Group & Phase Velocities (2B)

- 3-D Group & Phase Velocities

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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Phase Velocity (1)

At any point in space $\boldsymbol{x}$

$$s(\boldsymbol{x}, t) = A e^{j(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})}$$

oscillates with a temporal frequency ω

During one period of oscillation $T = \dfrac{2\pi}{\omega}$

in the direction of $\boldsymbol{k}$

The wave propagates forward

By one wavelength $\lambda = \dfrac{2\pi}{|\boldsymbol{k}|}$

Phase Velocity (2)

The speed of propagation

The speed at which planes of constant phase $\boldsymbol{k} \cdot \boldsymbol{x} = c$

Phase Velocity $|\boldsymbol{v}_p| = \frac{\lambda}{T} = \frac{\omega}{|\boldsymbol{k}|}$

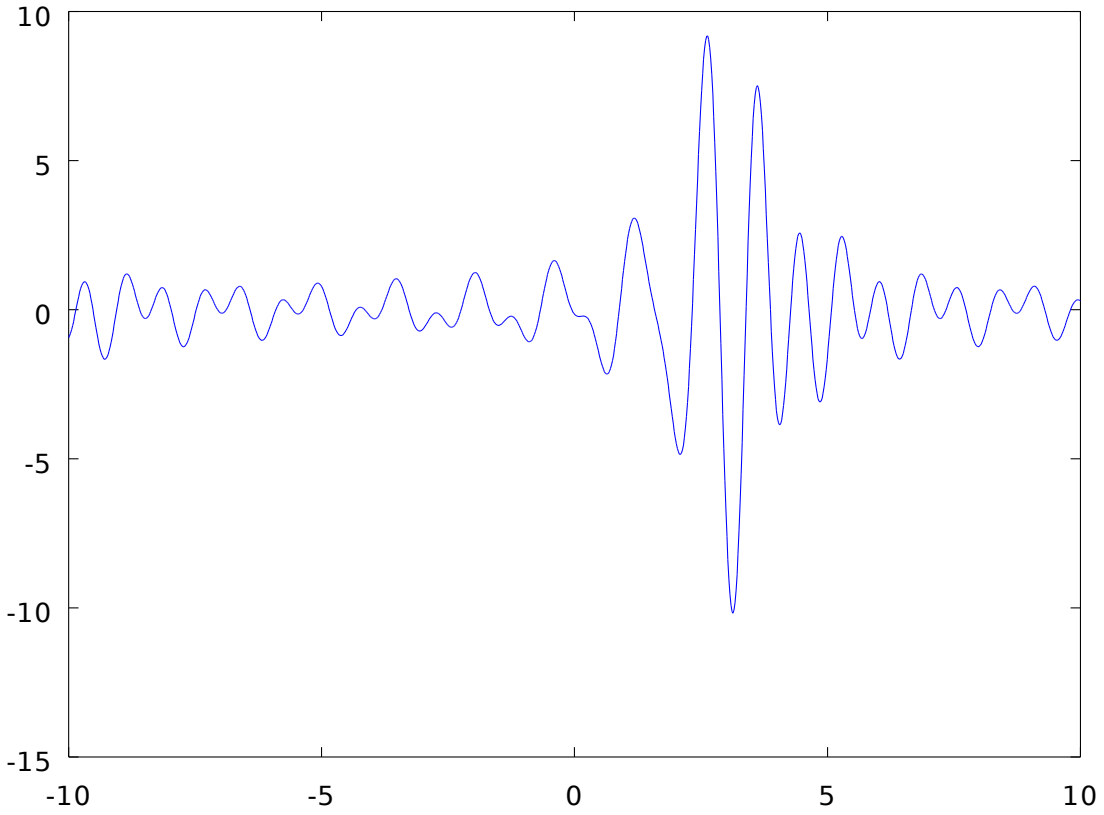
If the directions are the same $\boldsymbol{v}_p \quad \boldsymbol{k} \qquad \boldsymbol{v}_p = \frac{\omega \boldsymbol{k}}{|\boldsymbol{k}|^2}$

$$|\boldsymbol{v}_p| = \frac{\omega}{|\boldsymbol{k}|}$$

Acoustic Phonon Dispersion

$$\omega(k) = 2\sqrt{\frac{\gamma}{M}}\left|\sin\frac{k\,a}{2}\right|$$

Acoustic branch of vibrations in a crystal




```
x = linspace(-10, +10, 1000);

y = zeros(1, 1000);
for k= 1.0:0.1:2.0
   y = y + cos(4*k*(x-k));
end
plot(x, y);
```

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] http://www.mathpages.com/, Phase, Group, and Signal Velocity
[4] R. Barlow, www.hep.man.ac.uk/u/roger/PHYS10302/lecture15.pdf
[5] P. Hofmann, www.philiphofmann.net/book_material/notes/groupphasevelocity.pdf