Anti-aliasing Prefilter (6B)

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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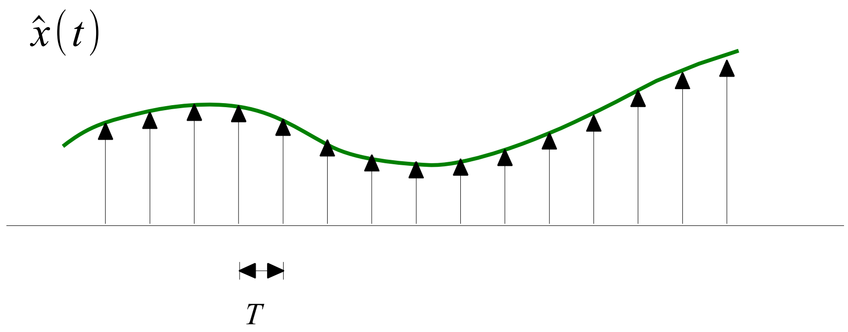
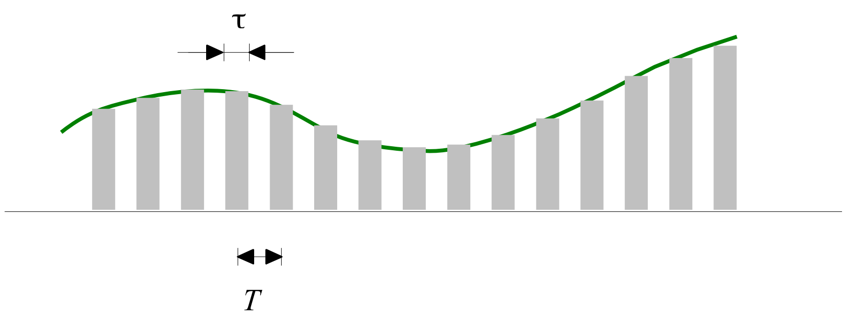
Sampler



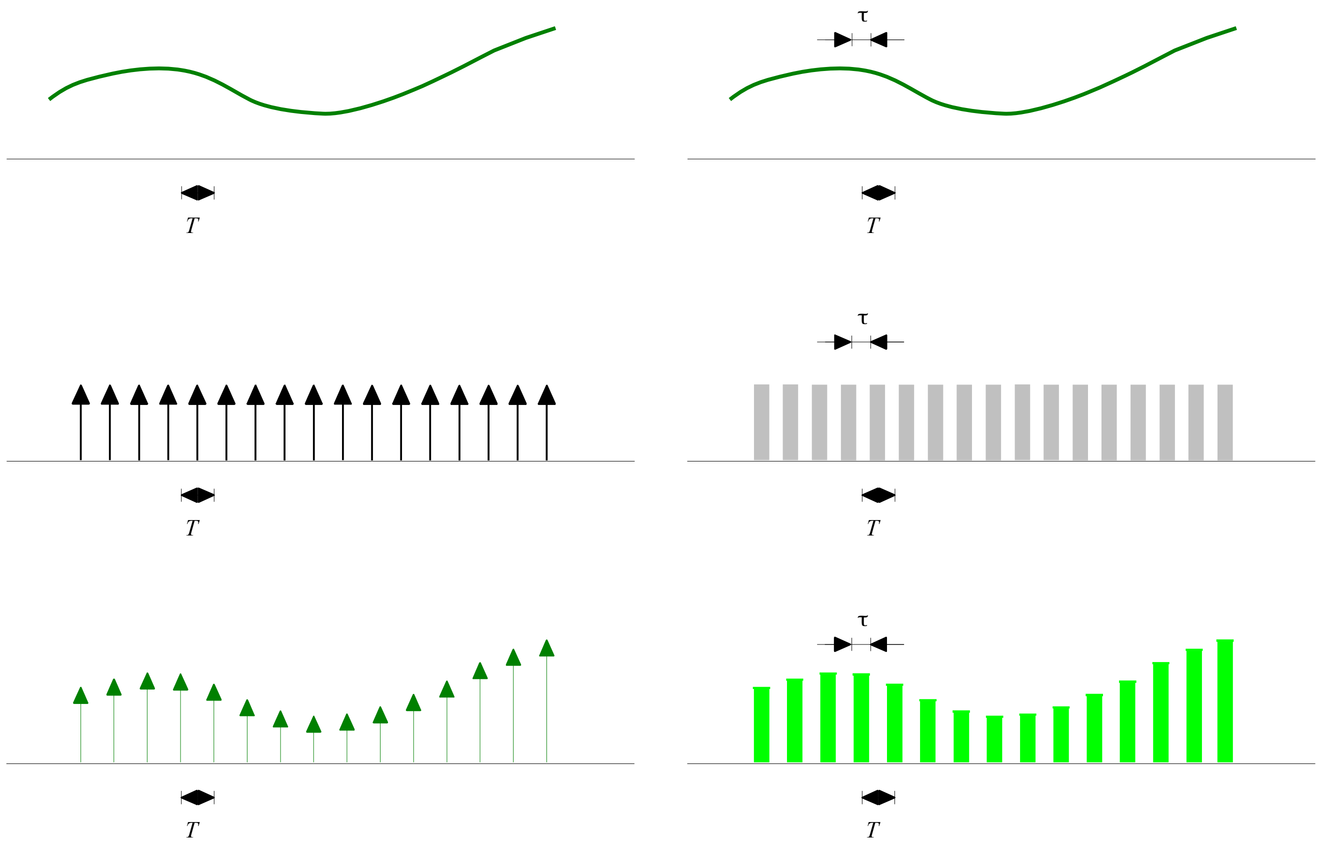


$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

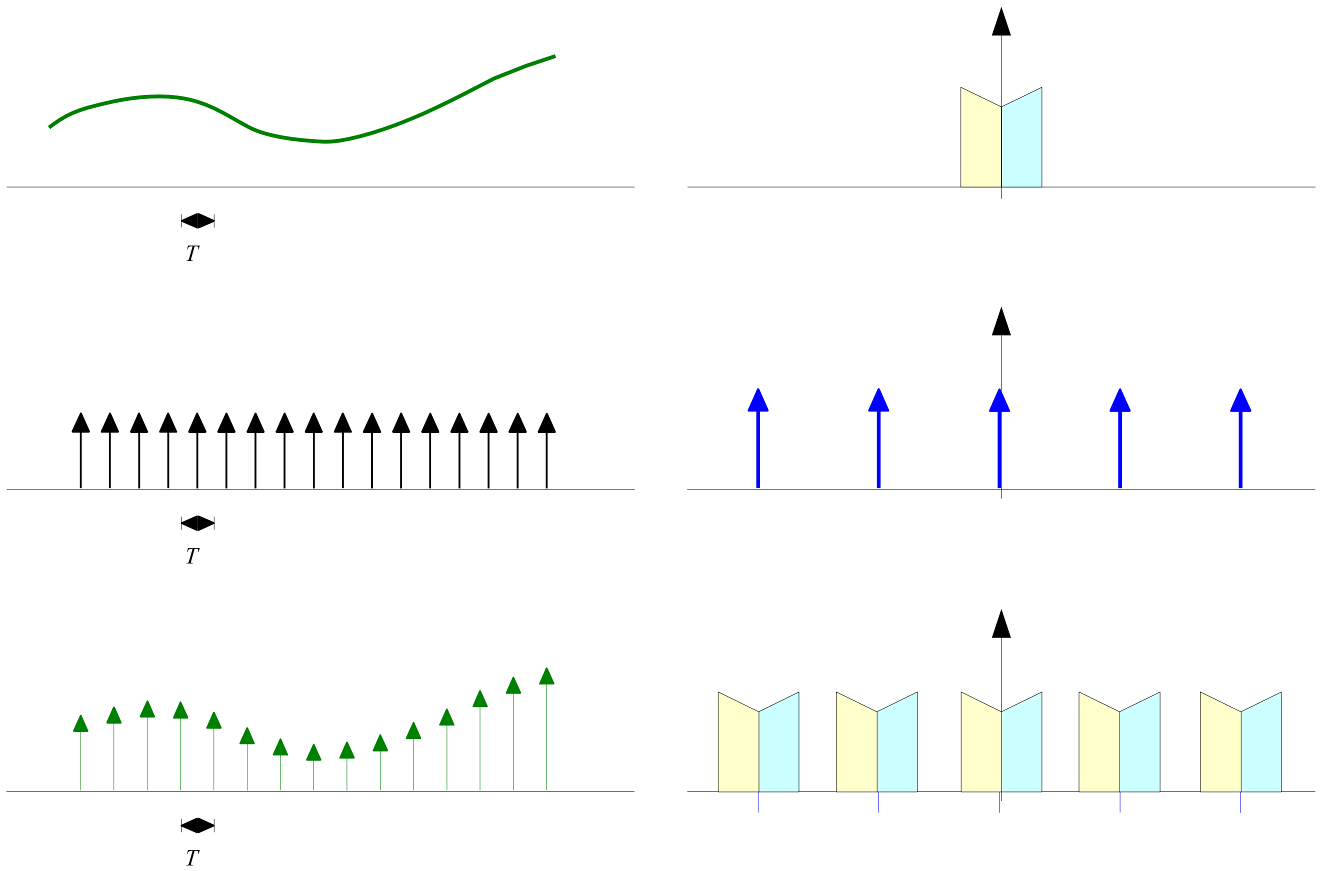
$$\hat{x}(t) \approx \sum_{n=-\infty}^{+\infty} x(nT)\,p(t-nT)$$

$$\hat{X}(f) = \int_{-\infty}^{+\infty} \hat{x}(t)\, e^{-j2\pi f t}\, dt$$

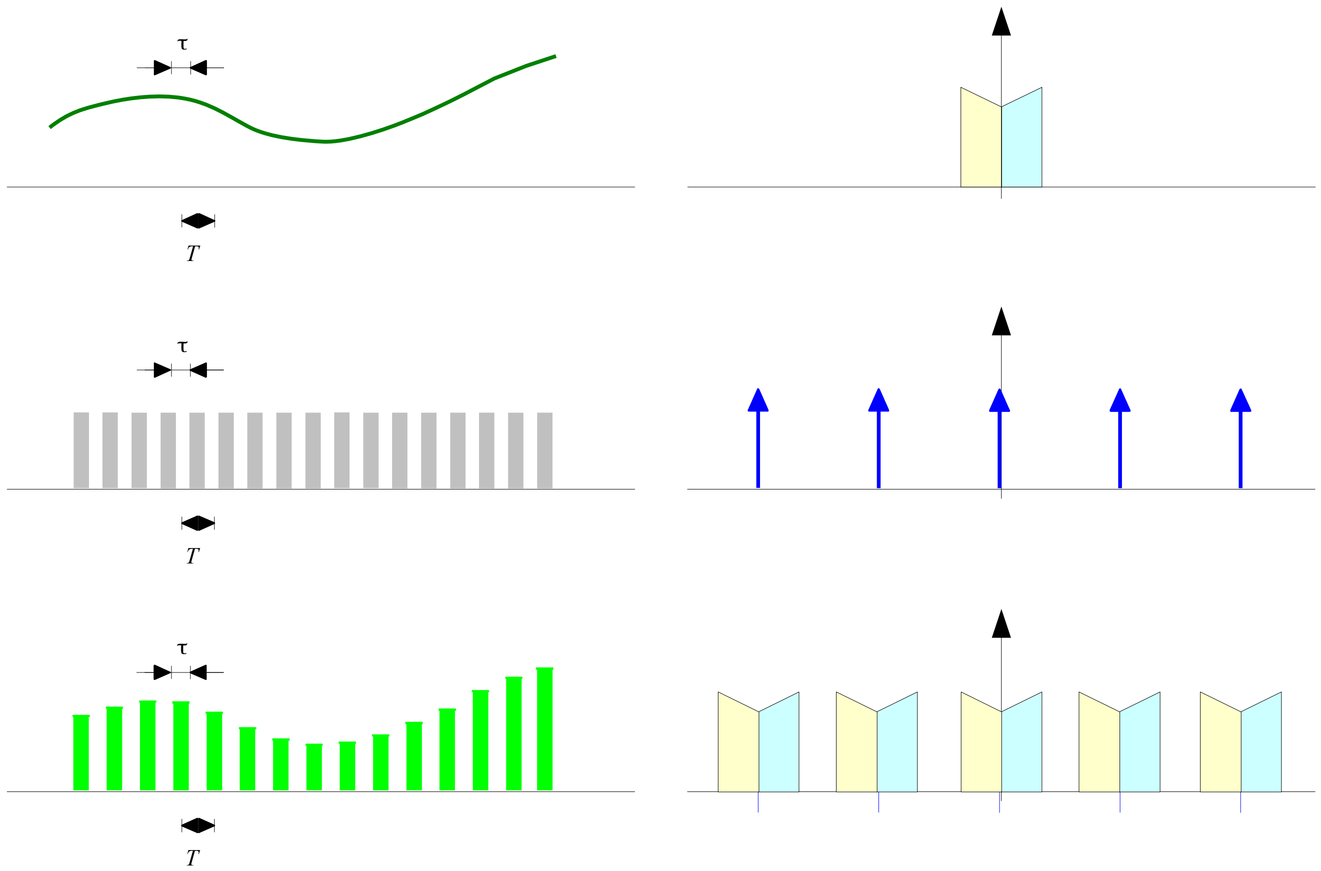
Sampler



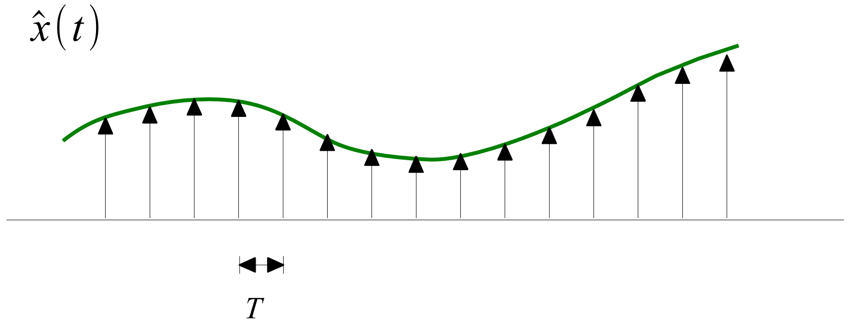
Sampler



Sampler



Discrete Time Fourier Transform



$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

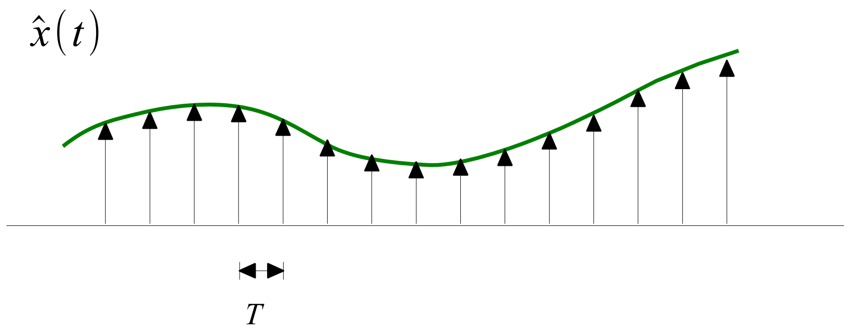
$$\hat{X}(f) = \int_{-\infty}^{+\infty} \hat{x}(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)\, e^{-j2\pi f t}\, dt$$

$$= \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n}\, dt$$

$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n}\, dt$$

Fourier Series



$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n}\, dt$$

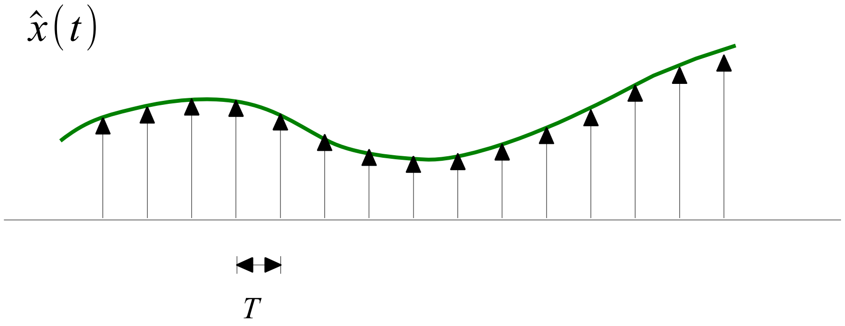
$$x(nT) = \frac{1}{f_s}\int_{-f_s/2}^{+f_s/2} \hat{X}(f)\, e^{+j2\pi f T n}\, df$$

$$= \int_{-\pi}^{+\pi} \hat{X}(\omega)\, e^{+j\omega n} \frac{d\omega}{2\pi}$$

$$\omega = 2\pi f / f_s$$

$$\frac{d\omega}{2\pi} = \frac{df}{f_s}$$

Numerical Approximation



$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT)\,e^{-j2\pi f T n}\,dt$$

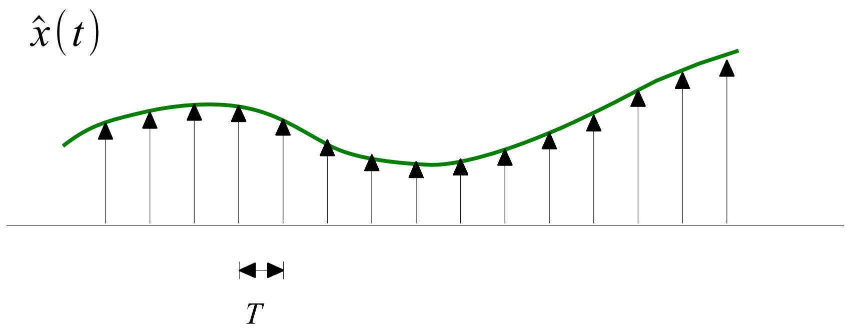
$$X(f) = \int_{-\infty}^{+\infty} x(t)\,e^{+j2\pi f t}\,dt$$

$$\approx \sum_{n=-\infty}^{+\infty} x(nT)\,e^{-j2\pi f T n}\cdot T$$

$$X(f) \approx T\,\hat{X}(f)$$

$$X(f) = \lim_{T\to 0} T\,\hat{X}(f)$$

Spectrum Replication (1)



$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

$$= x(t) \sum_{n=-\infty}^{+\infty} \delta(t-nT) = x(t)\,s(t)$$

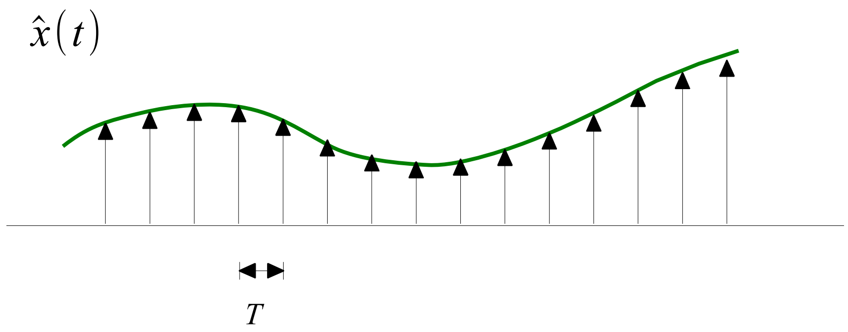
$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n}\, dt$$

$$s(t) = \sum_{n=-\infty}^{+\infty} \delta(t-nT) = \frac{1}{T} \sum_{m=-Infintiy}^{+\infty} e^{+j2\pi m f_s t}$$

$$\hat{x}(t) = x(t)\,s(t) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} x(t)\, e^{+j2\pi m f_s t}$$

$$\hat{X}(f) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} X(f - m f_s)$$

Spectrum Replication (2)



$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n}\, dt$$

$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

$$= x(t) \sum_{n=-\infty}^{+\infty} \delta(t-nT) = x(t)\,s(t)$$

$$\hat{X}(f) = \int_{-\infty}^{+\infty} X(f-f')\,S(f')\,df'$$

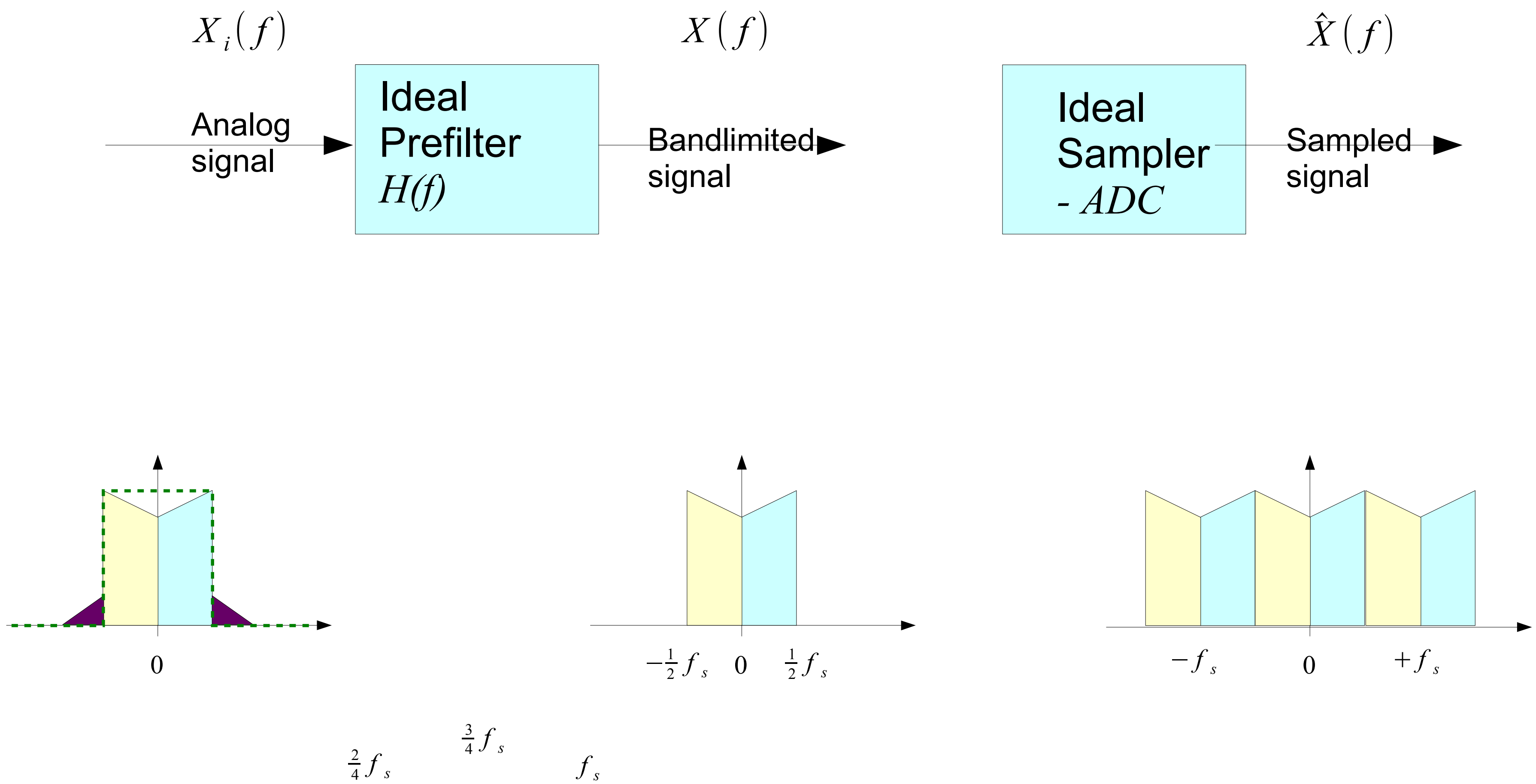
$$= \frac{1}{T} \sum_{m=-\infty}^{+\infty} \int_{-\infty}^{+\infty} X(f-f')\,\delta(f'-m f_s)\,df'$$

$$s(t) = \sum_{n=-\infty}^{+\infty} \delta(t-nT) = \frac{1}{T} \sum_{m=-Infintiy}^{+\infty} e^{+j2\pi m f_s t}$$

$$S(f) = \frac{1}{T} \sum_{m=-\infty}^{+\infty} \delta(f-m f_s)$$

$$\hat{x}(t) = x(t)\,s(t) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} x(t)\, e^{+j2\pi m f_s t}$$

$$\hat{X}(f) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} X(f-m f_s)$$

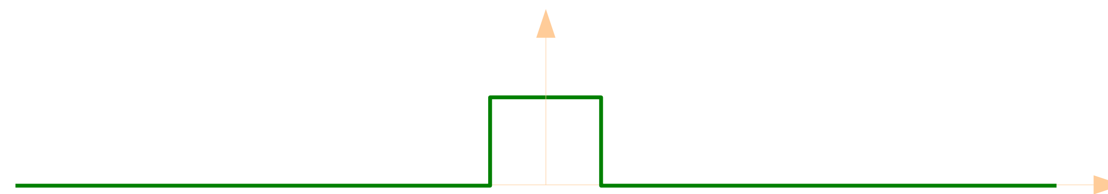


$X_i(f)$
Analog signal
Ideal Prefilter H(f)
$X(f)$
Bandlimited signal
Ideal Sampler - ADC
$\hat{X}(f)$
Sampled signal
0
$-\frac{1}{2}f_s$
0
$\frac{1}{2}f_s$
$-f_s$
0
$+f_s$
$\frac{2}{4}f_s$
$\frac{3}{4}f_s$
f_s

CTFT and CTFS (1)

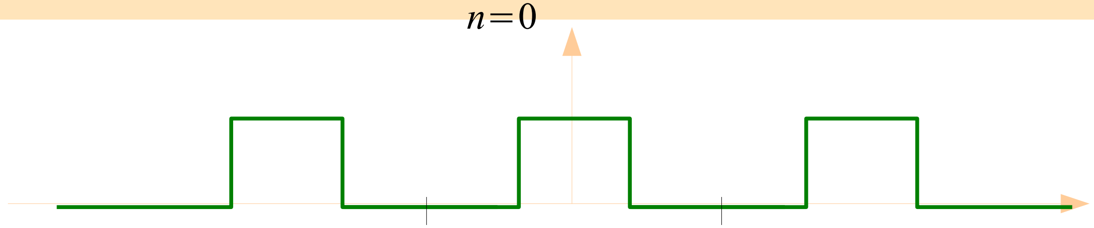
Continuous Time Fourier Transform — Aperiodic Continuous Time Signal

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \quad \Longleftrightarrow \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$



Continuous Time Fourier Series — Periodic Continuous Time Signal

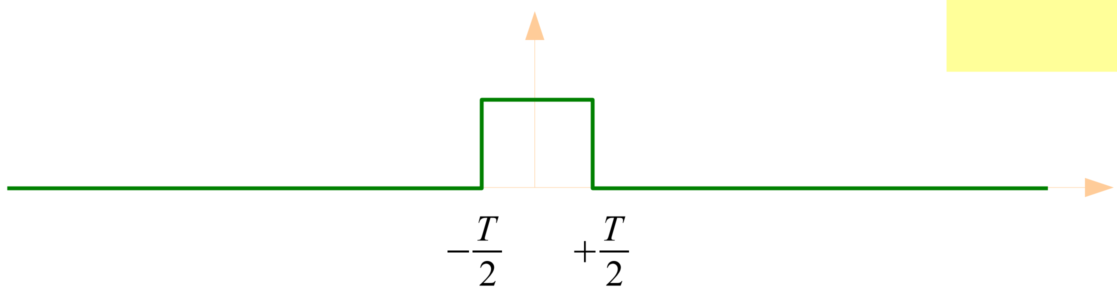
$$C_k = \frac{1}{T} \int_{0}^{T} x(t)\, e^{-jk\omega_0 t}\, dt \quad \Longleftrightarrow \quad x(t) = \sum_{n=0}^{\infty} C_k\, e^{+jk\omega_0 t}$$



CTFT and CTFS (2)

Aperiodic Continuous Time Signal

Continuous Time Fourier Transform



$$x(t)$$

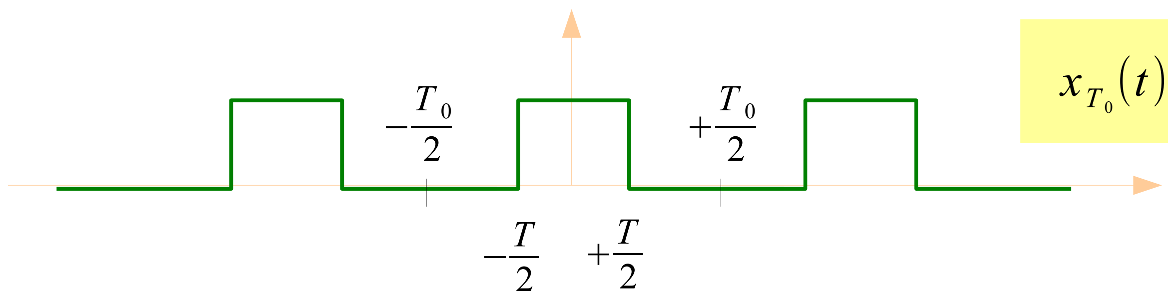
$$\text{As } T_0 \to \infty,$$

$$x_{T_0}(t) \to x(t)$$

$$\omega_0 = \frac{2\pi}{T_0} \to 0$$

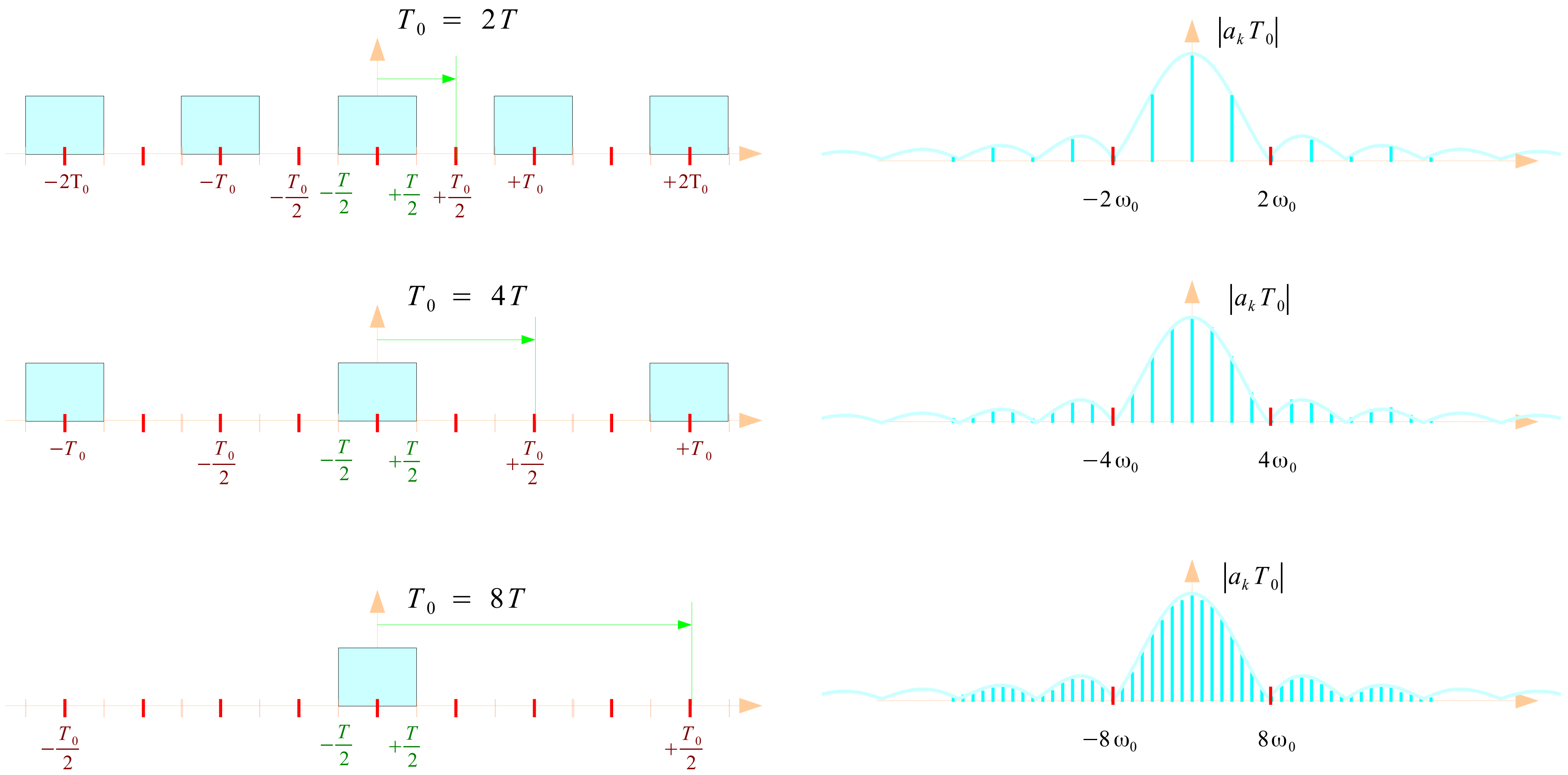
Periodic Continuous Time Signal

Continuous Time Fourier Series



$$x_{T_0}(t) = \sum_{n=-\infty}^{+\infty} x(t - nT_0)$$

CTFT and CTFS (3)



References

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