

Phasors

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Phase Lags and Leads

$$\frac{d}{dx} f(x) = \cos(x) \quad \text{leads} \quad f(x) = \sin(x)$$

$$\frac{d}{dx} f(x) = -\sin(x) \quad \text{leads} \quad f(x) = \cos(x)$$

$$\int f(x) dx = -\cos(x) + C \quad \text{lags} \quad f(x) = \sin(x)$$

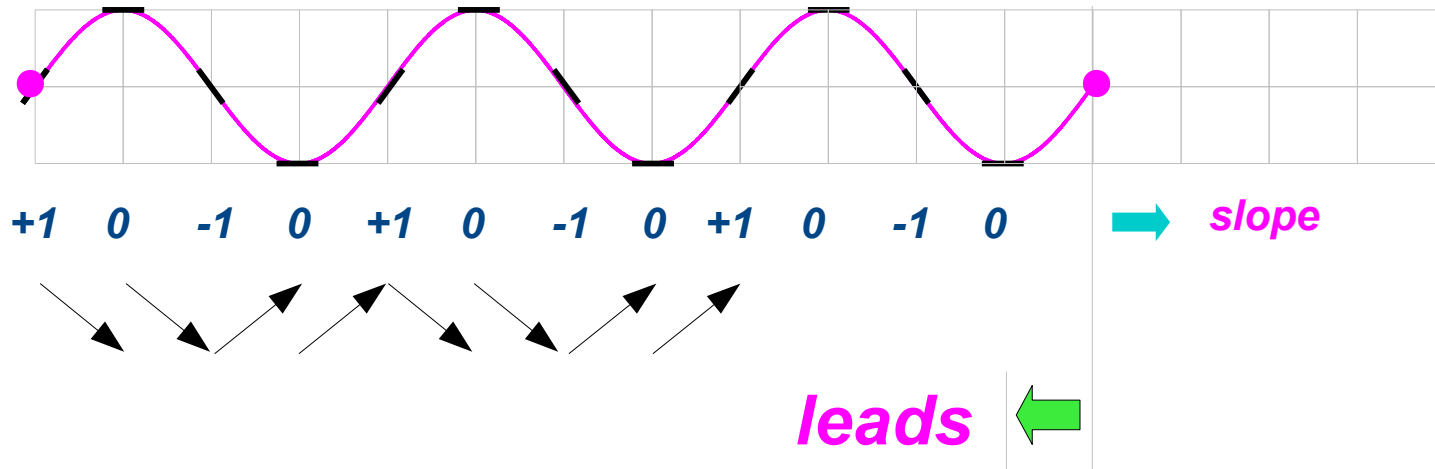
$$\int f(x) dx = \sin(x) + C \quad \text{lags} \quad f(x) = \cos(x)$$

$$\frac{d}{dx} f(x) \quad \text{leads} \quad f(x) \quad \text{by} \quad \frac{\pi}{2}$$

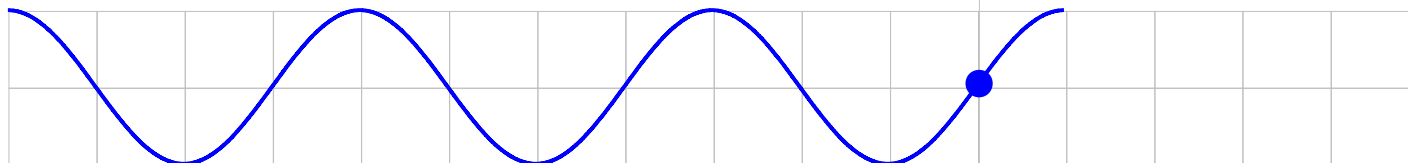
$$\int f(x) dx \quad \text{lags} \quad f(x) \quad \text{by} \quad \frac{\pi}{2}$$

Derivative of $\sin(x)$

$$f(x) = \sin(x)$$

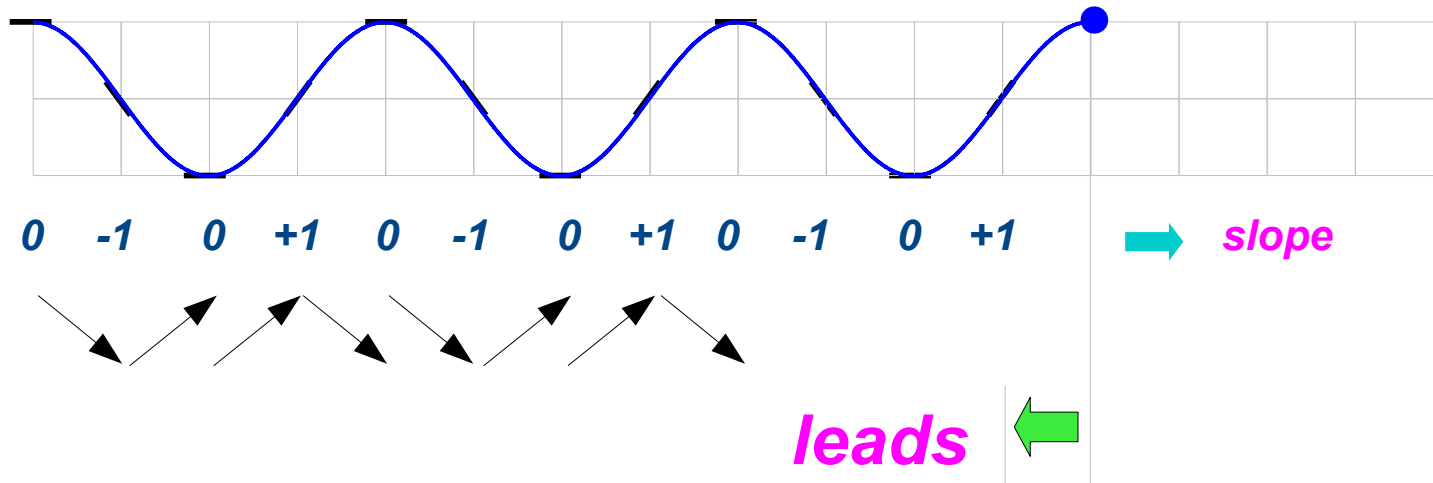


$$\frac{d}{dx} f(x) = \cos(x)$$

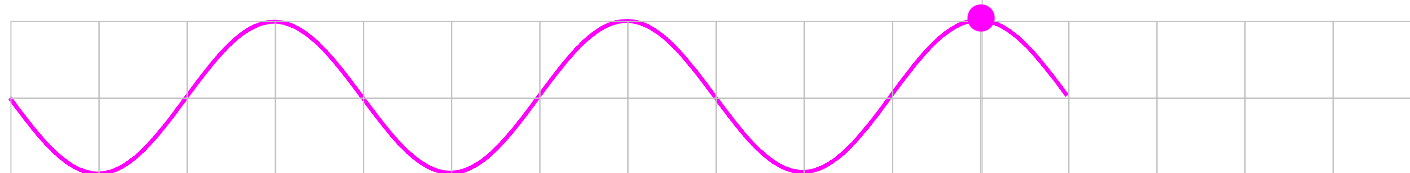


Derivative of $\cos(x)$

$$f(x) = \cos(x)$$




$$\frac{d}{dx} f(x) = -\sin(x)$$

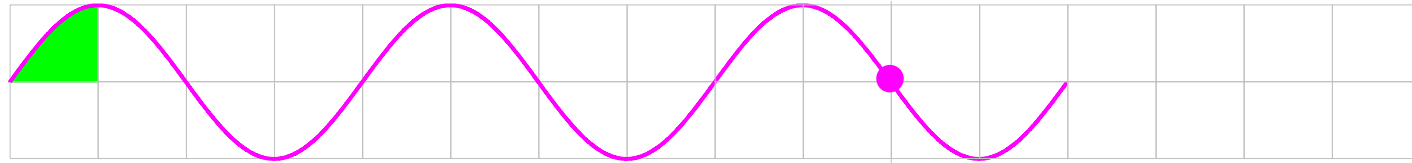


Integral of $\sin(x)$

$$f(x) = \sin(x)$$

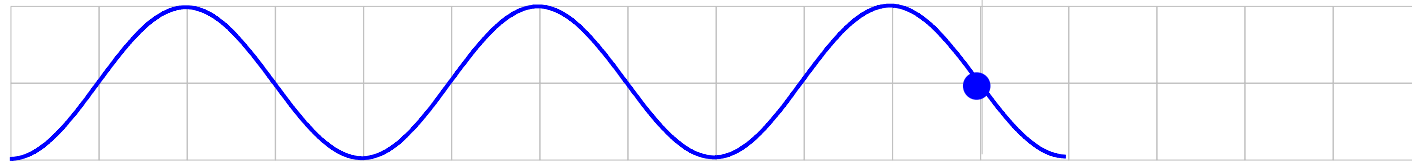


$$\int_0^{\pi/2} \sin(t) dt = 1$$




$C = 1$	0	1	2	1	0	1	2	1	0	1	2	1	→	area	$\int_0^x \sin(t) dt$
$C = 0$	-1	0	+1	0	-1	0	+1	0	-1	0	+1	0	→	area - 1	$\int_0^x \sin(t) dt - 1$
													→	lags	$= -\cos(x)$

$$\int f(x) dx = -\cos(x) + C$$

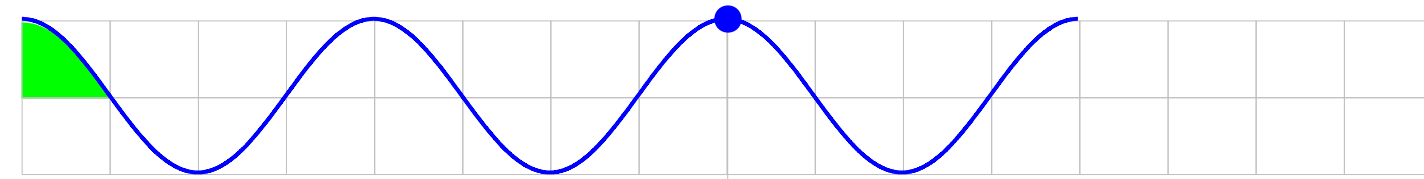


Integral of cos(x)

$$f(x) = \cos(x)$$



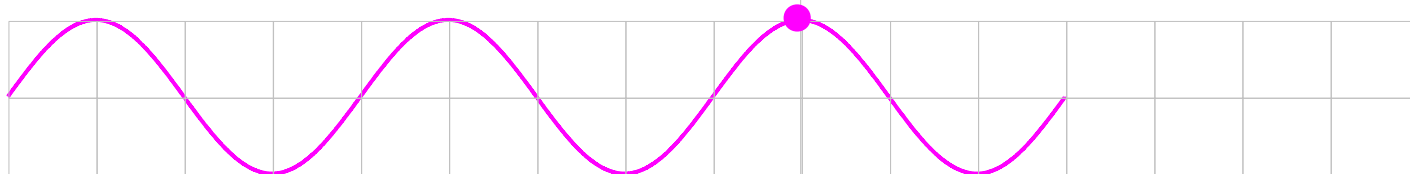
$$\int_0^{\pi/2} \cos(x) dx = 1$$



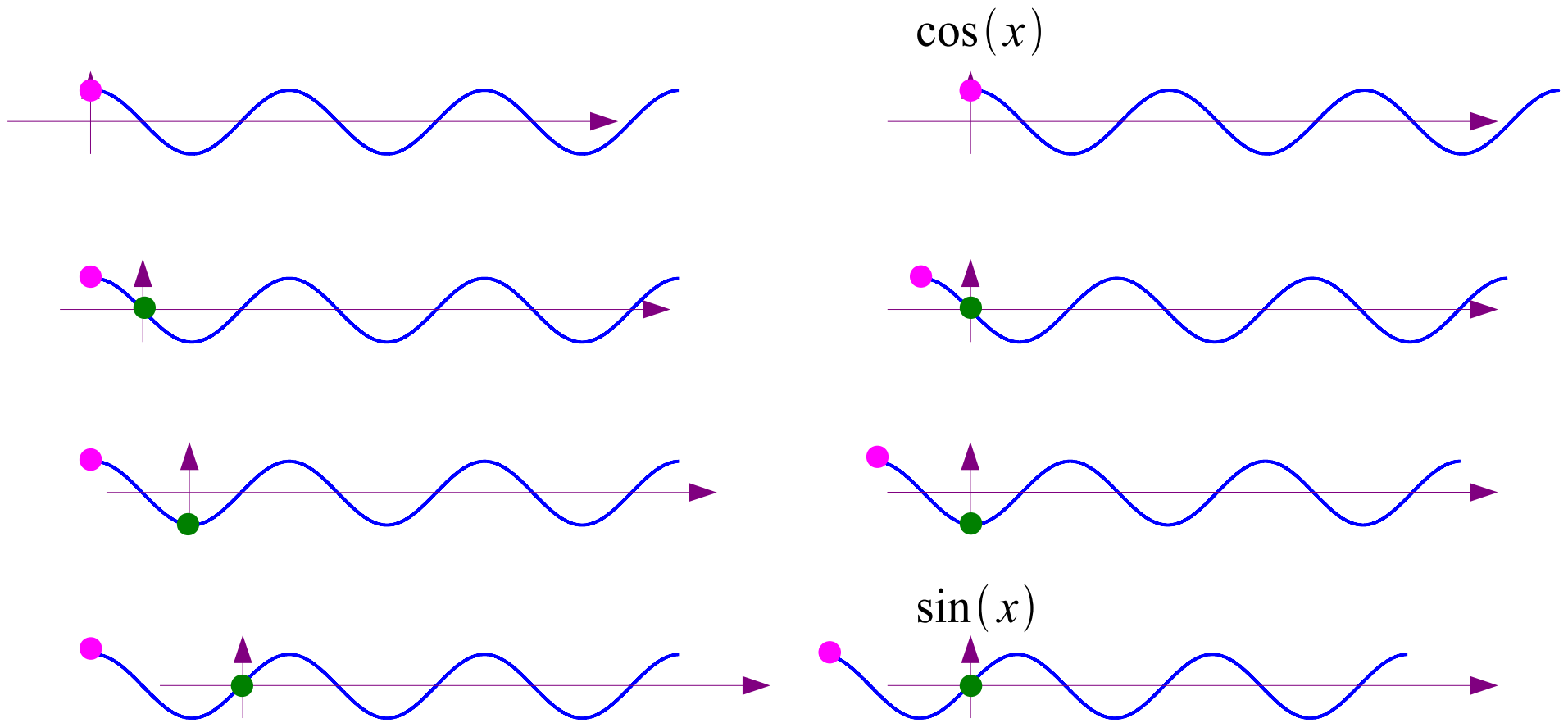
0 1 0 -1 0 1 0 -1 0 1 0 -1 → *area* $\int_0^x \cos(t) dt = -\sin(x)$

→ *lags*

$$\int f(x) dx = \sin(x) + C$$



Sinusoid



Same Amplitude
Same Angular Frequency

$$\left. \begin{array}{l} \cos(x) \Rightarrow 1 \cdot \cos(1 \cdot t) \\ \sin(x) \Rightarrow 1 \cdot \sin(1 \cdot t) \end{array} \right\}$$

$$A \cos(\omega t + \theta)$$

Sinusoid (Sine Waves)

$$A \cos(\omega t + \theta)$$

$$\left\{ \begin{array}{ll} \text{Amplitude} & A \\ \text{Angular Frequency} & \omega \\ \text{Angular Frequency} & \theta \end{array} \right.$$

1. Representation using Euler's Formula

$$A \cos(\omega t + \theta) = \frac{A}{2} \cdot e^{+i(\omega t + \theta)} + \frac{A}{2} \cdot e^{-i(\omega t + \theta)}$$

2. Representation using Real Part

$$A \cos(\omega t + \theta) = \operatorname{Re}\{A e^{i(\omega t + \theta)}\} = \operatorname{Re}\{A e^{i\theta} \cdot e^{i\omega t}\}$$

$$\rightarrow A e^{i\theta} \cdot e^{i\omega t}$$

$$\rightarrow A e^{i\theta}$$

$$\rightarrow A \angle \theta$$

Phasor

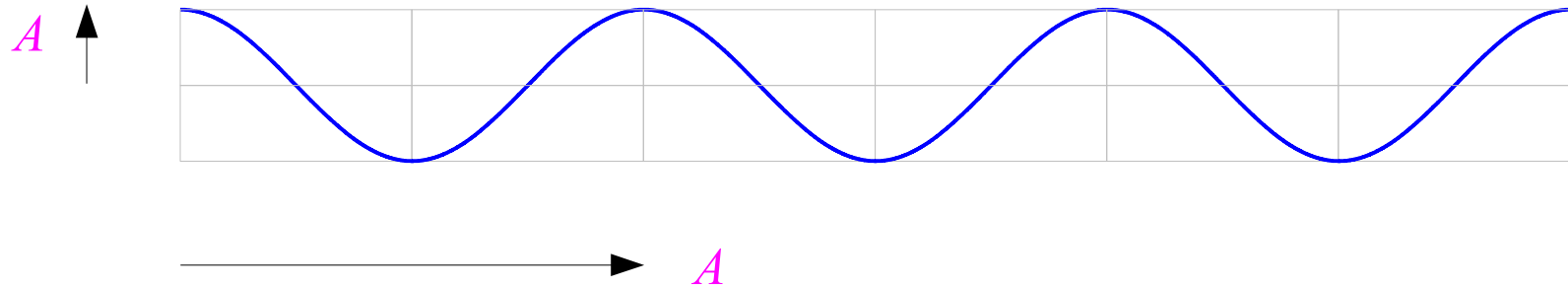
$$A \cos(\omega t + \theta)$$

$$A \cos(\omega t + \theta) = \Re \{ A e^{i(\omega t + \theta)} \}$$

$$= \Re \{ e^{i\omega t} \cdot A e^{i\theta} \}$$

$$A e^{i\theta}$$

$$A \angle \theta$$



References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003