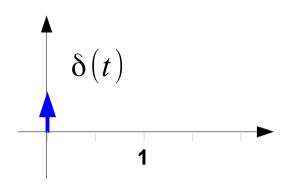
Convolution (1A)

•

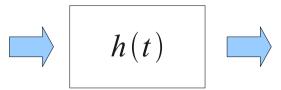
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Please send corrections (or suggestions) to youngwlim@hotmail.com.
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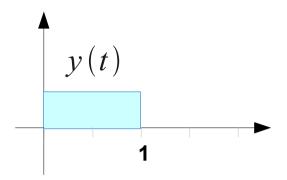
Convolution: delayed response of h(t)

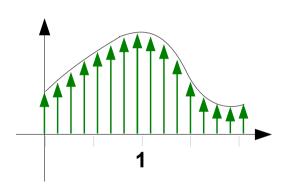
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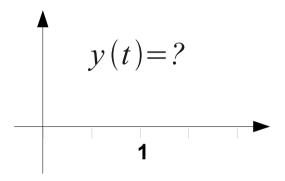


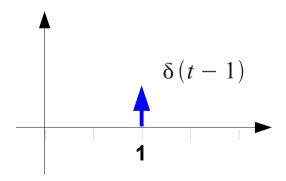




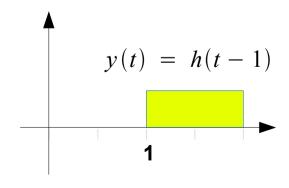


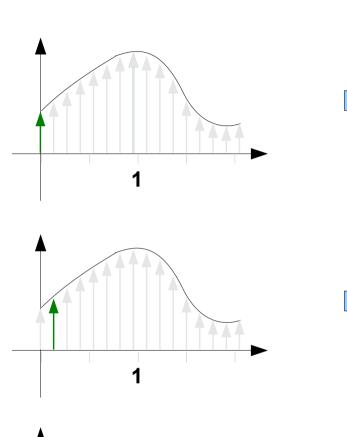


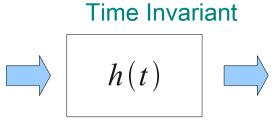


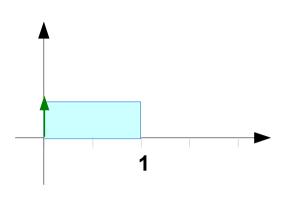


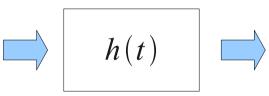




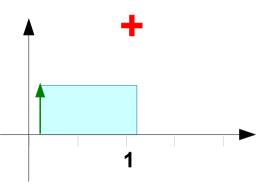


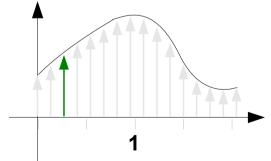


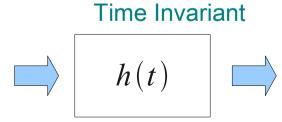


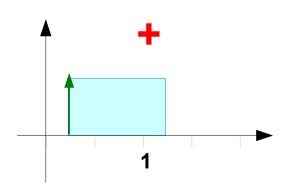


Time Invariant



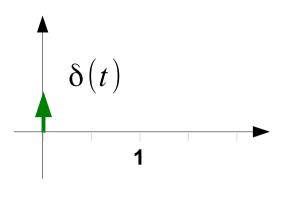






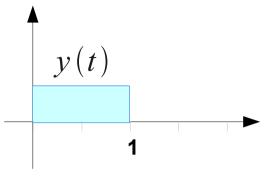
Convolution: delayed response of h(t)

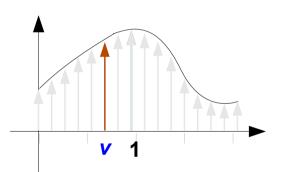
(3)











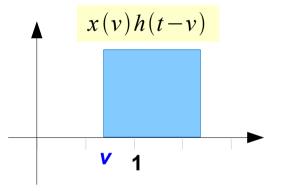
$$\delta(t-v)$$
 $h(t-v)$

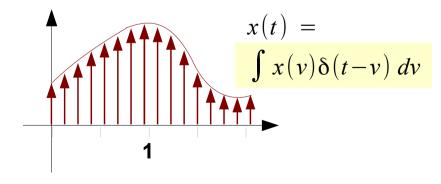
$$x(v) \delta(t-v)$$
 $x(v) h(t-v)$



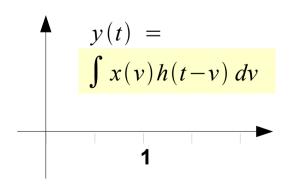
$$x(v) h(t-v)$$

input value at time v $\rightarrow x(v)$ delayed impulse response \rightarrow x(v) h(t - v)









Convolution: Commutative Law

$$x(t) \Longrightarrow h(t) \Longrightarrow y(t)$$

$$x(v) \qquad h(v) \xrightarrow{\text{Flip}} h(-v) \xrightarrow{\text{Shift}} h(t-v)$$

$$\int x(v)h(t-v) dv = y(t)$$

$$\int h(v)x(t-v) dv = y(t)$$

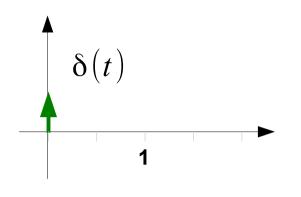
$$h(v) \qquad x(v) \xrightarrow{\text{Flip}} x(-v) \xrightarrow{\text{Shift}} x(t-v)$$

$$h(t) \Longrightarrow x(t) \Longrightarrow y(t)$$

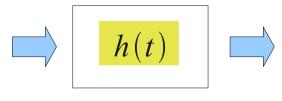
$$LTI$$

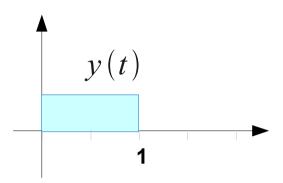
Convolution: delayed response of x(t)

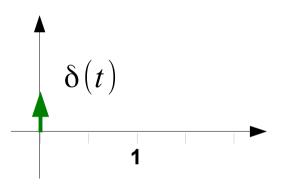
(1)



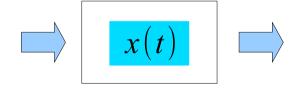


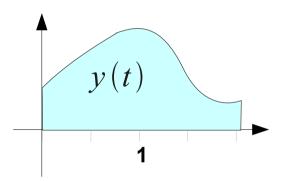


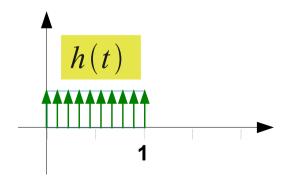




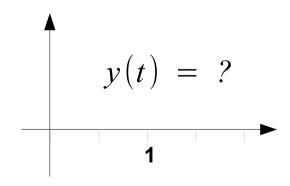


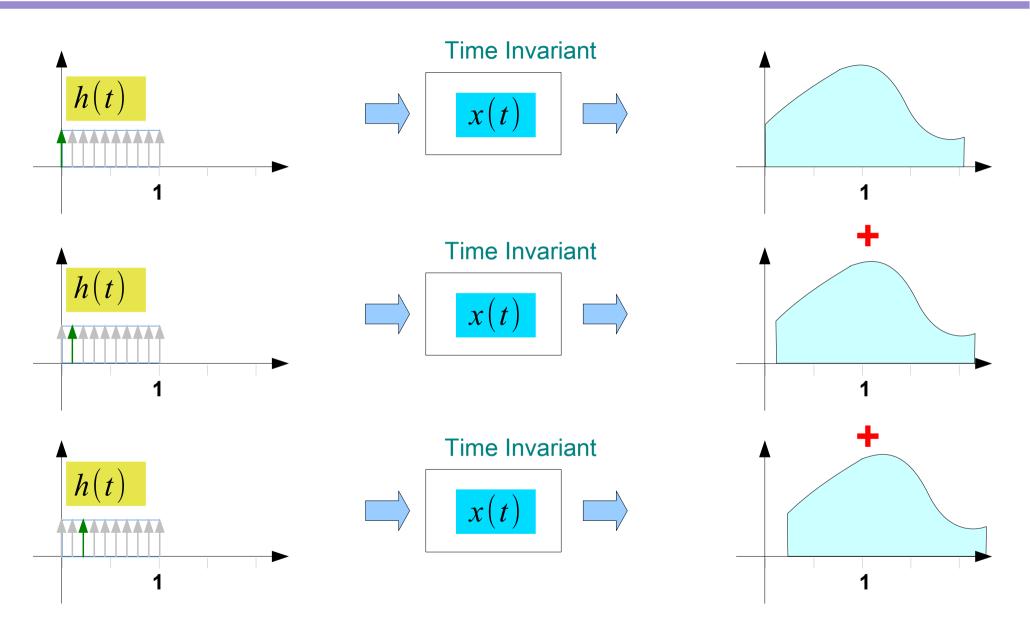






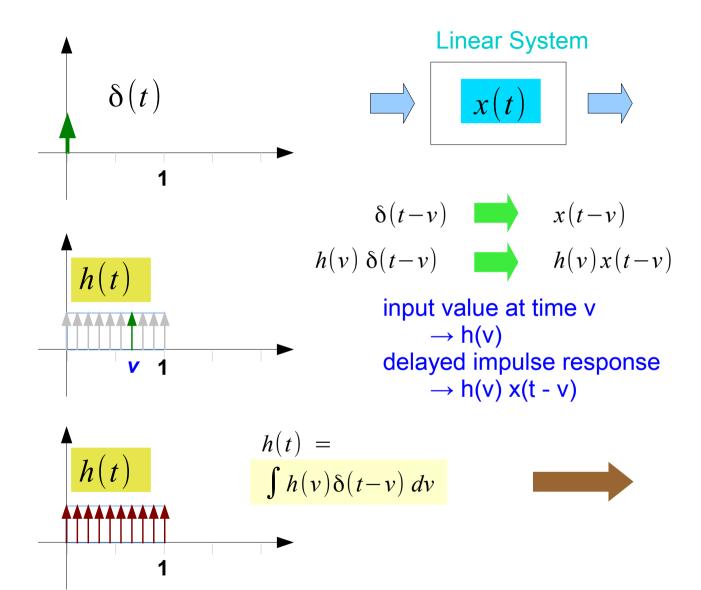
delayed response by 1

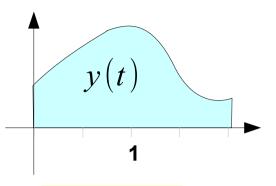


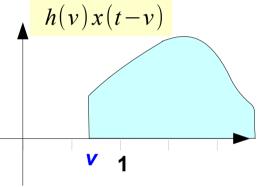


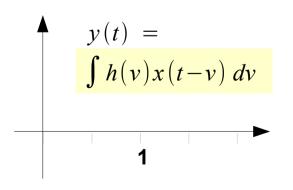
Convolution: delayed response of x(t)

(3)

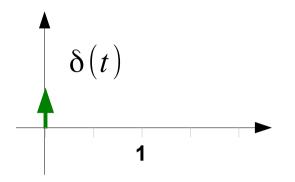




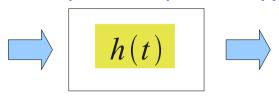




FIR and IIR



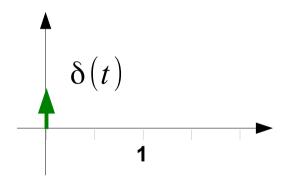




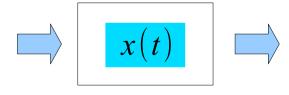
y(t)

Finite Impulse Response

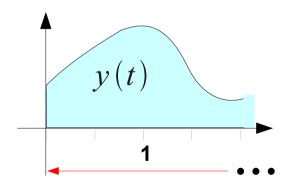
Finite Duration



Impulse response x(t)



Infinite Impulse Response



Infinite Duration

Frequency Response

$$y(t) = h(t) * x(t) = \int_{-\infty}^{+\infty} h(\tau) x(t - \tau) d\tau$$

$$x(t) = A e^{j\Phi} e^{j\omega t} \qquad h(t) \qquad y(t) = H(jw) \cdot A e^{j\Phi} e^{j\omega t}$$

$$y(t) = \int_{-\infty}^{+\infty} h(\tau) A e^{j\Phi} e^{jw(t-\tau)} d\tau$$

$$= \int_{-\infty}^{+\infty} h(\tau) A e^{j\Phi} e^{jwt} e^{-j\omega\tau} d\tau$$

$$= A e^{j\Phi} e^{jwt} \cdot \int_{-\infty}^{+\infty} h(\tau) e^{-j\omega\tau} d\tau$$

$$= x(t) \cdot H(jw)$$

Frequency Response

$$y(t) = h(t) * x(t) = \int_{-\infty}^{+\infty} h(\tau) x(t - \tau) d\tau$$

$$x(t) \qquad h(t) \qquad y(t)$$

$$\delta(t) \qquad h(t) \qquad h(t)$$

$$A e^{j\Phi} e^{j\Theta t} \qquad h(t) \qquad H(j\Theta) A e^{j\Phi} e^{j\Theta t}$$

$$single frequency component : \Theta$$

$$H(j\omega) = \int_{-\infty}^{+\infty} h(\tau) e^{-j\omega\tau} d\tau$$

References

- [1] http://en.wikipedia.org/
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003