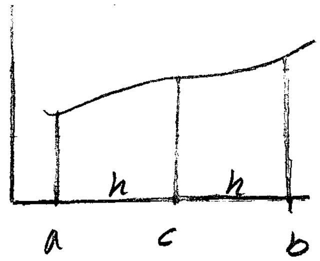
Derive Composite Trap & Simpson's rule from simple Trap & Simpson's rule

Trap.



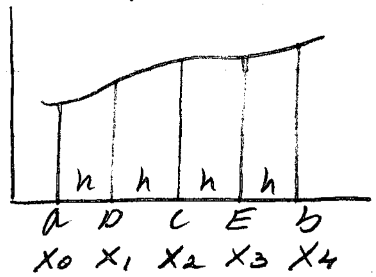
$$\text{7.11 (1)} \quad I_1 = \frac{b-a}{2}\left(f(a)+f(b)\right)$$

$$\text{(2 bin)} \quad I_2 = \frac{\overset{h}{(c-a)}}{2}\left(f(a)+f(c)\right)+\frac{\overset{h}{(b-c)}}{2}\left(f(c)+f(b)\right)$$

(n=2)

$$I_2 = \frac{h}{2}\left(f(a)+2f(c)+f(b)\right)$$

$h = c-a = b-c$



$$\text{(4 bin)} \quad I_4 = \frac{\overset{h}{(D-a)}}{2}\left(f(a)+f(D)\right)+\frac{\overset{h}{(C-D)}}{2}\left(f(c)+f(D)\right)$$

$$+\frac{\overset{h}{(E-c)}}{2}\left(f(c)+f(E)\right)+\frac{\overset{h}{(b-E)}}{2}\left(f(E)+f(b)\right)$$

(n=4)

$$I_4 = \frac{h}{2}\left(f(a)+2f(D)+2f(c)+2f(E)+f(b)\right)$$

$h = D-a = C-D = E-C = b-E$

when n bins are added this can be extended such

$$I_n = h\left(2f(x_0)+f(x_1)+f(x_2)\ldots+f(x_{n-1})\right)+2f(x_n)$$

Simpson's simple

$$I_2 = \frac{h}{3}\left(f(x_0)+4f(x_1)+f(x_2)\right)$$

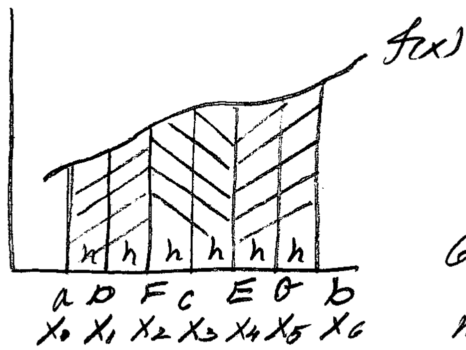
$$= \frac{h}{3}\left(f(a)+4f(c)+f(b)\right)$$

$$I_4 = \frac{h}{3}\left(f(a)+4f(D)+f(c)\right)+\frac{h}{3}\left(f(c)+4f(E)+f(b)\right)$$

$$= \frac{h}{3}\left(f(a)+\underline{4f(D)}+\underline{2f(c)}+\underline{4f(E)}+f(b)\right)$$

to check the repetativness we can do I_6

$$I_6 = \frac{h}{3}(f_0+4f_1+f_2)$$
$$+\frac{h}{3}(f_2+4f_3+f_4)$$
$$+\frac{h}{3}(f_4+4f_5+f_6)$$
$$= \frac{h}{3}(f_0+4f_1+2f_2+4f_3+2f_4+4f_5+f_6)$$



6 bins

$n=6$

so the pattern we see now can be translated to n bins

$$h=(x_1-x_0)=(x_2-x_1)=(x_3-x_2)$$
$$=(x_4-x_3)=(x_5-x_4)=(x_6-x_5)$$

$$I_n = \frac{h}{3}(f_0+4f_1+2f_2+4f_3+2f_{n-2}+4f_{n-1}+f_n)$$