

LMS Overview (1A)

- N

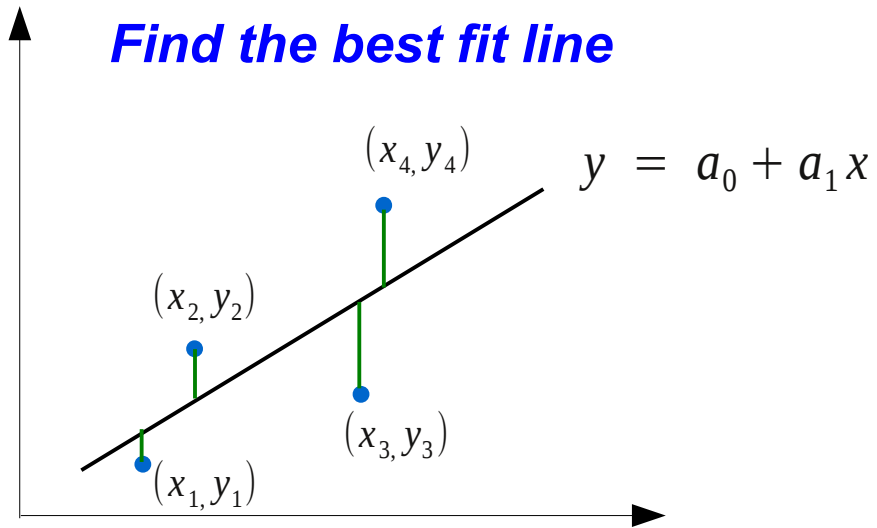
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Linear Regression (1)



a_0, a_1 *unknowns*

(x_i, y_i) *measured data*

random

$$S_r = \sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n (y_i - (a_0 + a_1 x_i))^2$$

Linear Regression (2)

Sum of the square of the residuals

$$S_r = \sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n (y_i - (a_0 + a_1 x_i))^2$$

a_0, a_1 *unknowns*

(x_i, y_i) *measured data*

random

Minimum Condition

$$\frac{\partial S_r}{\partial a_0} = 2 \sum_{i=1}^n (y_i - a_0 - a_1 x_i)(-1) = 0$$



$$\sum_{i=1}^n a_0 + \sum_{i=1}^n a_1 x_i = \sum_{i=1}^n y_i$$

$$\frac{\partial S_r}{\partial a_1} = 2 \sum_{i=1}^n (y_i - a_0 - a_1 x_i)(-x_i) = 0$$



$$\sum_{i=1}^n a_0 x_i + \sum_{i=1}^n a_1 x_i^2 = \sum_{i=1}^n y_i x_i$$

Linear Regression (3)

$$\sum_{i=1}^n a_0 + \sum_{i=1}^n a_1 x_i = \sum_{i=1}^n y_i$$

$$n \cdot a_0 + \sum_{i=1}^n a_1 x_i = \sum_{i=1}^n y_i$$

$$a_0 = \frac{1}{n} \sum_{i=1}^n y_i - \frac{1}{n} \sum_{i=1}^n a_1 x_i$$

$$\sum_{i=1}^n a_0 x_i + \sum_{i=1}^n a_1 x_i^2 = \sum_{i=1}^n y_i x_i$$

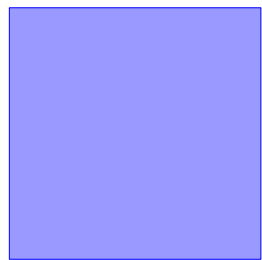
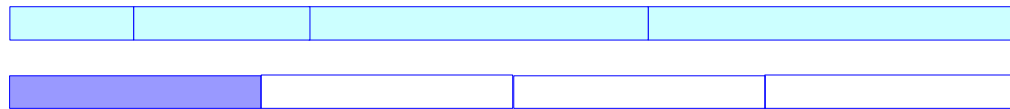
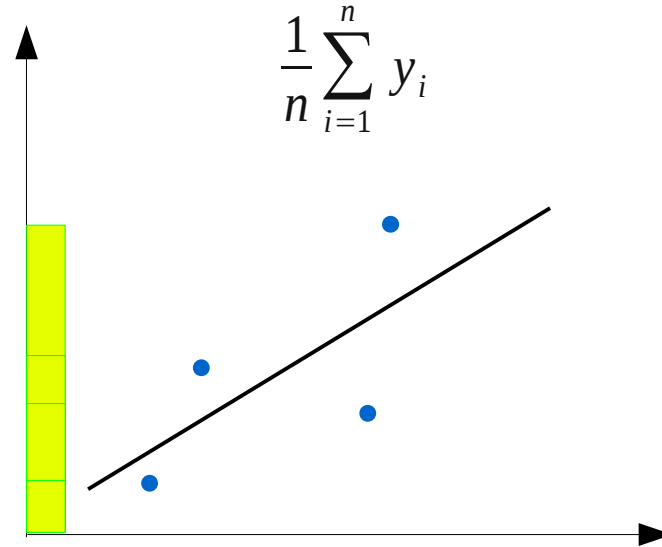
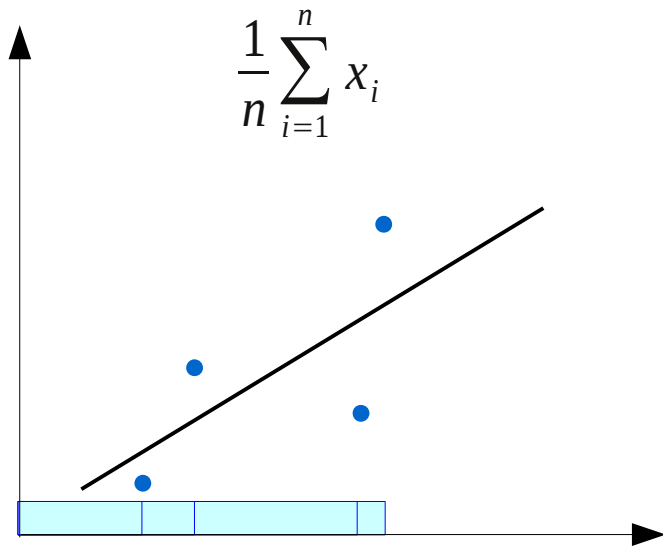
$$\left(\sum_{i=1}^n x_i \right) \left(\frac{1}{n} \sum_{i=1}^n y_i - \frac{1}{n} \sum_{i=1}^n a_1 x_i \right) + \sum_{i=1}^n a_1 x_i^2 = \sum_{i=1}^n y_i x_i$$

$$\frac{1}{n} \left(\sum_{i=1}^n x_i \right) \left(\sum_{i=1}^n y_i \right) - \frac{1}{n} \left(\sum_{i=1}^n x_i \right)^2 a_1 + \left(\sum_{i=1}^n x_i^2 \right) a_1 = \left(\sum_{i=1}^n y_i x_i \right)$$

$$n \left(\sum_{i=1}^n x_i^2 \right) a_1 - \left(\sum_{i=1}^n x_i \right)^2 a_1 = n \left(\sum_{i=1}^n y_i x_i \right) - \left(\sum_{i=1}^n x_i \right) \left(\sum_{i=1}^n y_i \right)$$

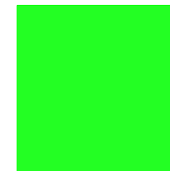
$$a_1 = \frac{n \left(\sum_{i=1}^n y_i x_i \right) - \left(\sum_{i=1}^n x_i \right) \left(\sum_{i=1}^n y_i \right)}{n \left(\sum_{i=1}^n x_i^2 \right) - \left(\sum_{i=1}^n x_i \right)^2}$$

Mean Values of x_i, y_i

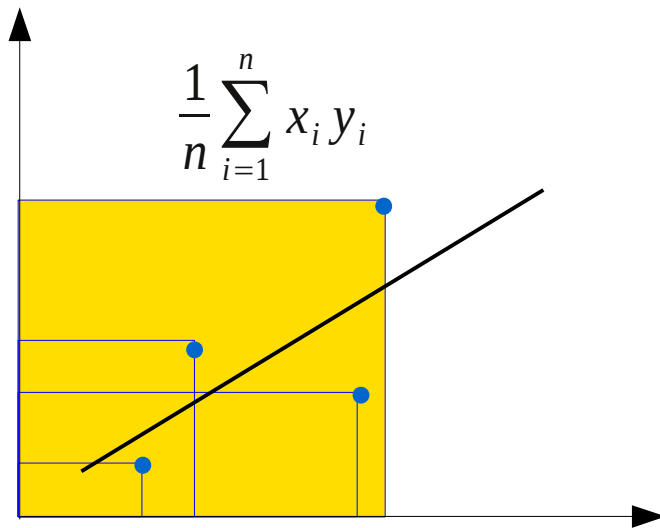
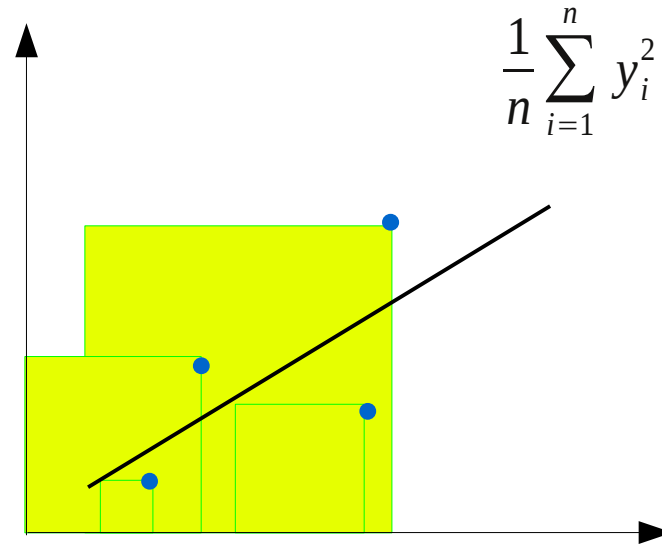
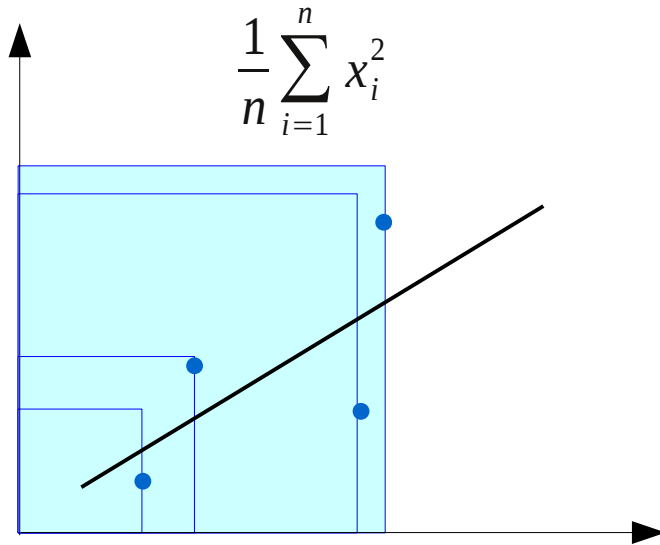


$$\left(\frac{1}{n} \sum_{i=1}^n x_i \right)^2$$

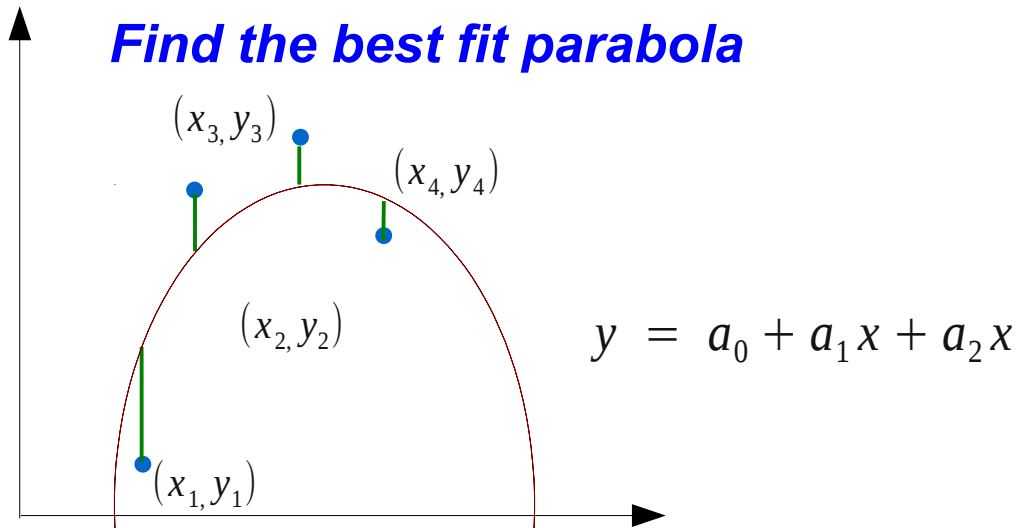
$$\left(\frac{1}{n} \sum_{i=1}^n y_i \right)^2$$



Mean Values of x_i^2 , y_i^2 , $x_i y_i$



Non-Linear Regression (1)



a_0, a_1, a_2 *unknowns*

(x_i, y_i) *measured data*

random

$$S_r = \sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n (y_i - (a_0 + a_1x_i + a_2x_i^2))^2$$

Non-Linear Regression (2)

Sum of the square of the residuals

$$S_r = \sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n (y_i - (a_0 + a_1 x_i + a_2 x_i^2))^2$$

Minimum Condition

$$\frac{\partial S_r}{\partial a_0} = 2 \sum_{i=1}^n (y_i - a_0 - a_1 x_i - a_2 x_i^2)(-1) = 0$$

$$\frac{\partial S_r}{\partial a_1} = 2 \sum_{i=1}^n (y_i - a_0 - a_1 x_i - a_2 x_i^2)(-x_i) = 0$$

$$\frac{\partial S_r}{\partial a_2} = 2 \sum_{i=1}^n (y_i - a_0 - a_1 x_i - a_2 x_i^2)(-x_i^2) = 0$$

a_0, a_1, a_2 *unknowns*

(x_i, y_i) *measured data*

random

Find the best fit parabola

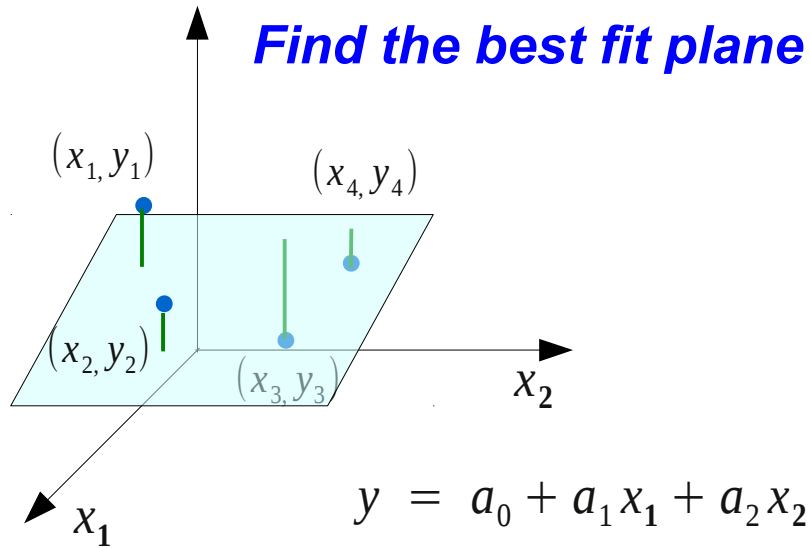
Non-Linear Regression (3)

$$\left(\sum_{i=1}^n 1 \right) \cdot a_0 + \left(\sum_{i=1}^n x_i \right) \cdot a_1 + \left(\sum_{i=1}^n x_i^2 \right) \cdot a_2 = \left(\sum_{i=1}^n y_i \right)$$

$$\left(\sum_{i=1}^n x_i \right) \cdot a_0 + \left(\sum_{i=1}^n x_i^2 \right) \cdot a_1 + \left(\sum_{i=1}^n x_i^3 \right) \cdot a_2 = \left(\sum_{i=1}^n x_i y_i \right)$$

$$\left(\sum_{i=1}^n x_i^2 \right) \cdot a_0 + \left(\sum_{i=1}^n x_i^3 \right) \cdot a_1 + \left(\sum_{i=1}^n x_i^4 \right) \cdot a_2 = \left(\sum_{i=1}^n x_i^2 y_i \right)$$

Multivariate Regression (1)



a_0, a_1, a_2 *unknowns*
 $(x_{i,1}, x_{i,2}, y_i)$ *measured data*

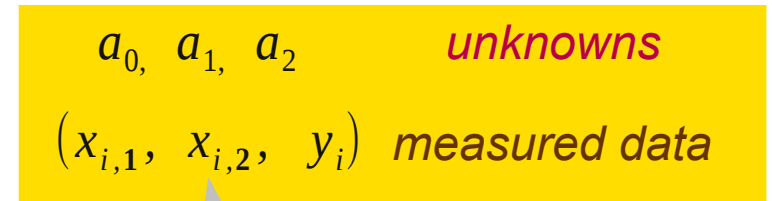
random

$$S_r = \sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n (y_i - (a_0 + a_1 x_{i,1} + a_2 x_{i,2}))^2$$

Multivariate Regression (2)

Sum of the square of the residuals

$$S_r = \sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n (y_i - (a_0 + a_1 x_{i,1} + a_2 x_{i,2}))^2$$



random

Minimum Condition

$$\frac{\partial S_r}{\partial a_0} = 2 \sum_{i=1}^n (y_i - a_0 - a_1 x_{i,1} - a_2 x_{i,2})(-1) = 0$$

$$\frac{\partial S_r}{\partial a_1} = 2 \sum_{i=1}^n (y_i - a_0 - a_1 x_{i,1} - a_2 x_{i,2})(-x_{i,1}) = 0$$

$$\frac{\partial S_r}{\partial a_2} = 2 \sum_{i=1}^n (y_i - a_0 - a_1 x_{i,1} - a_2 x_{i,2})(-x_{i,2}) = 0$$

Multivariate Regression (3)

$$\left(\sum_{i=1}^n 1\right) \cdot a_0 + \left(\sum_{i=1}^n x_{i,1}\right) \cdot a_1 + \left(\sum_{i=1}^n x_{i,2}\right) \cdot a_2 = \left(\sum_{i=1}^n y_i\right)$$

$$\left(\sum_{i=1}^n x_{i,1}\right) \cdot a_0 + \left(\sum_{i=1}^n x_{i,1}^2\right) \cdot a_1 + \left(\sum_{i=1}^n x_{i,1}x_{i,2}\right) \cdot a_2 = \left(\sum_{i=1}^n x_{i,1}y_i\right)$$

$$\left(\sum_{i=1}^n x_{i,2}\right) \cdot a_0 + \left(\sum_{i=1}^n x_{i,1}x_{i,2}\right) \cdot a_1 + \left(\sum_{i=1}^n x_{i,2}^2\right) \cdot a_2 = \left(\sum_{i=1}^n x_{i,2}y_i\right)$$

Least Square

Sum of the square of the residuals

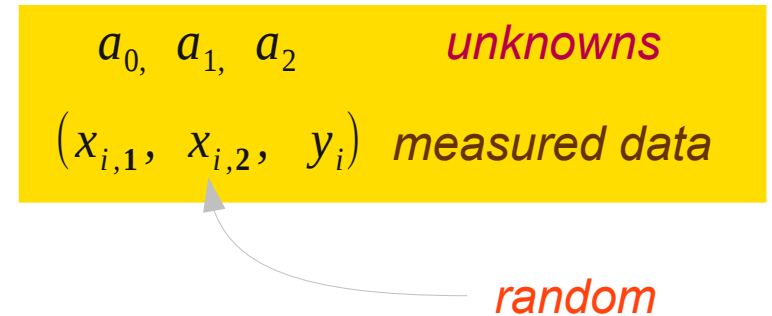
$$S_r = \sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n \left(y_i - \sum_{j=1}^n x_{ij} \right)^2$$

Minimum Condition

$$\frac{\partial S_r}{\partial a_0} = 2 \sum_{i=1}^n (y_i - a_0 - a_1 x_{i,1} - a_2 x_{i,2})(-1) = 0$$

$$\frac{\partial S_r}{\partial a_1} = 2 \sum_{i=1}^n (y_i - a_0 - a_1 x_{i,1} - a_2 x_{i,2})(-x_{i,1}) = 0$$

$$\frac{\partial S_r}{\partial a_2} = 2 \sum_{i=1}^n (y_i - a_0 - a_1 x_{i,1} - a_2 x_{i,2})(-x_{i,2}) = 0$$



References

- [1] <http://en.wikipedia.org/>
- [2] <http://numericalmethods.eng.usf.edu/>
- [3] S.C. Chapra, Applied Numerical Methods W/ml Engineering And Science