$x[n] = x(nT_s)$$

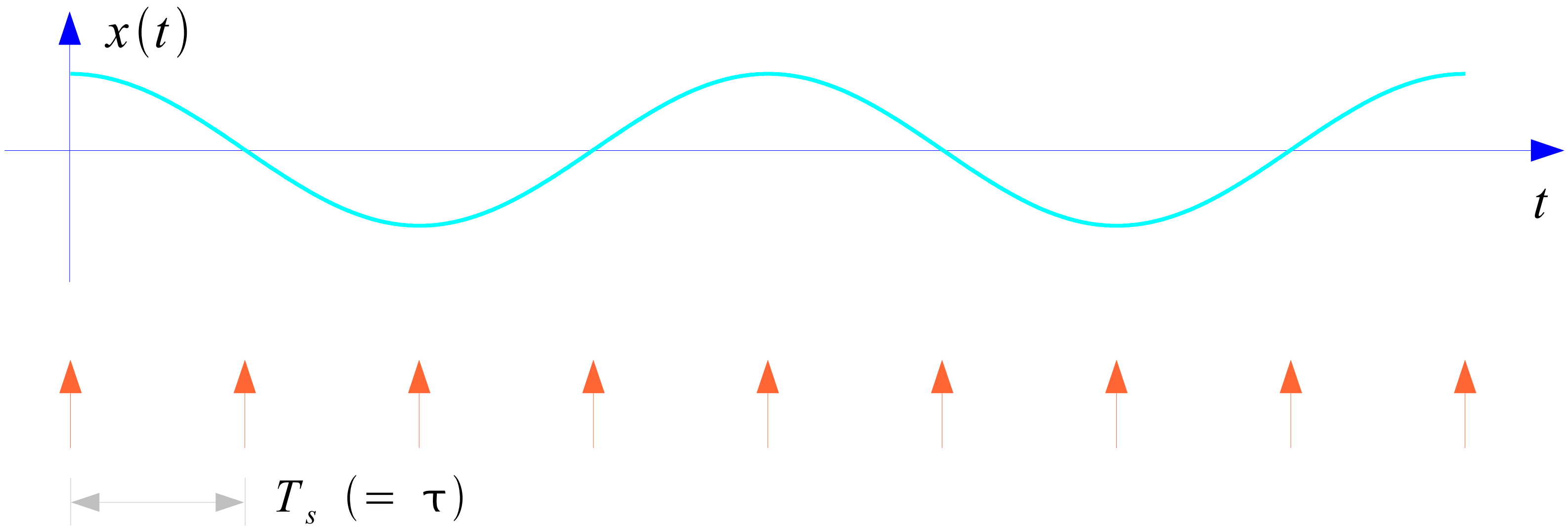
Dimensionless sequence

Normalized Radian Frequency

$$\hat{\omega} = \omega \cdot T_s \quad (rad)$$

Dimensionless quantity

Sampling Frequency



Sampling Time T_s

Sampling Frequency

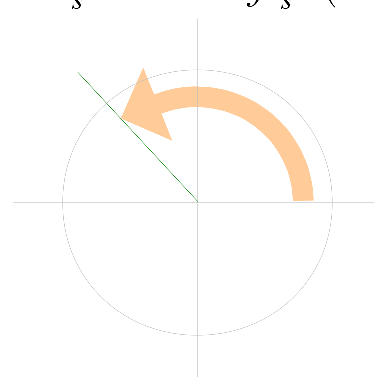
$$f_s = \frac{1}{T_s}$$

Sampling Angular Frequency

$$\omega_s = 2\pi f_s \quad (rad/sec)$$

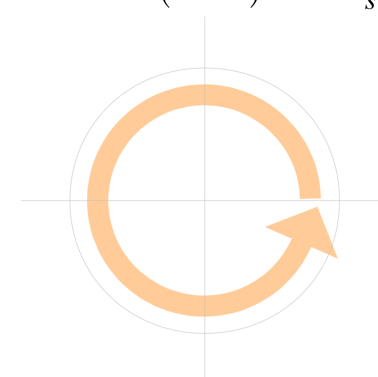
For 1 second

$$\omega_s = 2\pi f_s \quad (rad/sec)$$



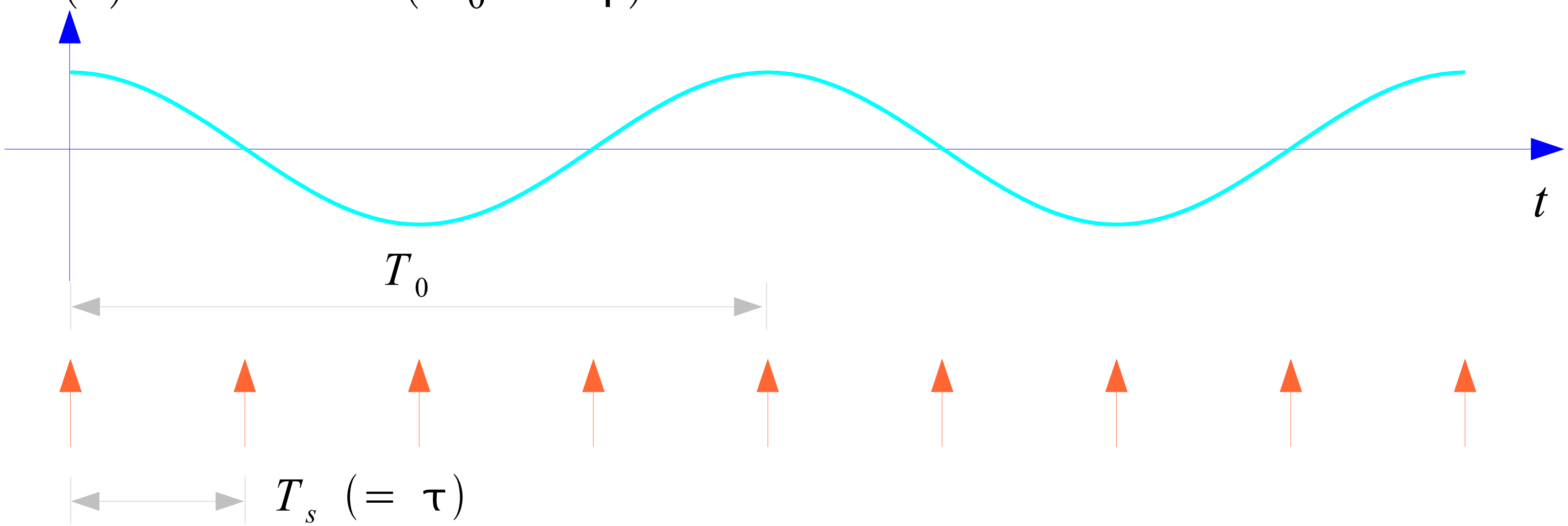
For 1 revolution

$$2\pi\ (rad)\ /\ T_s\ (sec)$$



Signal Frequency

$$x(t) = A\cos(\omega_0 t + \phi)$$



Signal Frequency

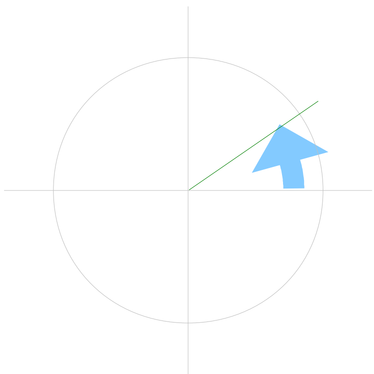
$$f_0 = \frac{1}{T_0}$$

Signal Angular Frequency

$$\omega_0 = 2\pi f_0 \; (rad/sec)$$

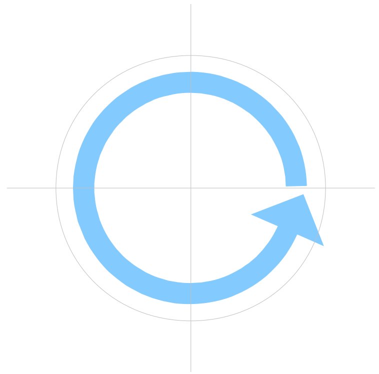
For 1 second

$$\omega_0 = 2\pi f_0 \; (rad/sec)$$



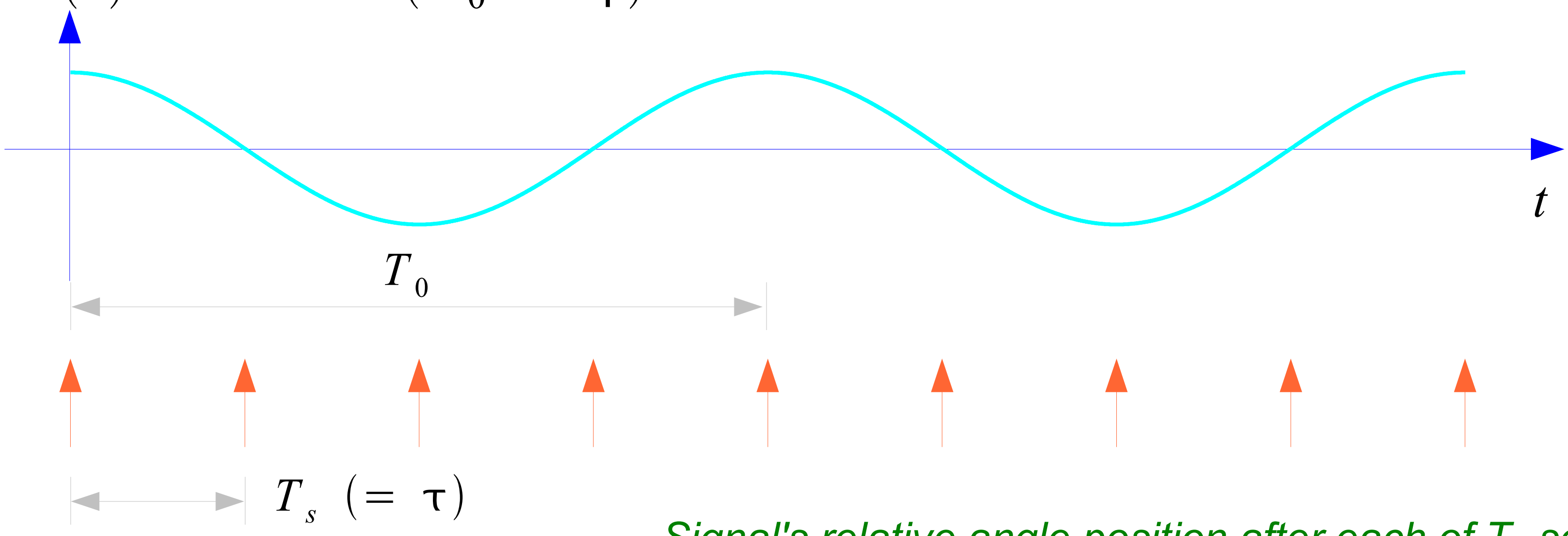
For 1 revolution

$$2\pi \, (rad) \;/\; T_0 \, (sec)$$



Normalize Radian Frequency

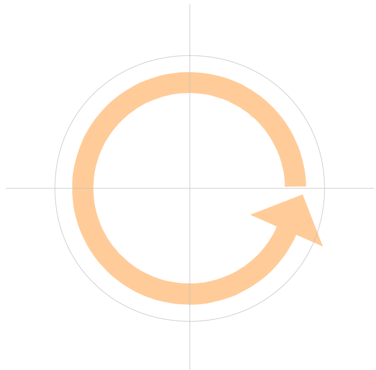
$$x(t) = A\cos(\omega_0 t + \phi)$$



Signal's relative angle position after each of T_s second

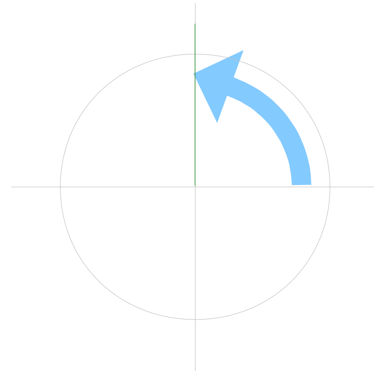
For 1 revolution

$$2\pi\,(rad)\;/\;T_s\,(sec)$$



For T_s second

$$\hat{\omega} = \omega_0 \cdot T_s \;\;(rad)$$



$$\hat{\omega} = \omega\, T_s$$

$$\hat{\omega} = \frac{\omega}{f_s}$$

Sampling

$$\omega_s = 2\pi f_s \ (rad/sec)$$

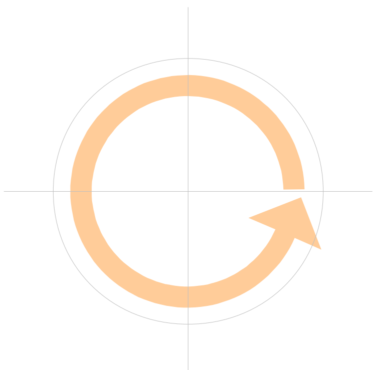
$$A \cos(\omega_1 t + \phi)$$

$$\omega_1 = \frac{\omega_s}{2}$$

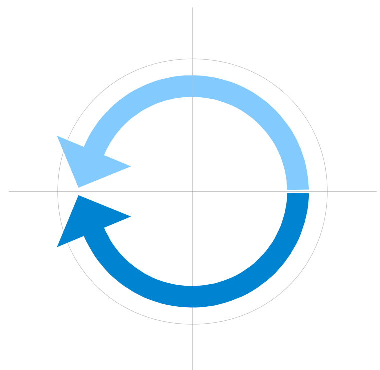
$$A \cos(\omega_2 t + \phi)$$

$$\omega_2 = \frac{\omega_s}{4}$$

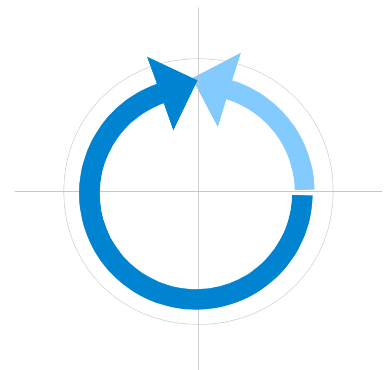
$$2\pi\ (rad)\ /\ T_s\ (sec)$$



$$\hat{\omega} = \pi\ (rad)$$



$$\hat{\omega} = \frac{\pi}{2}\ (rad)$$



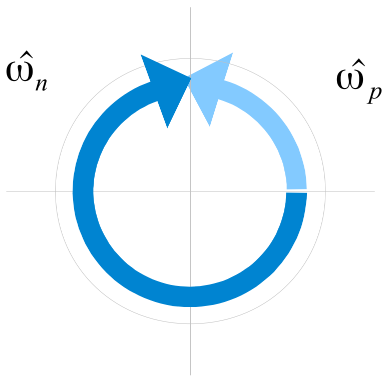
- **Negative Angles**

$$\hat{\omega} = -\pi\ (rad)$$

$$\omega_1 = -\frac{\omega_s}{2}$$

$$\hat{\omega} = -\frac{3\pi}{2}\ (rad)$$

$$\omega_2 = -\frac{3\omega_s}{2}$$



$+ \qquad -$

$$\omega_p - \omega_n = 2\pi$$

$+ \qquad\qquad -$

$$\omega_p = 2\pi + \omega_n$$

$- \qquad +$

$$\omega_n = \omega_p - 2\pi$$

Sampling

$$\omega_s = 2\pi f_s \ (rad/sec)$$

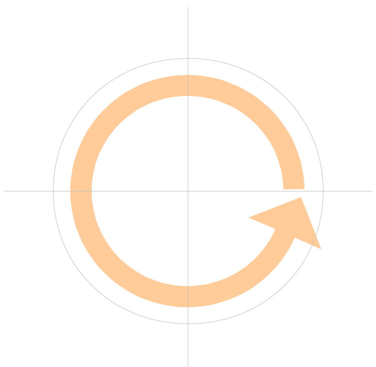
$$A\cos(\omega_1 t + \phi)$$

$$\omega_1 = \frac{\omega_s}{2}$$

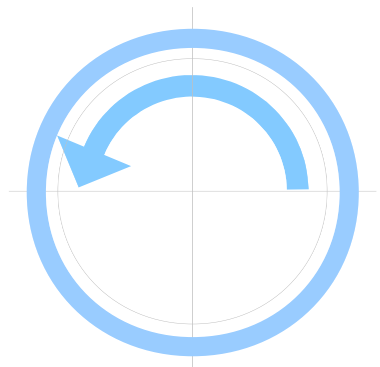
$$A\cos(\omega_2 t + \phi)$$

$$\omega_2 = \frac{\omega_s}{4}$$

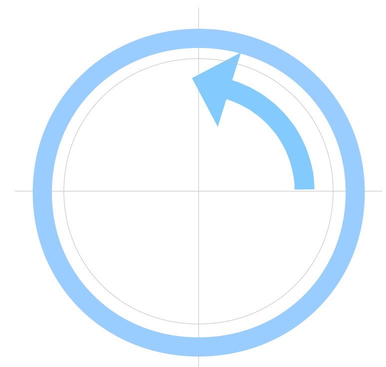
$$2\pi\,(rad)\ /\ T_s\,(sec)$$



$$\hat{\omega} = \pi\,(rad)$$



$$\hat{\omega} = \frac{\pi}{2}\,(rad)$$



- **Co-terminal Angles**

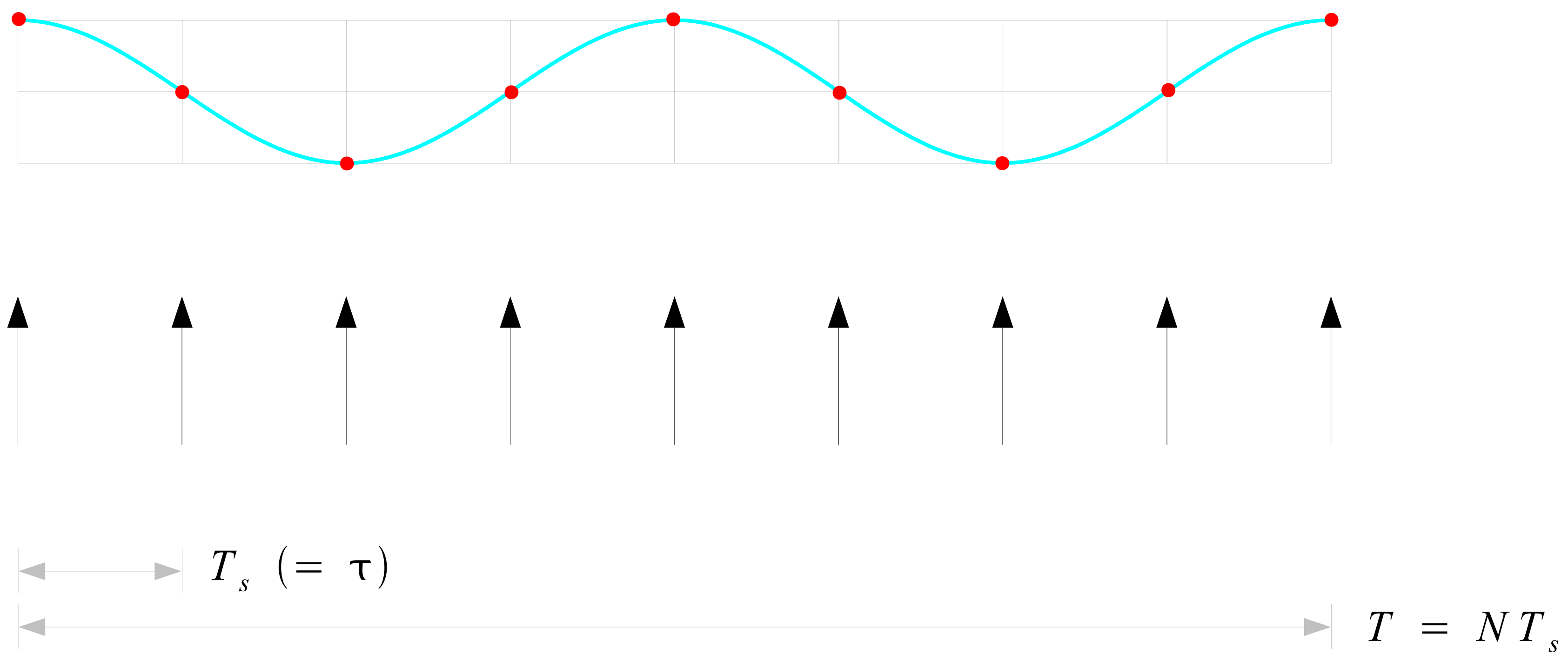
$$\hat{\omega} = \pi + 2\pi\,(rad)$$

$$\omega_1 = \frac{\omega_s}{2} + \omega_s$$

$$\hat{\omega} = \frac{\pi}{2} + 2\pi\,(rad)$$

$$\omega_2 = \frac{\omega_s}{4} + \omega_s$$

Sampling



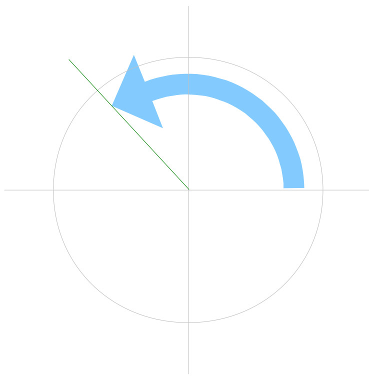
Sampling Time T_s

Sequence Time Length $T = N T_s$

Sampling Frequency $f_s = \frac{1}{T_s}$ *(samples per second)*

Sampling

$$\omega_s = 2\pi f_s \ (rad/sec)$$



$$\omega_1 = 2\pi f_1$$

$$\omega_1 = \frac{\omega_s}{2} \ (rad/sec)$$

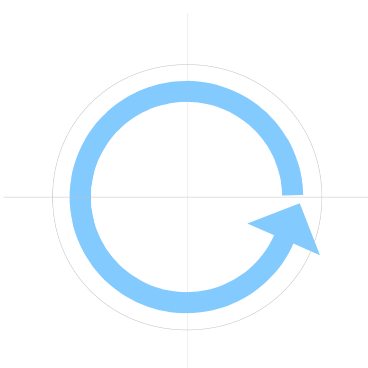
$$f_1 = \frac{f_s}{2} \ (rad/sec)$$

$$\omega_2 = 2\pi f_2$$

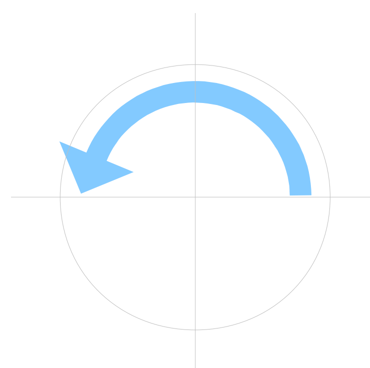
$$\omega_2 = -\frac{\omega_s}{2} \ (rad/sec)$$

$$f_2 = -\frac{f_s}{2} \ (rad/sec)$$

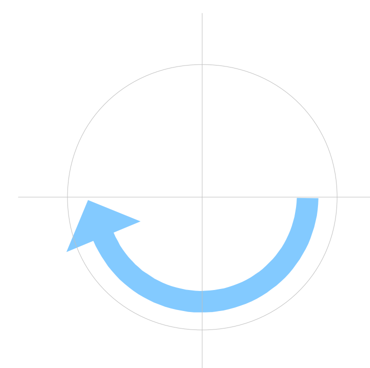
$$2\pi \ (rad) \ / \ T_s \ (sec)$$



$$\pi \ (rad) \ / \ T_s \ (sec)$$

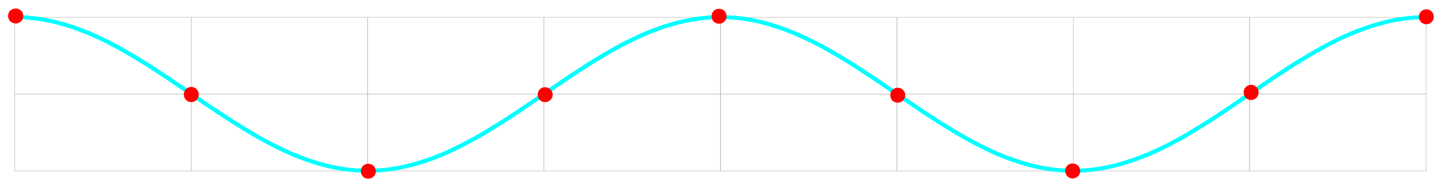


$$-\pi \ (rad) \ / \ T_s \ (sec)$$

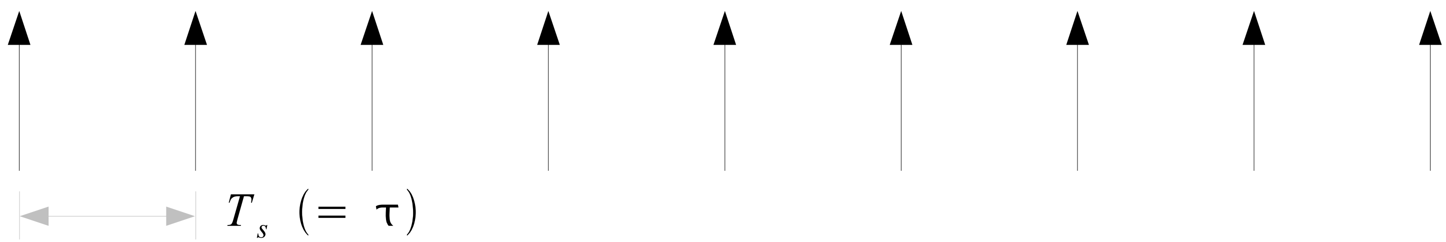


Sampling

$\omega_1 = 2\pi f_1 \ (rad/sec)$

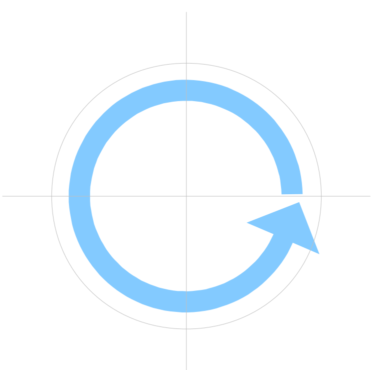


$\omega_s = 2\pi f_s \ (rad/sec)$

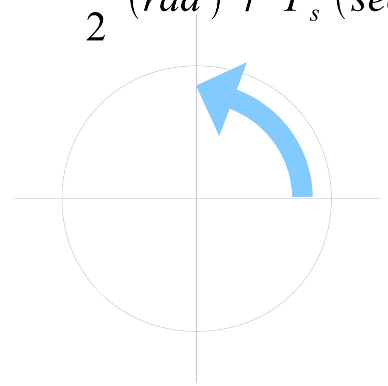


$T_s \ (= \tau)$

$2\pi \ (rad) \ / \ T_s \ (sec)$



$\frac{\pi}{2} \ (rad) \ / \ T_s \ (sec)$



For the period of T_s

Angular displacement $\frac{\pi}{2} \ (rad)$

$$\hat{\omega} = \omega \cdot T_s \ (rad)$$

$$= 2\pi f_1 \cdot T_s \ (rad)$$

$$= 2\pi \frac{f_s}{4} \cdot T_s \ (rad)$$

$$= \frac{\pi}{2} \ (rad)$$

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2/28/11

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A “graphical interpretation” of the DFT and FFT, by Steve Mann