DLTI Difference Equation

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Causal LTI Systems (1)

$$a_N y[n-N]+\cdots+a_1 y[n-1]+a_0 y[n] = b_M x[n-M]+\cdots+b_1 x[n-1]+b_0 x[n]$$

$$y[n+N] + a_1 y[n+N-1] + \cdots + a_{N-1} y[n+1] + a_N y[n]$$
$$= b_{N-M} x[n+M] + b_{N-M+1} x[n+M-1] + \cdots + b_{N-1} x[n+1] + b_N x[n]$$

$$y[n+N] + a_1 y[n+N-1] + \cdots + a_{N-1} y[n+1] + a_N y[n]$$
$$= b_0 x[n+M] + b_1 x[n+M-1] + \cdots + b_{N-1} x[n+1] + b_N x[n]$$

$$y[n]+a_1 y[n-1]+\cdots+a_{N-1} y[n-N+1]+a_N y[n-N] = b_0 x[n]+b_1 x[n-1]+\cdots+b_{N-1} x[n-N+1]+b_N x[n-N]$$

$$(E^N+a_1 E^{N-1}+\cdots+a_{N-1} E+a_N) y(t) = (b_0 E^M+b_{N-M+1} E^{M-1}+\cdots+b_{N-1} E+b_N) x(t)$$

$$Q(E) y(t) = P(E) x(t)$$

Closed Form h[n] (1)

$$y[n]+a_1 y[n-1]+\cdots+a_{N-1} y[n-N+1]+a_N y[n-N] = b_0 x[n]+b_1 x[n-1]+\cdots+b_{N-1} x[n-N+1]+b_N x[n-N]$$

$$(E^N+a_1 E^{N-1}+\cdots+a_{N-1} E+a_N) y(t) = (b_0 E^M+b_{N-M+1} E^{M-1}+\cdots+b_{N-1} E+b_N) x(t)$$

$$Q(E) y(t) = P(E) x(t)$$

h[n] : system response to input δ[n]

When n < 0, h[n] = 0

When n > 0, h[n] must be made up of characteristic modes

When the input is zero, only the characteristic modes can be sustained

When n = 0, it may have non-zero value A_0

$$h[n] = A_0 \delta[n] + y_c[n] u[n]$$

linear combination of the characteristic modes

Closed Form h[n] (2)

$$y[n]+a_1 y[n-1]+\cdots+a_{N-1} y[n-N+1]+a_N y[n-N] \;=\; b_0 x[n]+b_1 x[n-1]+\cdots+b_{N-1} x[n-N+1]+b_N x[n-N]$$

$$(E^N+a_1 E^{N-1}+\cdots+a_{N-1} E+a_N)\, y(t) \;=\; (b_0 E^M+b_{N-M+1} E^{M-1}+\cdots+b_{N-1} E+b_N)\, x(t)$$

$$Q(E)\, y(t) \;=\; P(E)\, x(t)$$

$$Q(E)\, y(t) \;=\; P(E)\, x(t) \;\Rightarrow\; Q(E)\, h(t) \;=\; P(E)\, \delta(t)$$

causal h[n]

$$h[n] \;=\; \underline{A_0 \delta[n] \;+\; y_c[n]\, u[n]}$$

initial condition $h[-1]=h[-2]=\cdots=h[-N]=0$

$$Q(E)\,(\underline{A_0\delta[n] \;+\; y_c[n]\, u[n]}) \;=\; P(E)\,\delta(t)$$

y_c is made up of characteristic modes

$$Q(E)\,(y_c[n]\, u[n]) \;=\; 0$$

$$Q(E)\,(A_0\delta[n]) \;=\; P(E)\,\delta(t)$$

$$A_0(\delta[n]+a_1\delta[n-1]+\cdots+a_{N-1}\delta[n-N+1]+a_N\delta[n-N]) \;=\; b_0\delta[n]+b_1\delta[n-1]+\cdots+b_{N-1}\delta[n-N+1]+b_N\delta[n-N]$$

n=0

$$A_0\, a_N \;=\; b_N \qquad A_0 \;=\; \frac{a_N}{b_N}$$

$$h[n] \;=\; \frac{b_N}{a_N}\delta[n] \;+\; y_c[n]\, u[n]$$

Example (1)

$$y[n+2]-0.6\,y[n+1]-0.16\,y[n] = 5\,x[n+2]$$

$$\left(E^2 - 0.6\,E - 0.16\right)\,y(t) = 5\,E^2\,x[n]$$

initial condition $y[-1]=0,\; y[-2] = \frac{25}{4}$

input $x(t) = 4^{-n}\,u[n]$

Characteristic polynomial

$$\gamma - 0.6\,\gamma - 0.16 = (\gamma+0.2)(\gamma-0.8)$$

Characteristic Equation $(\gamma+0.2)(\gamma-0.8) = 0$

Characteristic Roots $\gamma = -0.2, \quad \gamma = 0.8$

Zero Input Response $y_0[n]$

$$y_0[n] = C_1(-0.2)^n + C_2(0.8)^n$$

$$y_0[-1] = -5\,C_1 + \frac{5}{4}C_2 = 0 \qquad C_1 = \frac{1}{5}$$

$$y_0[-2] = 25\,C_1 + \frac{25}{16}C_2 = \frac{25}{4} \qquad C_2 = \frac{4}{5}$$

$$y_0[n] = \frac{1}{5}(-0.2)^n + \frac{4}{5}(0.8)^n$$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] B.P. Lathi, Linear Systems and Signals (2nd Ed)