

Group & Phase Velocities (2B)

- 3-D Group & Phase Velocities

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Phase Velocity (1)

At any point in space \mathbf{x}

$$s(\mathbf{x}, t) = A e^{j(\omega t - \mathbf{k} \cdot \mathbf{x})}$$

oscillates with a temporal frequency ω

During one period of oscillation $T = \frac{2\pi}{\omega}$

in the direction of \mathbf{k}

The wave propagates forward

By one wavelength $\lambda = \frac{2\pi}{|\mathbf{k}|}$

Phase Velocity (2)

The speed of propagation

The speed at which planes of constant phase $\mathbf{k} \cdot \mathbf{x} = c$

Phase Velocity

$$|\mathbf{v}_p| = \frac{\lambda}{T} = \frac{\omega}{|\mathbf{k}|}$$

If the directions are the same

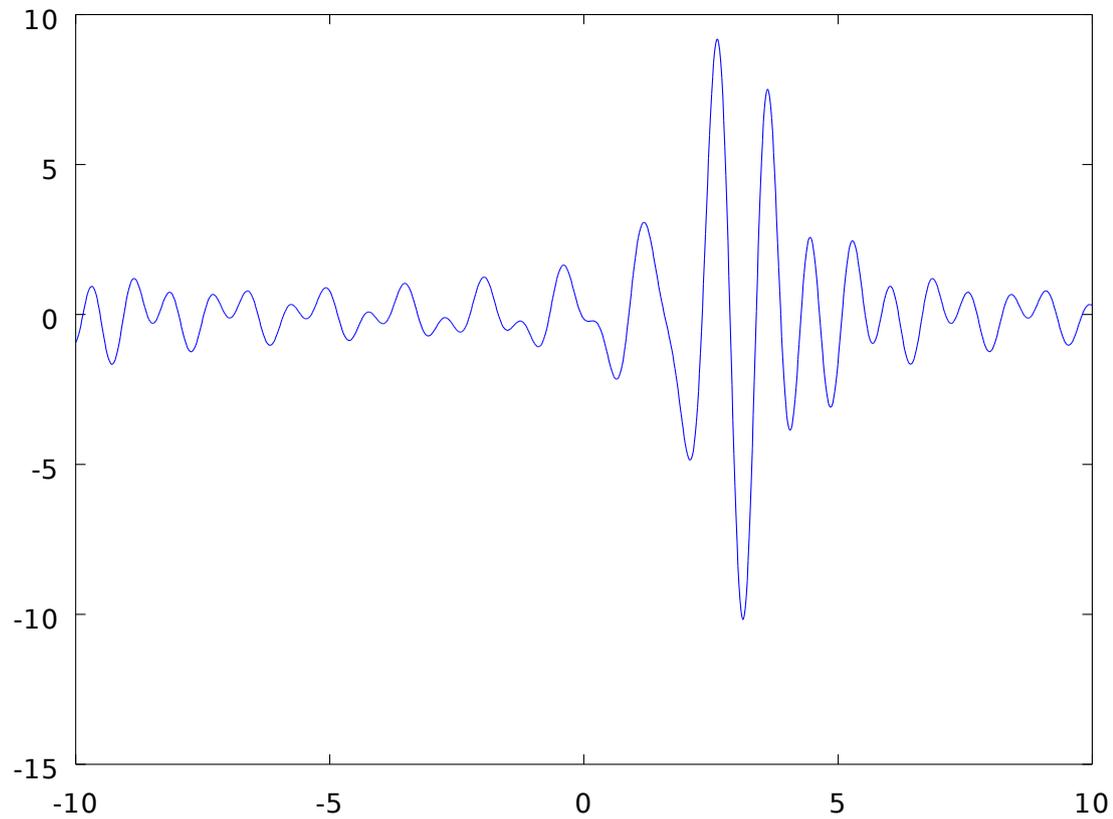
$$\mathbf{v}_p \parallel \mathbf{k} \quad \mathbf{v}_p = \frac{\omega \mathbf{k}}{|\mathbf{k}|^2}$$

$$|\mathbf{v}_p| = \frac{\omega}{|\mathbf{k}|}$$

Acoustic Phonon Dispersion

$$\omega(k) = 2\sqrt{\frac{\gamma}{M}} \left| \sin \frac{ka}{2} \right|$$

Acoustic branch of vibrations in a crystal



```
x = linspace(-10, +10, 1000);
```

```
y = zeros(1, 1000);
```

```
for k= 1.0:0.1:2.0
```

```
    y = y + cos(4*k*(x-k));
```

```
end
```

```
plot(x, y);
```

References

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