# Bandpass Sampling (2B)

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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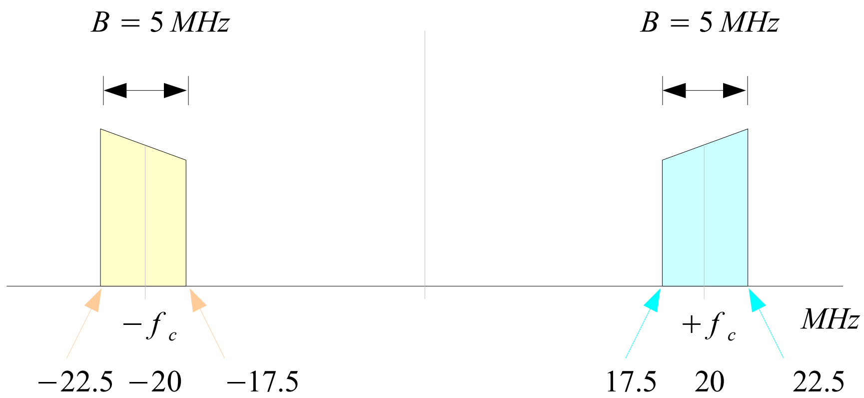
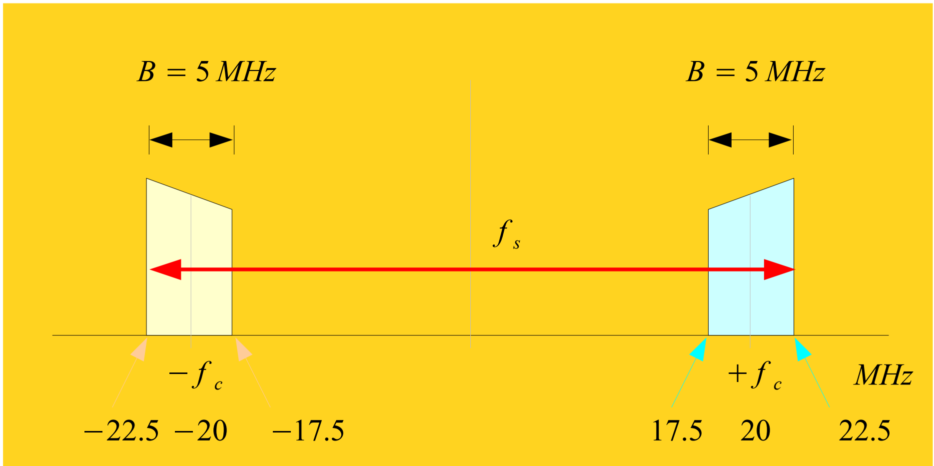
# Band-limited Signal

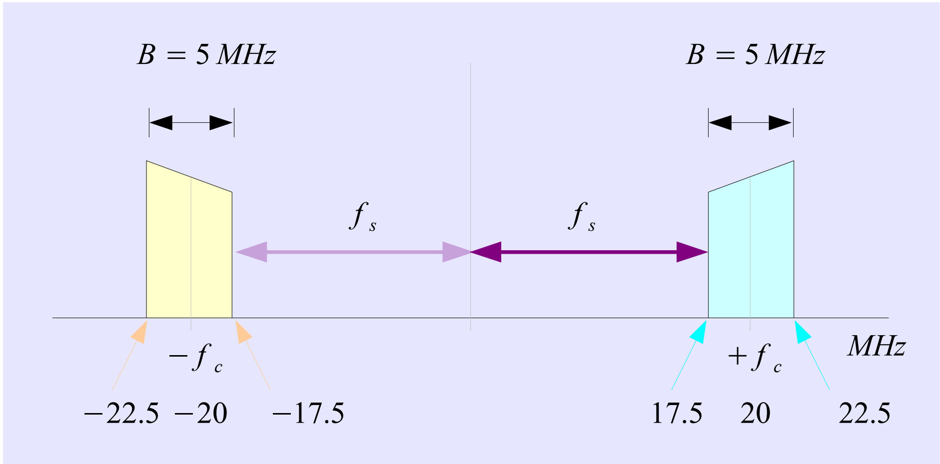




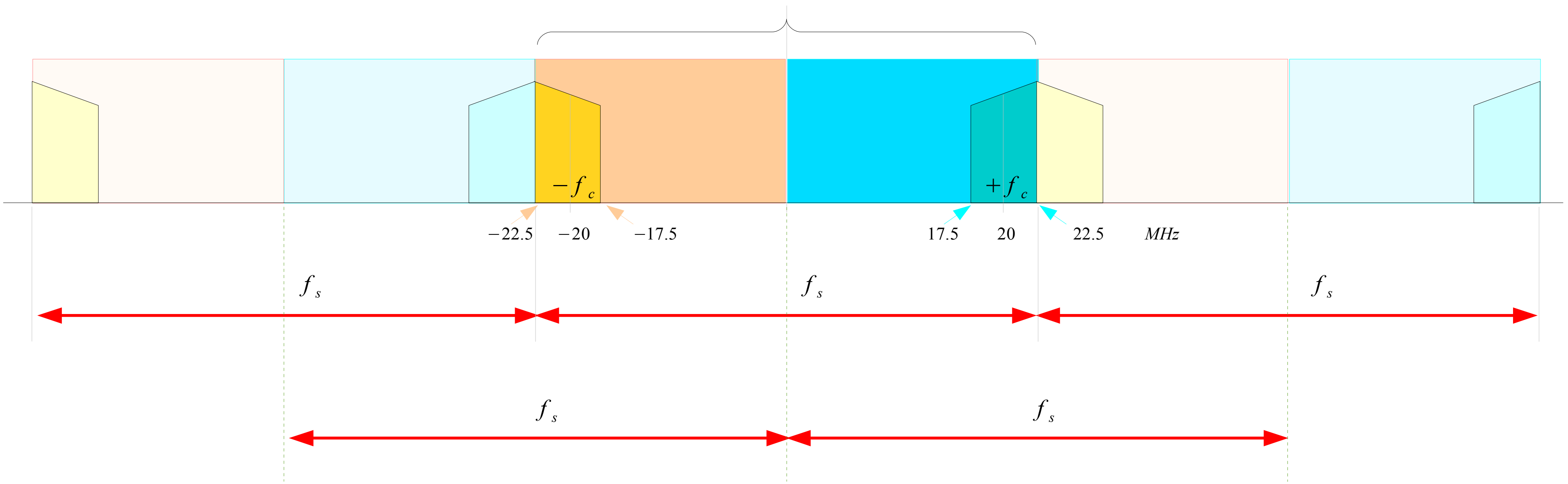


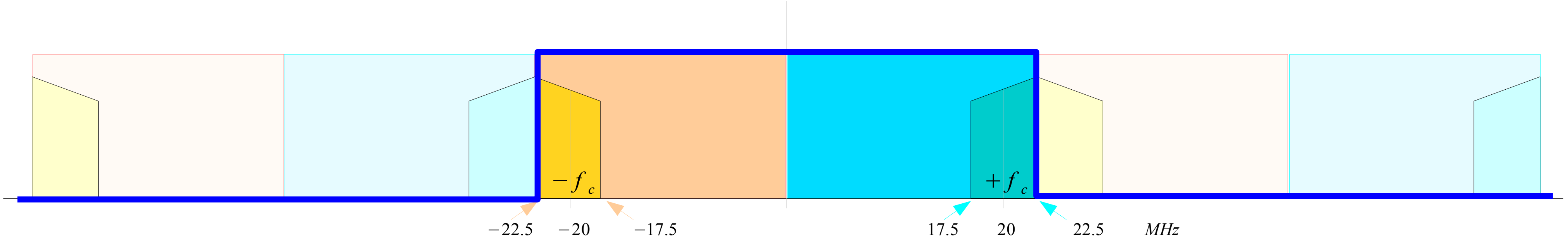

- Lowpass Sampling

- Bandpass Sampling
- IF filtering
- Harmonic Sampling
- Sub-Nyquist Sampling

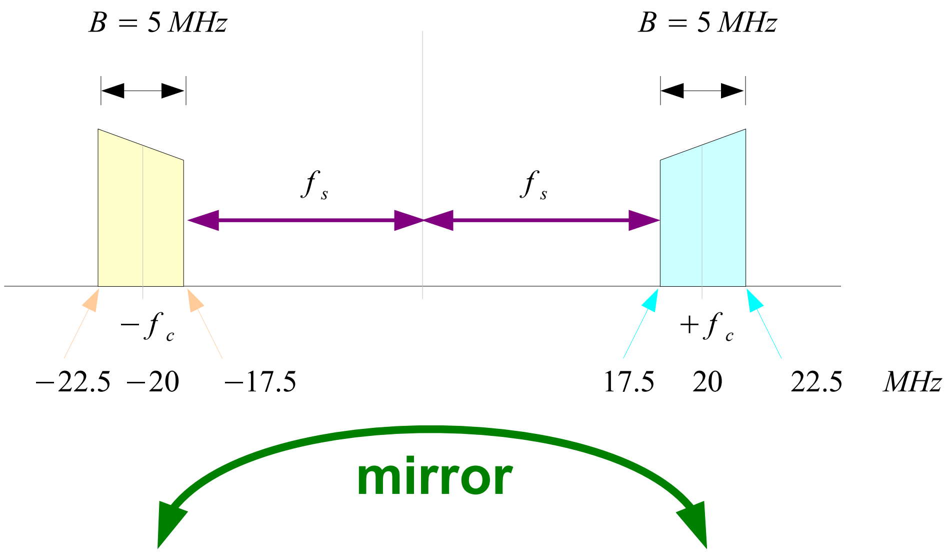


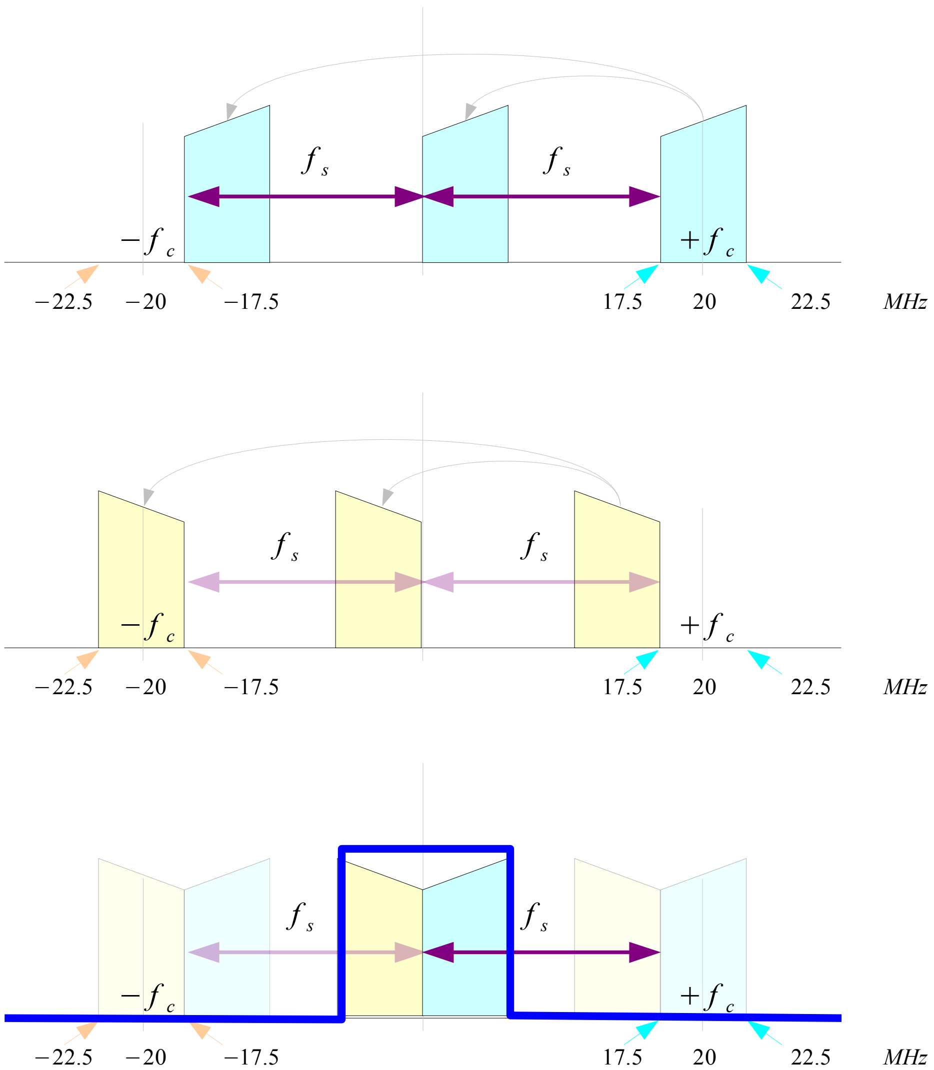
# Low-pass Signal Sampling



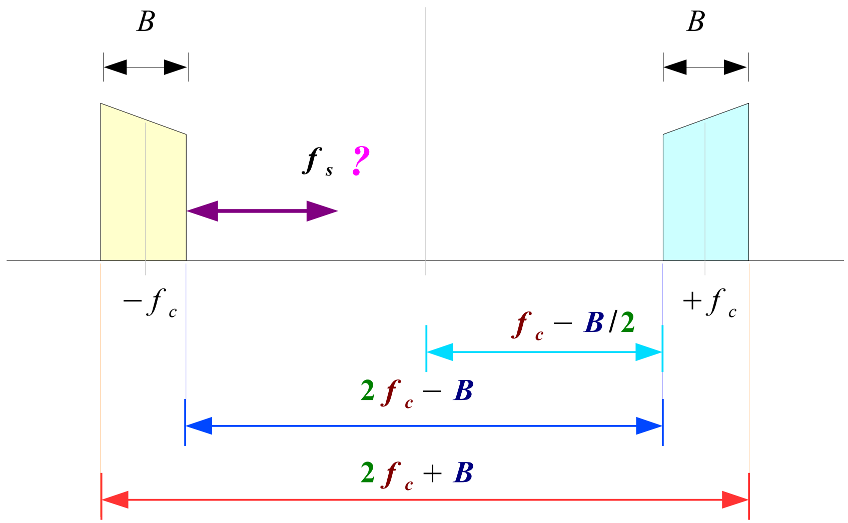
# Band-pass Signal Sampling




- **Bandpass Sampling**
- **IF filtering**
- **Harmonic Sampling**
- **Sub-Nyquist Sampling**

# Sampling Frequency $f_s$ (1)




- **Bandpass Sampling**
- **IF filtering**
- **Harmonic Sampling**
- **Sub-Nyquist Sampling**

*Assume there are m multiples of* $f_s$

*Given an integer m*

$$2f_c - B = m \cdot f_s$$

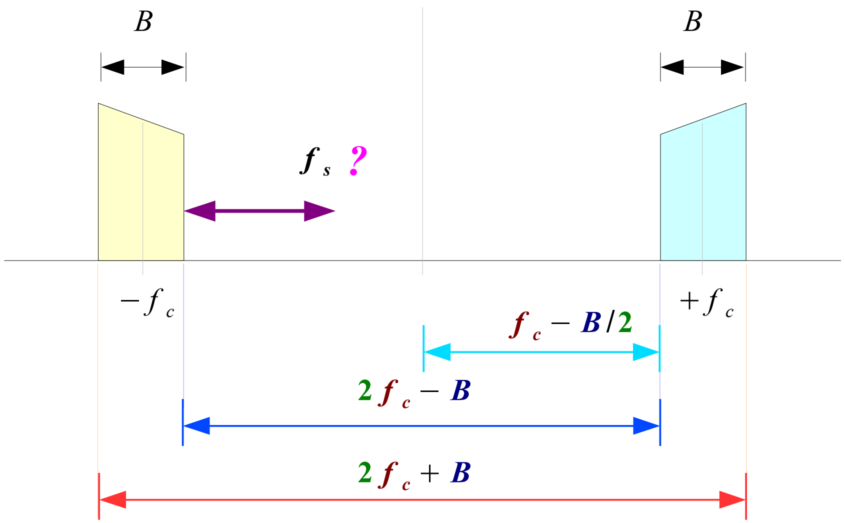
***Max*** $f_s$ ***condition***

$f_s$ *can be decreased according to the following condition without introducing aliasing problems*

$$2f_c + B = (m+1) \cdot f_s$$

***Min*** $f_s$ ***condition***

# Sampling Frequency $f_s$ (2)




*When $m = 6$*

$$\max f_s = \frac{2f_c - B}{6}$$

$$\min f_s = \frac{2f_c + B}{7}$$

$$\min f_s \quad \frac{2f_c + B}{7} \le f_s \le \frac{2f_c - B}{6} \quad \max f_s$$

*Assume there are m multiples of $f_s$*

*Given an integer m*

$$2f_c - B = m \cdot f_s$$

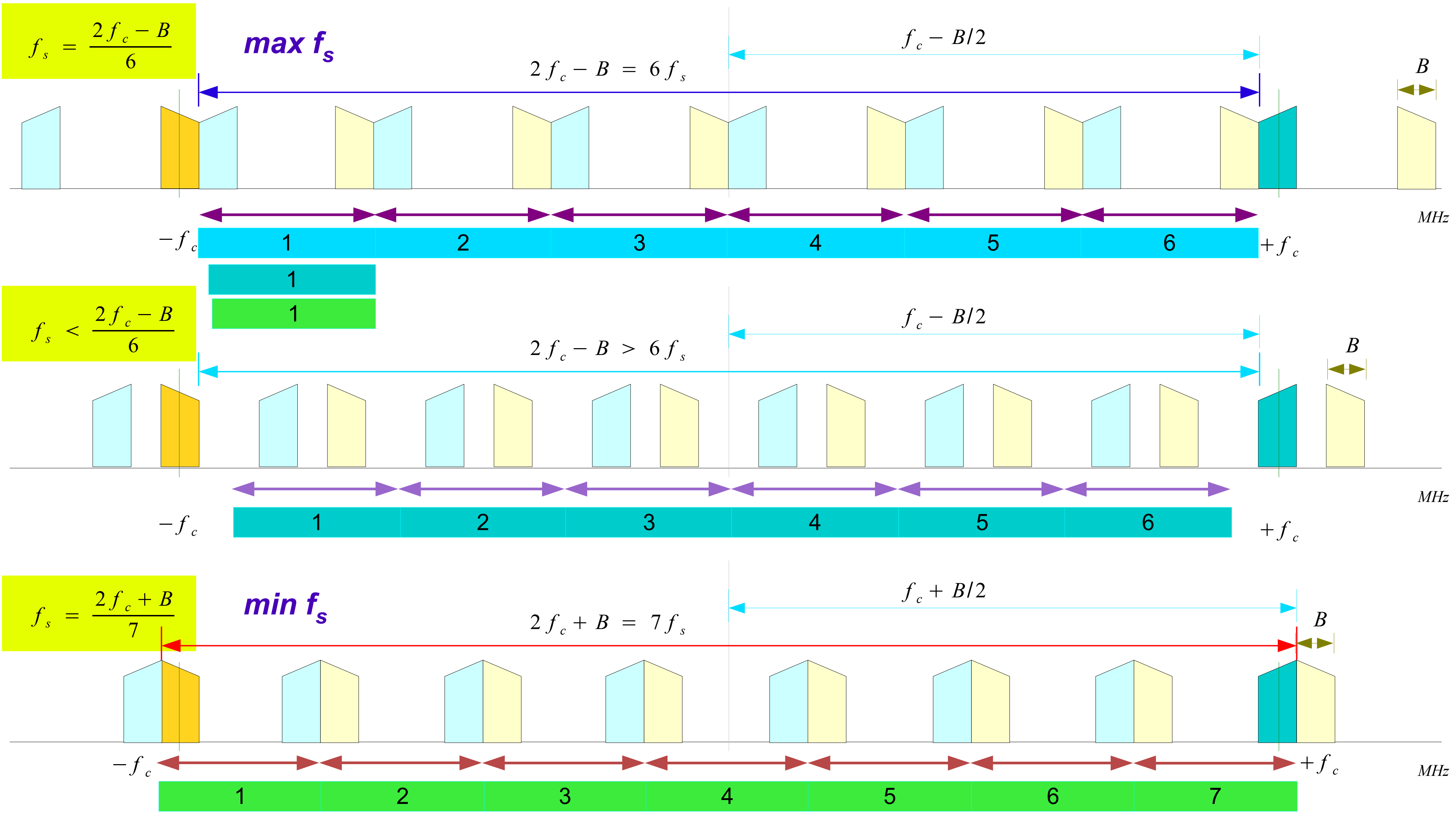
**Max $f_s$ condition**

*$f_s$ can be decreased according to the following condition without introducing aliasing problems*

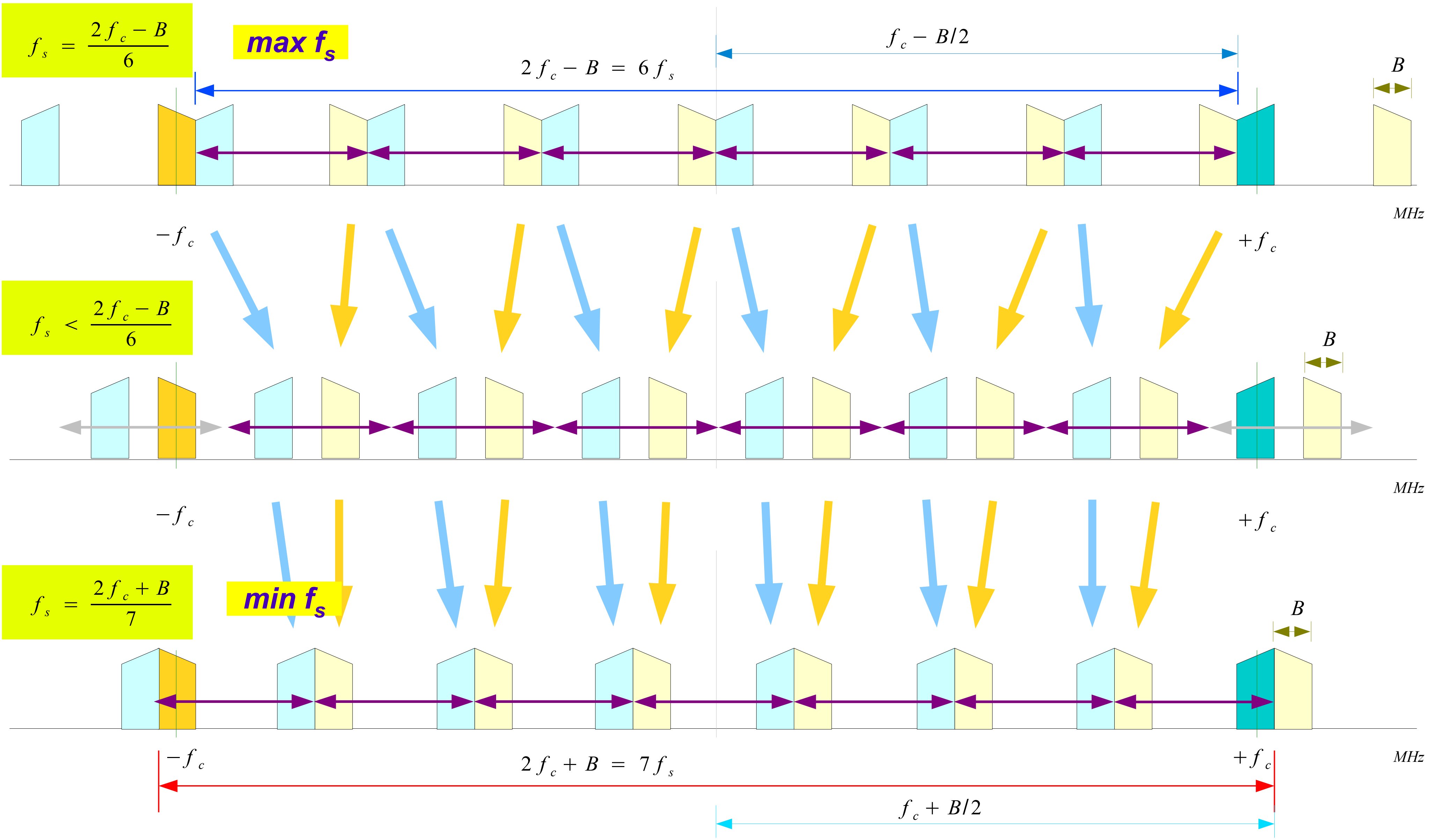
$$2f_c + B = (m+1) \cdot f_s$$

**Min $f_s$ condition**

# Sampling Frequency $f_s$ (3)



# Sampling Frequency $f_s$ (4)



# Range of $f_s$ (1)

| For a given $m$ | $\dfrac{2f_c + B}{m+1} \le f_s \le \dfrac{2f_c - B}{m}$ | Nyquist Criterion | $2B \le f_s$ |
|---|---|---|---|

$f_c = 20\ MHz$

$B = 5\ MHz$

min $f_s$ ← $\dfrac{2f_c + B}{m+1}$; max $f_s$ ← $\dfrac{2f_c - B}{m}$

Optimum Sampling Frequency

$m = 1$ ⇨ $\dfrac{2\cdot 20 + 5}{1+1} = 22.5 \le f_s \le \dfrac{2\cdot 20 - 5}{1} = 35$ ⇨ $f_s = 22.5\ MHz \quad (10 \le f_s)$

$m = 2$ ⇨ $\dfrac{2\cdot 20 + 5}{2+1} = 15 \le f_s \le \dfrac{2\cdot 20 - 5}{2} = 17.5$ ⇨ $f_s = 17.5\ MHz \quad (10 \le f_s)$

$m = 3$ ⇨ $\dfrac{2\cdot 20 + 5}{3+1} = 11.25 \le f_s \le \dfrac{2\cdot 20 - 5}{3} = 11.67$ ⇨ $f_s = 11.25\ MHz \quad (10 \le f_s)$

$m = 4$ ⇨ $\dfrac{2\cdot 20 + 5}{4+1} = 9 \ge \dfrac{2\cdot 20 - 5}{4} = 8.75$ ⇨ X

$m = 5$ ⇨ $\dfrac{2\cdot 20 + 5}{5+1} = 7.5 \ge \dfrac{2\cdot 20 - 5}{5} = 7.0$ ⇨ X

$f_c = 20\ MHz$

$B = 5\ MHz$

# Range of $f_s$ (2)

$$\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}$$

$$\frac{f_c + B/2}{B} = R$$

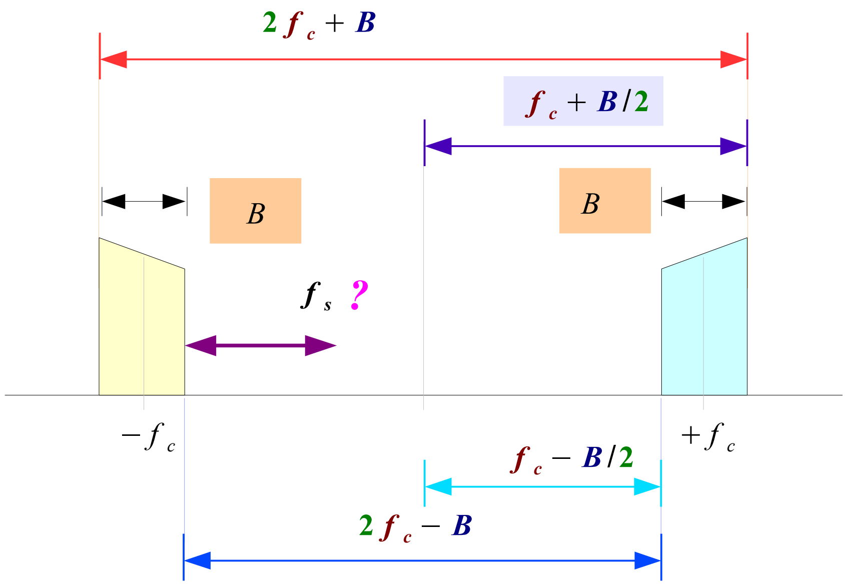
→ $\frac{\text{highest signal frequency}}{\text{bandwidth B}}$

$$\frac{2f_c + B}{(m+1)B} = f(m,R)$$

→ $\frac{\text{minimum sampling rate}}{\text{bandwidth B}}$

$$\frac{2(f_c + B/2)}{(m+1)B} = \frac{2R}{m+1} = f(m,R)$$

| | | | |
|---|---|---|---|
| $m = 1$ | $f(1,R) = R$ | $m = 5$ | $f(5,R) = \frac{1}{3}R$ |
| $m = 2$ | $f(2,R) = \frac{2}{3}R$ | $m = 6$ | $f(6,R) = \frac{2}{7}R$ |
| $m = 3$ | $f(3,R) = \frac{1}{2}R$ | $m = 7$ | $f(7,R) = \frac{1}{4}R$ |
| $m = 4$ | $f(4,R) = \frac{2}{5}R$ | $m = 8$ | $f(8,R) = \frac{2}{9}R$ |



# Range of $f_s$ (3)

$$\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}$$

$$\frac{f_c + B/2}{B} = R$$ → $\frac{\text{highest signal frequency}}{\text{bandwidth B}}$

$$\frac{2f_c + B}{(m+1)B} = f(m, R)$$ → $\frac{\text{minimum sampling rate}}{\text{bandwidth B}}$

$$\frac{2f_c + B}{(m+1)B} = f(m, R) = \frac{2R}{m+1}$$

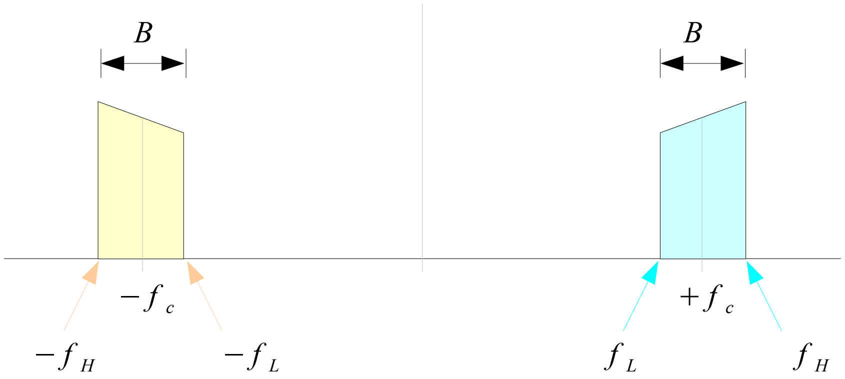
$$m + 1 = k$$

$$f_H = f_c + B/2$$

$$R = f_H / B$$

$$f_{s,min} = \frac{2f_c + B}{m+1} = \frac{2f_H}{k}$$

$$f(m, R) = \frac{2f_H}{kB} = \frac{2R}{k}$$



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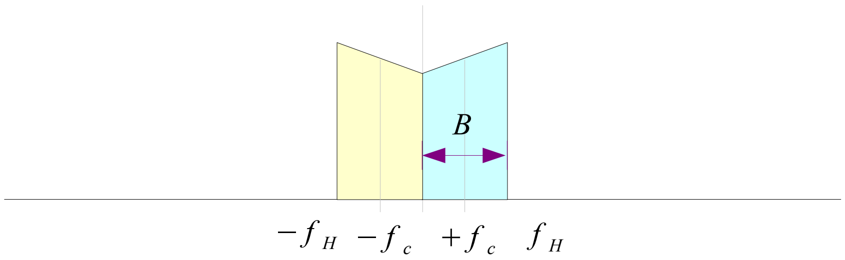
$k = 1 \quad (m = 0)$

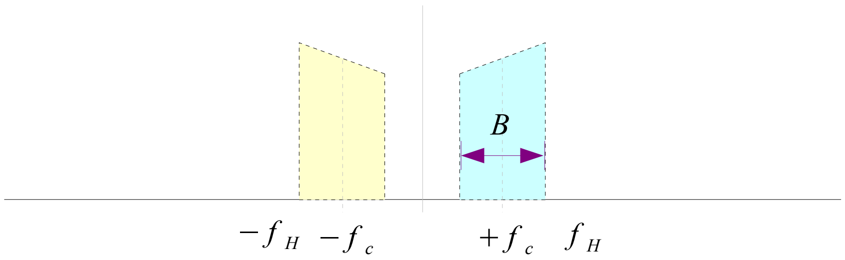



$$f_H = f_c + B/2 = 1\,B$$

$$f_H / B = R = 1$$

$$f_{s,min} = \frac{2 f_H}{k} = \frac{2 f_H}{m+1} = 2\,B$$

$k = 1 \quad (m = 0)$

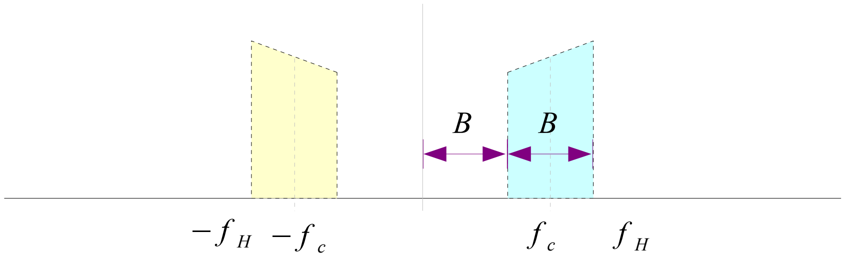



$$f_H = f_c + B/2 = 1.5\,B$$

$$f_H / B = R = 1.5$$

$$f_{s,min} = \frac{2 f_H}{k} = \frac{2 f_H}{m+1} = 3\,B$$

$k = 1 \quad (m = 0)$

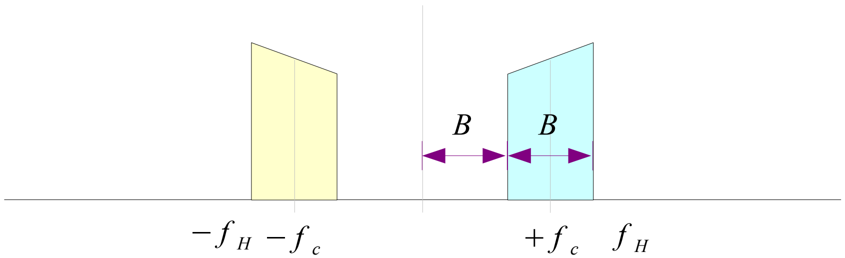



$$f_H = f_c + B/2 = 2\,B$$

$$f_H / B = R = 2$$

$$f_{s,min} = \frac{2 f_H}{k} = \frac{2 f_H}{m+1} = 4\,B$$

$k = 2 \quad (m = 1)$

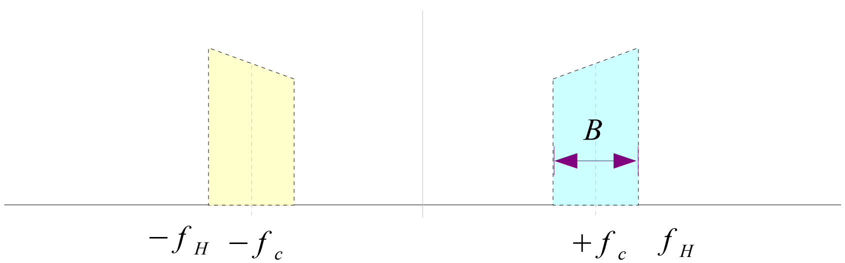



$$f_H = f_c + B/2 = 2B$$

$$f_H / B = R = 2$$

$$f_{s,min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = 2B$$

$k = 2 \quad (m = 1)$

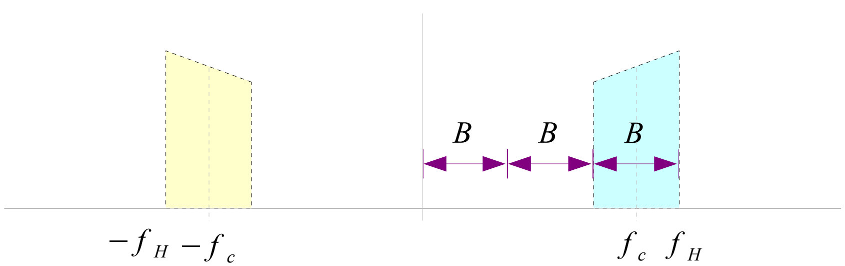



$$f_H = f_c + B/2 = 2.5B$$

$$f_H / B = R = 2.5$$

$$f_{s,min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = 2.5B$$

$k = 2 \quad (m = 3)$

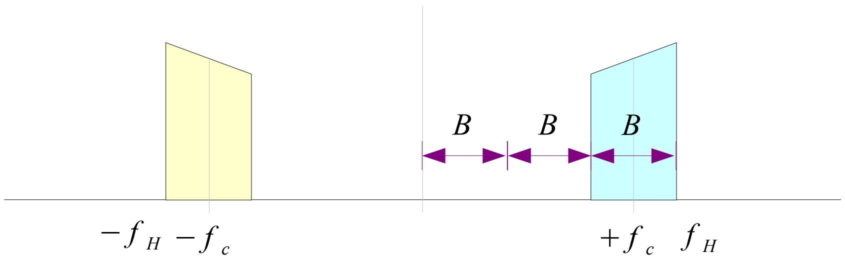



$$f_H = f_c + B/2 = 3B$$

$$f_H / B = R = 3$$

$$f_{s,min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = 3B$$

$k = 3 \quad (m = 2)$

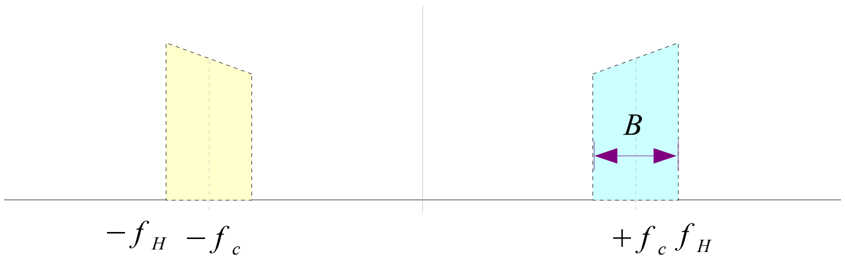



$$f_H = f_c + B/2 = 3B$$

$$f_H / B = R = 3$$

$$f_{s,min} = \frac{2 f_H}{k} = \frac{2 f_H}{m+1} = 2B$$

$k = 3 \quad (m = 2)$

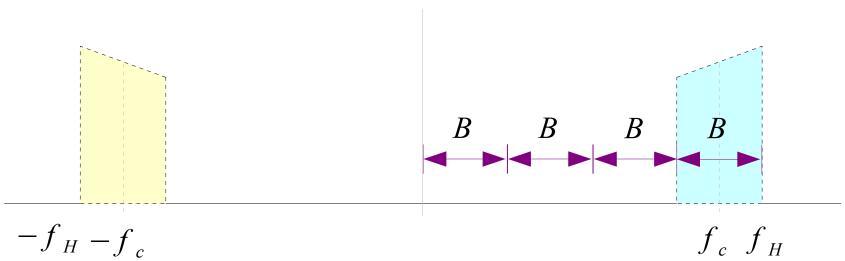



$$f_H = f_c + B/2 = 3.5B$$

$$f_H / B = R = 3.5$$

$$f_{s,min} = \frac{2 f_H}{k} = \frac{2 f_H}{m+1} = \frac{7}{3} B$$

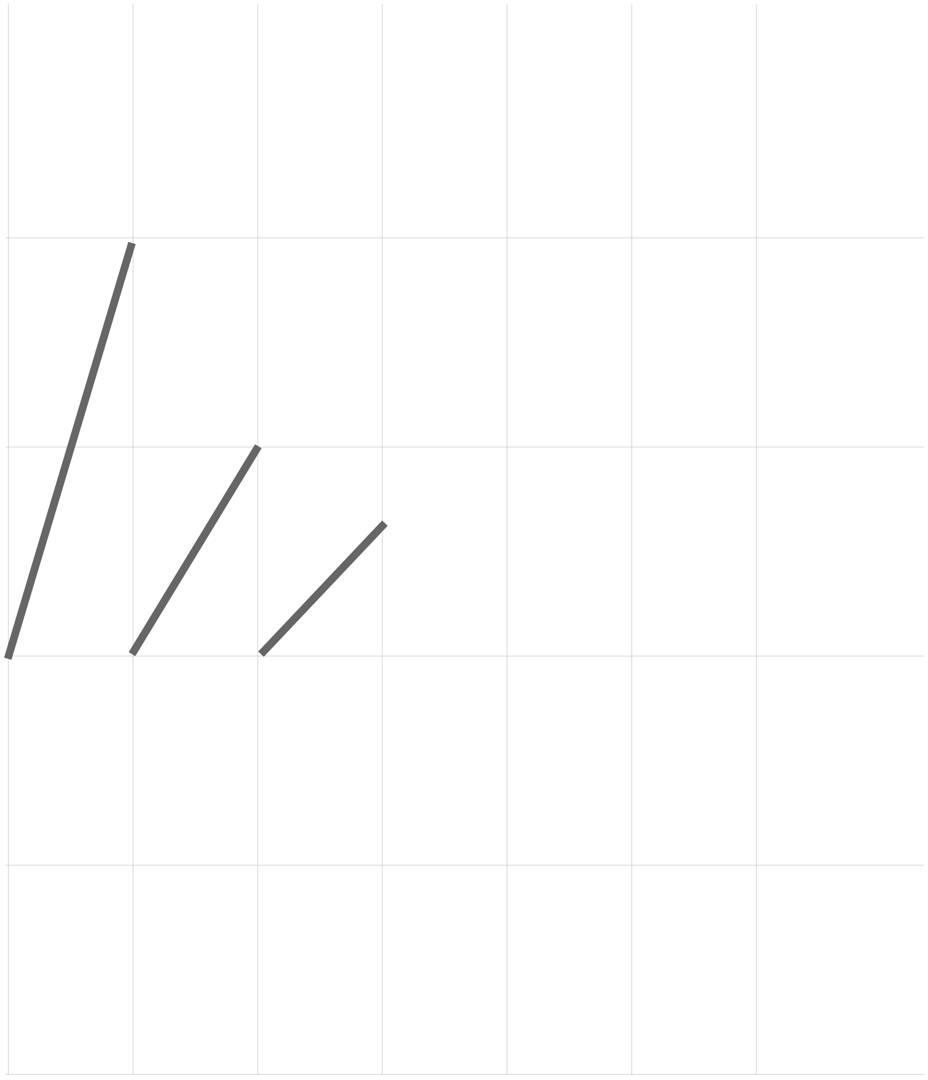
$k = 3 \quad (m = 2)$




$$f_H = f_c + B/2 = 4B$$

$$f_H / B = R = 4$$

$$f_{s,min} = \frac{2 f_H}{k} = \frac{2 f_H}{m+1} = \frac{8}{3} B$$



## References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A “graphical interpretation” of the DFT and FFT, by Steve Mann
[4] R. G. Lyons, Understanding Digital Signal Processing, 1997