

# EigenSpaces (5A)

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# EigenValues and EigenVectors

$n \times n$

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \lambda \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

$$A \mathbf{x} = \lambda \mathbf{x}$$

$$\begin{pmatrix} a_{11} - \lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} - \lambda & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} - \lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$(A - \lambda I) \mathbf{x} = \mathbf{0}$$

characteristic Equation

$$\det(A - \lambda I) = 0$$

# A $n \times n$ Matrix $\mathbf{A}$ (1)

1.  $\mathbf{A}$  is **invertible**
2.  $\mathbf{Ax} = \mathbf{0}$  has only the **trivial** solution
3. The **RREF**( $\mathbf{A}$ ) =  $\mathbf{I}_n$
4.  $\mathbf{A}$  can be written as a product of **elementary matrix**
5.  $\mathbf{Ax} = \mathbf{b}$  is **consistent** for every  $n \times 1$   $\mathbf{b}$
6.  $\mathbf{Ax} = \mathbf{b}$  has **exactly one solution** for every  $n \times 1$   $\mathbf{b}$
7. **det**( $\mathbf{A}$ )  $\neq 0$
8. The column vectors are **linearly independent**
9. The row vectors are **linearly independent**
10. The column vectors **span**  $\mathbb{R}^n$
11. The row vectors **span**  $\mathbb{R}^n$
12. The column vectors form a **basis** for  $\mathbb{R}^n$
13. The row vectors form a **basis** for  $\mathbb{R}^n$
14. **rank**( $\mathbf{A}$ ) =  $n$
15. **nullity**( $\mathbf{A}$ ) =  $0$
16. The **orthogonal complement** of the null space is  $\mathbb{R}^n$
17. The **orthogonal complement** of the row space is  $\{\mathbf{0}\}$

# A $n \times n$ Matrix A (2)

18. The range of  $T_A$  is  $\mathbb{R}^n$
19.  $T_A$  is one-to-one
20.  $\lambda=0$  is not the eigenvalue of A

## References

- [1] <http://en.wikipedia.org/>
- [2] Anton, et al., Elementary Linear Algebra, 10<sup>th</sup> ed, Wiley, 2011
- [3] Anton, et al., Contemporary Linear Algebra,