

# CTFT

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- Continuous Time Fourier Transform

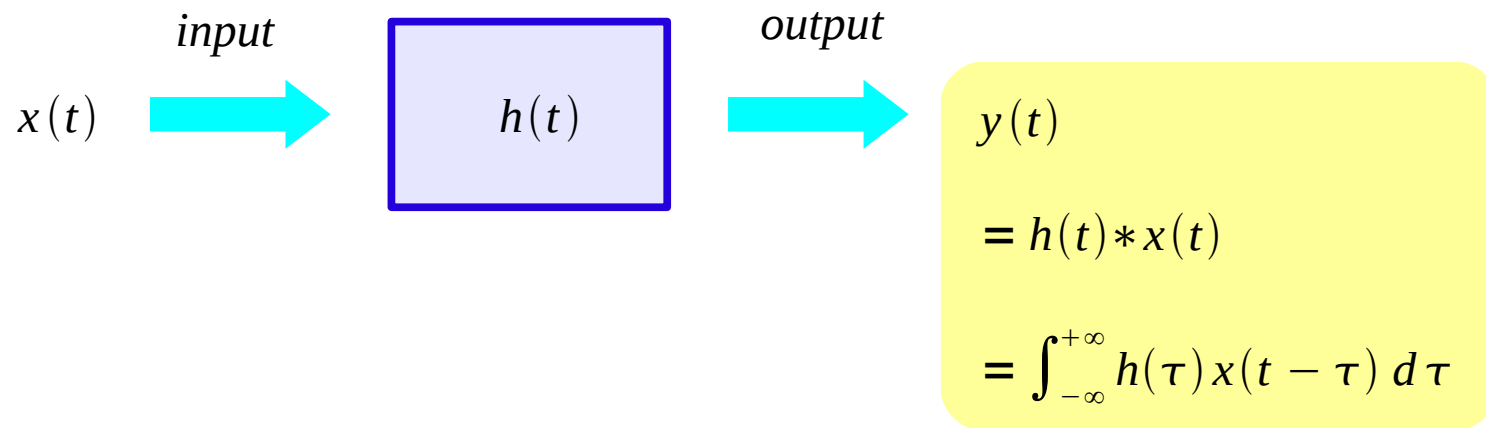
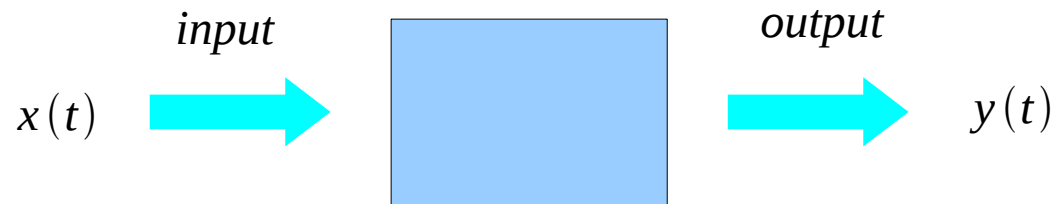
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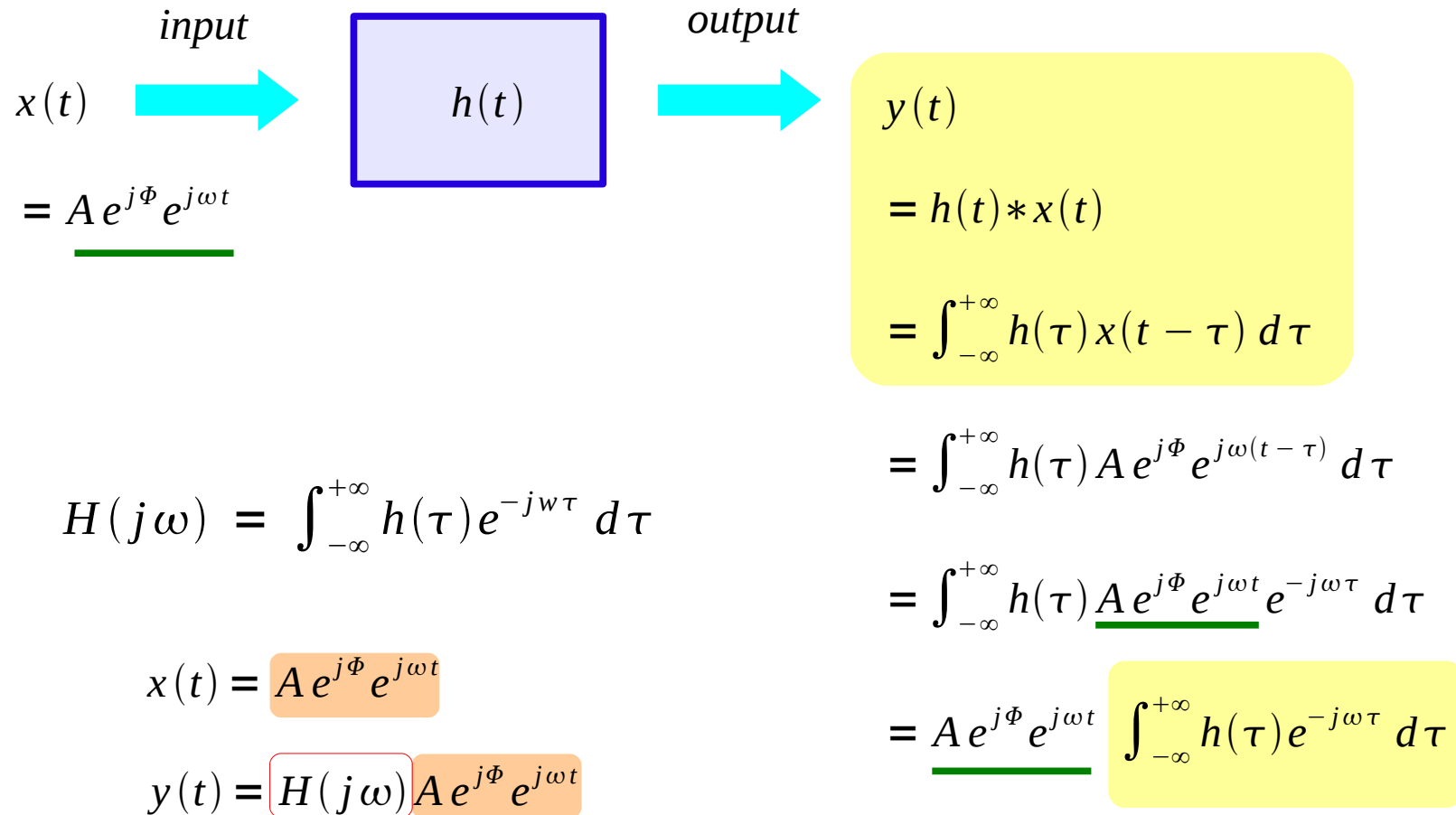
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# Impulse Response



# Frequency Response



# CTFS and CTFT

## Continuous Time Fourier Series

$$C_n = \frac{1}{T} \int_0^T x(t) e^{-jn\omega_0 t} dt \quad \longleftrightarrow \quad x(t) = \sum_{n=-\infty}^{\infty} C_n e^{+jn\omega_0 t}$$

## Continuous Time Fourier Transform

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt \quad \longleftrightarrow \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{+j\omega t} d\omega$$

# From CTFS to CTFT

## Continuous Time Fourier Series

$$C_n = \frac{1}{T} \int_0^T x(t) e^{-jn\omega_0 t} dt \quad \longleftrightarrow \quad x(t) = \sum_{n=0}^{\infty} C_n e^{+jn\omega_0 t}$$

$$C_n = \frac{1}{T_0} \int_{-T_0/2}^{+T_0/2} x_{T_0}(t) e^{-jn\omega_0 t} dt$$

$$x_{T_0}(t) = \sum_{n=0}^{\infty} C_n e^{+jn\omega_0 t} \cdot \frac{2\pi}{2\pi} \cdot \frac{T_0}{T_0}$$

$$C_n T_0 = \int_{-T_0/2}^{+T_0/2} x_{T_0}(t) e^{-jn\omega_0 t} dt$$

$$x_{T_0}(t) = \frac{1}{2\pi} \sum_{n=0}^{\infty} C_n T_0 e^{+jn\omega_0 t} \cdot \frac{2\pi}{T_0}$$

$$T_0 \rightarrow \infty \quad C_n T_0 \rightarrow X(j\omega) \quad x_{T_0} \rightarrow x(t) \quad \omega = \frac{2\pi}{T_0} \rightarrow d\omega$$

## Continuous Time Fourier Transform

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt \quad \longleftrightarrow \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{+j\omega t} d\omega$$

# CTFT of Time Domain Impulse

## Continuous Time Fourier Transform

$$x(t) = A\delta(t) \quad \longleftrightarrow \quad X(j\omega) = A$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

$$\begin{aligned} X(j\omega) &= \int_{-\infty}^{+\infty} A\delta(t) e^{-j\omega t} dt \\ &= \int_{-\infty}^{+\infty} A\delta(t) e^0 dt \\ &= A \int_{-\infty}^{+\infty} \delta(t) dt = A \end{aligned}$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{+j\omega t} d\omega$$

$$\begin{aligned} x(t) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} A e^{+j\omega t} d\omega \\ &= \frac{A}{2\pi} \int_{-\infty}^{+\infty} e^{+j\omega t} d\omega = A\delta(t) \end{aligned}$$

# CTFT of Frequency Domain Impulse

## Continuous Time Fourier Transform

$$X(j\omega) = 2\pi \delta(\omega) \longleftrightarrow x(t) = 1$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{+j\omega t} d\omega$$

$$\begin{aligned} x(t) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} 2\pi \delta(\omega) e^{+j\omega t} d\omega \\ &= \int_{-\infty}^{+\infty} \delta(\omega) e^0 d\omega = 1 \end{aligned}$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

$$X(j\omega) = \int_{-\infty}^{+\infty} e^{-j\omega t} dt = 2\pi \delta(\omega)$$



# CTFS of Impulse Train

## Continuous Time Fourier Series

$$C_n = \frac{1}{T} \int_0^T x(t) e^{-jn\omega_0 t} dt$$

$$x(t) = \sum_{n=0}^{\infty} C_n e^{+jn\omega_0 t}$$

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s)$$

$$\begin{aligned} C_n &= \frac{1}{T_s} \int_{-T_s/2}^{+T_s/2} \delta(t) e^{-jn\omega_s t} dt \\ &= \frac{1}{T_s} \int_{-T_s/2}^{+T_s/2} \delta(t) dt = \frac{1}{T_s} \end{aligned}$$

$$\begin{aligned} p(t) &= \sum_{n=-\infty}^{+\infty} C_n e^{jn\omega_s t} \\ &= \sum_{n=-\infty}^{+\infty} \frac{1}{T_s} e^{jn\omega_s t} \end{aligned}$$

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s) = \sum_{n=-\infty}^{+\infty} \frac{1}{T_s} e^{jn\omega_s t}$$

# CTFT of Impulse Train

## Continuous Time Fourier Transform

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s)$$

$$P(j\omega) = \int_{-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \delta(t - nT_s) e^{-jn\omega t} dt = \int_{-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \frac{1}{T_s} e^{jn\omega_s t} e^{-jn\omega t} dt$$

$$= \sum_{n=-\infty}^{+\infty} \frac{1}{T_s} \int_{-\infty}^{+\infty} e^{-j(\omega - n\omega_s)t} dt$$

$$= \sum_{n=-\infty}^{+\infty} \left( \frac{2\pi}{T_s} \right) \delta(\omega - n\omega_s)$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

$$p(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \left( \frac{2\pi}{T_s} \right) \delta(\omega - n\omega_s) e^{+j\omega t} d\omega = \sum_{n=-\infty}^{+\infty} \frac{1}{T_s} \int_{-\infty}^{+\infty} \delta(\omega - n\omega_s) e^{+j\omega t} d\omega$$

$$= \sum_{n=-\infty}^{+\infty} \frac{1}{T_s} e^{jn\omega_s t} = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s)$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{+j\omega t} d\omega$$

# Other Convention

## Continuous Time Fourier Transform {unitary, angular frequency}

$$X(j\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt \quad \longleftrightarrow$$

$$x(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} X(j\omega) e^{+j\omega t} d\omega$$

## Continuous Time Fourier Transform {non-unitary, angular frequency}

$$X(j\omega) = 1 \cdot \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt \quad \longleftrightarrow$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{+j\omega t} d\omega$$

## References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, R.W. Schafer, M.A. Yoder, "Signal Processing First", 2003, Pearson Education