Phasors

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Phase Lags and Leads

$$\frac{d}{dx} f(x) = \cos(x)$$ **leads** $$f(x) = \sin(x)$$

$$\frac{d}{dx} f(x) = -\sin(x)$$ **leads** $$f(x) = \cos(x)$$

$$\int f(x)\, dx = -\cos(x) + C$$ **lags** $$f(x) = \sin(x)$$

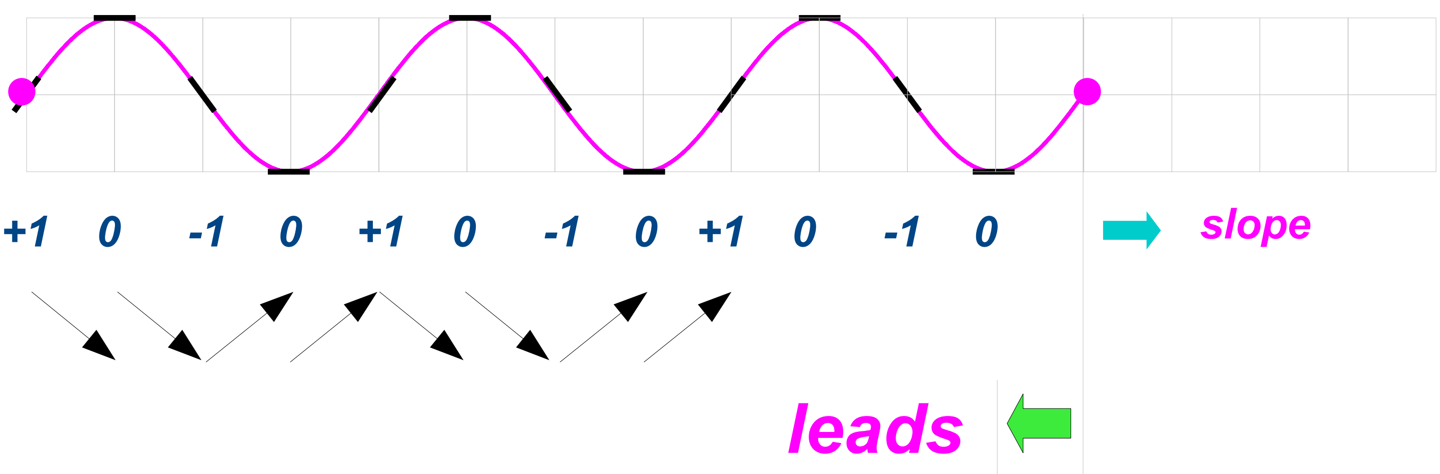
$$\int f(x)\, dx = \sin(x) + C$$ **lags** $$f(x) = \cos(x)$$

$\frac{d}{dx} f(x)$ **leads** $f(x)$ **by** $\frac{\pi}{2}$

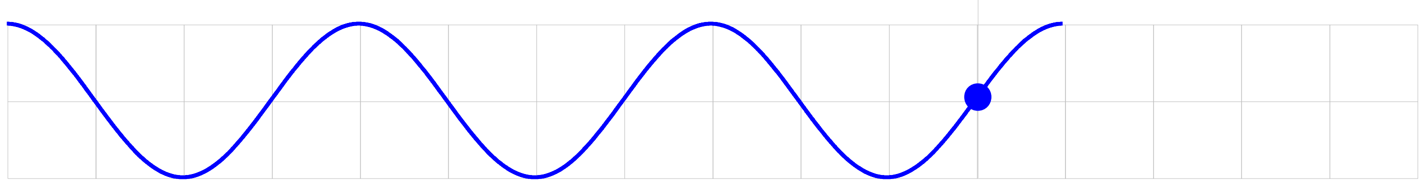
$\int f(x)\, dx$ **lags** $f(x)$ **by** $\frac{\pi}{2}$

Derivative of sin(x)

$$f(x) = \sin(x)$$

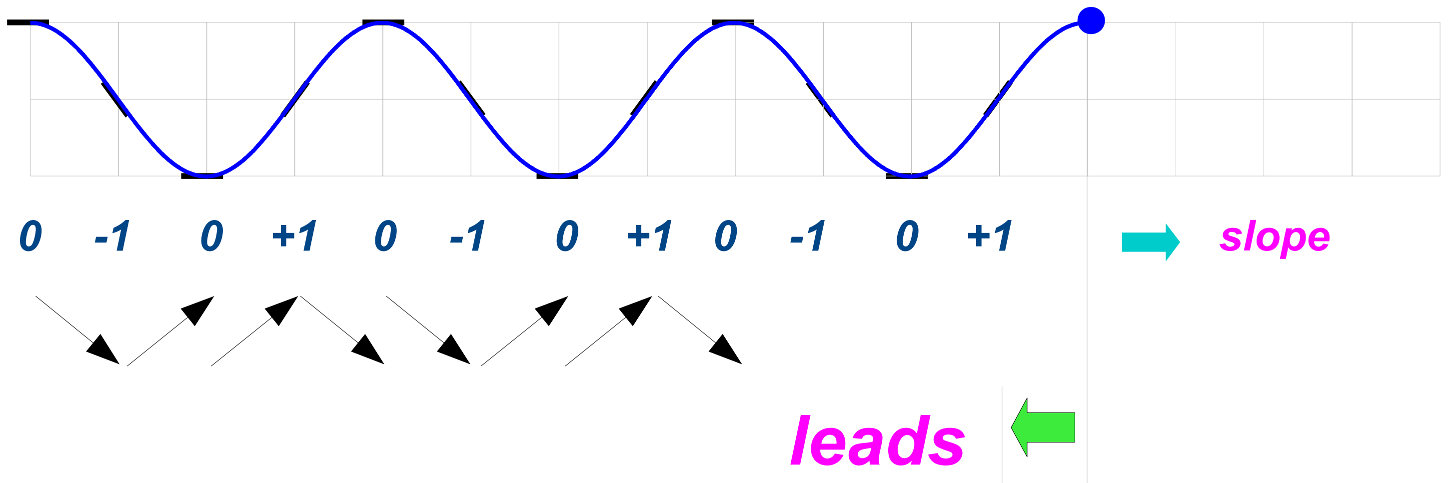


$$\frac{d}{dx} f(x) = \cos(x)$$

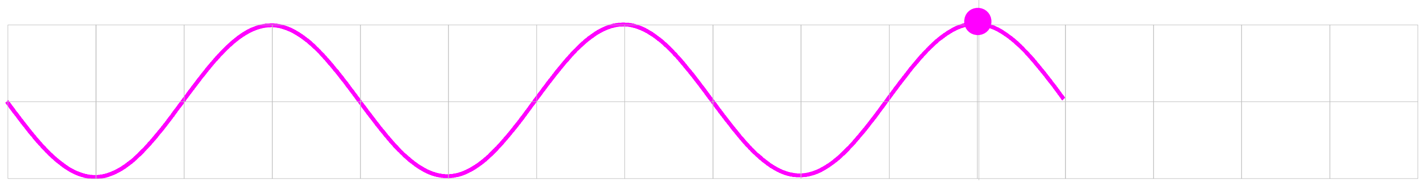


Derivative of cos(x)

$$f(x) = \cos(x)$$

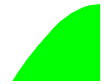


$$\frac{d}{dx} f(x) = -\sin(x)$$

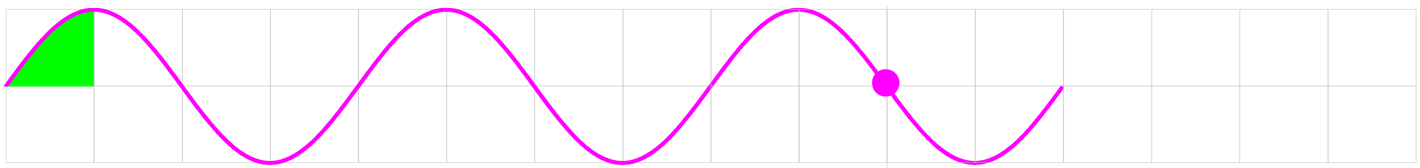


Integral of sin(x)

$$f(x) = \sin(x)$$



$$\int_0^{\pi/2} \sin(t)\,dt = 1$$

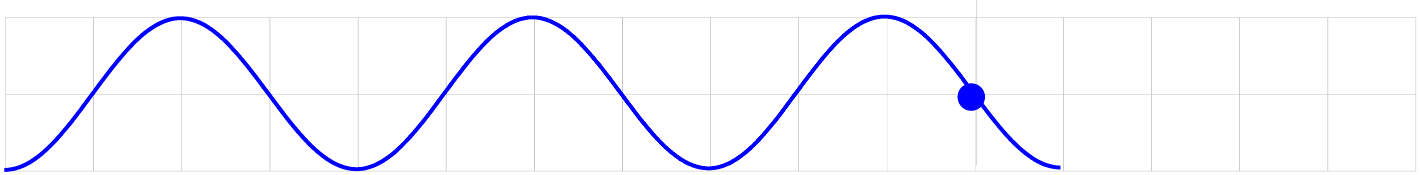


$C = 1$ 0 1 2 1 0 1 2 1 0 1 2 1 → **area** $\int_0^x \sin(t)\,dt$

$C = 0$ -1 0 +1 0 -1 0 +1 0 -1 0 +1 0 → **area - 1** $\int_0^x \sin(t)\,dt - 1$

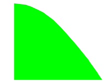
→ **lags** $= -\cos(x)$

$$\int f(x)\,dx = -\cos(x) + C$$

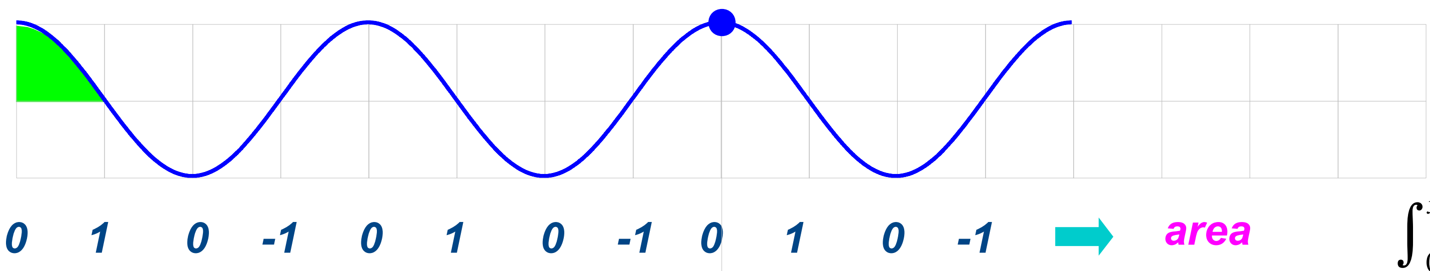


Integral of cos(x)

$$f(x) = \cos(x)$$



$$\int_0^{\pi/2} \cos(x)\, dx = 1$$



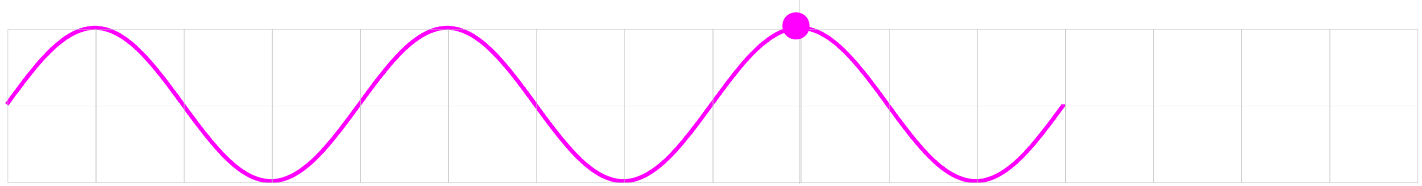
area

$$\int_0^{x} \cos(t)\, dt$$

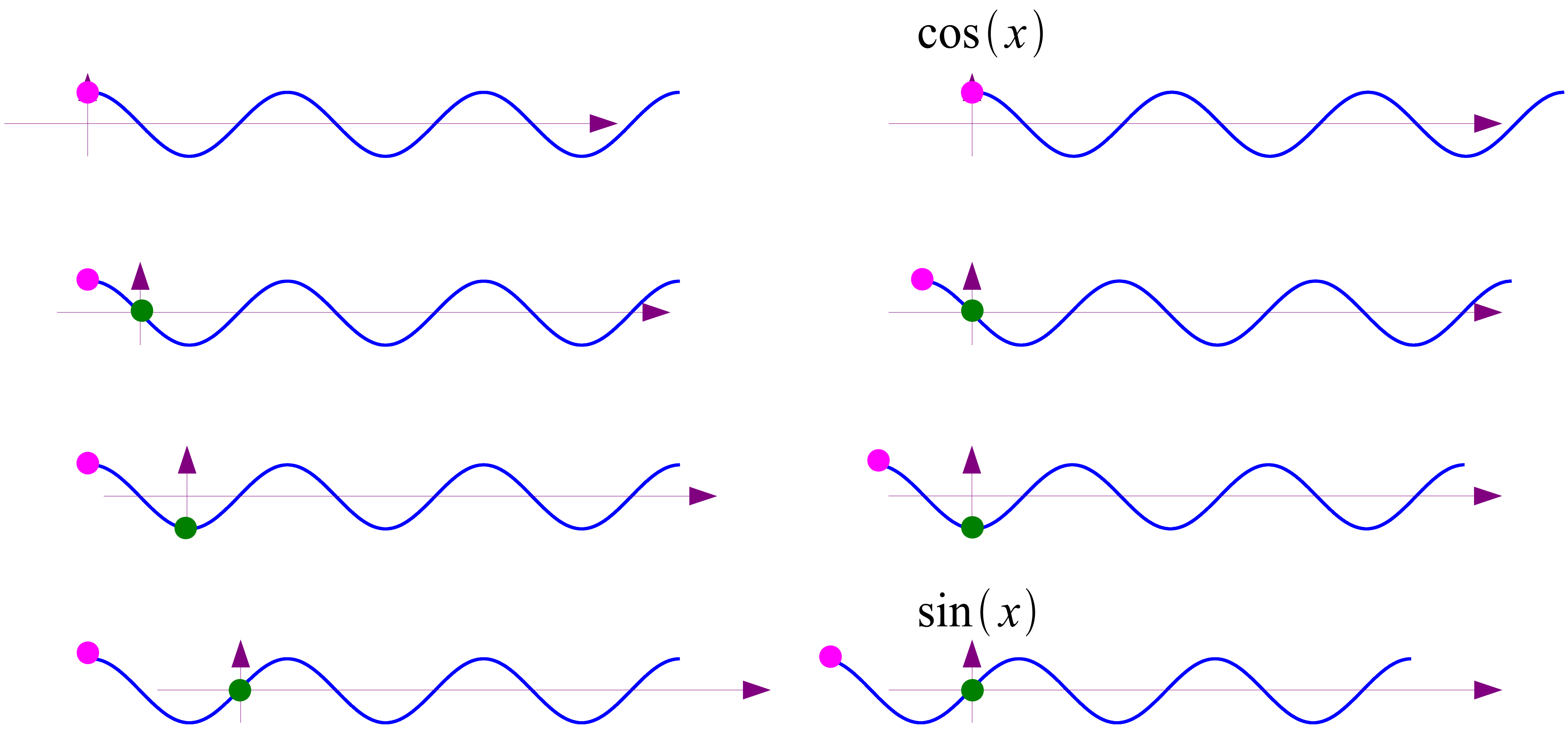
$$= -\sin(x)$$

lags

$$\int f(x)\, dx = \sin(x) + C$$



Sinusoid



Same Amplitude

Same Angular Frequency

$$\left.\begin{aligned}\cos(x) &\Rightarrow 1\cdot\cos(1\cdot t)\\ \sin(x) &\Rightarrow 1\cdot\sin(1\cdot t)\end{aligned}\right\}\qquad A\cos(\omega t + \theta)$$

Phasor

Sinusoid (Sine Waves)

$$A\cos(\omega t + \theta)$$

- ***Amplitude*** A
- ***Angular Frequency*** ω
- ***Angular Frequency*** θ

1. Representation using Euler's Formula

$$A\cos(\omega t + \theta) = \frac{A}{2}\cdot e^{+i(\omega t+\theta)} + \frac{A}{2}\cdot e^{-i(\omega t+\theta)}$$

2. Representation using Real Part

$$A\cos(\omega t + \theta) = Re\{A e^{i(\omega t + \theta)}\} = Re\{A e^{i\theta}\cdot e^{i\omega t}\}$$

➡ $A e^{i\theta}\cdot e^{i\omega t}$

➡ $A e^{i\theta}$

➡ $A \measuredangle \theta$

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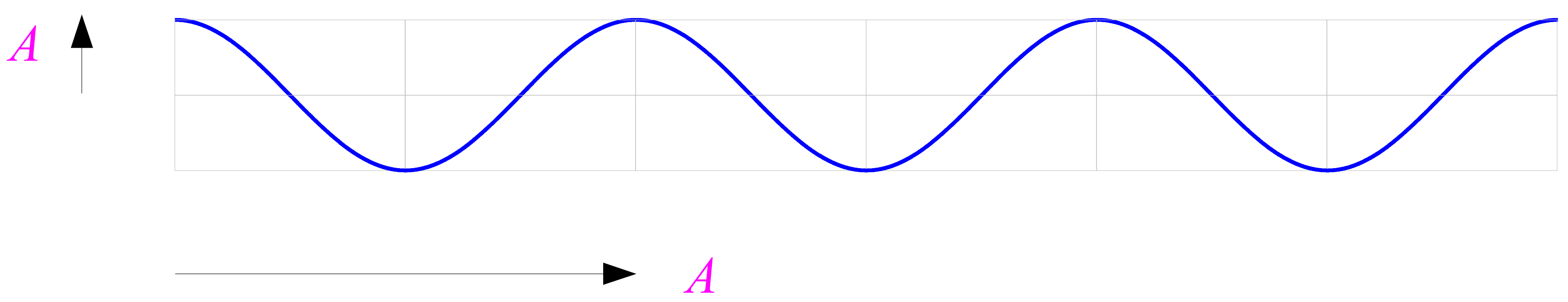
Phasor

$$A\cos(\omega t + \theta)$$

$$A\cos(\omega t + \theta) = \Re\{A e^{i(\omega t + \theta)}\}$$

$$= \Re\{e^{i\omega t} \cdot A e^{i\theta}\}$$

$$A e^{i\theta} \qquad\qquad A \measuredangle \theta$$



References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003