Hyperbolic Functions (1A)

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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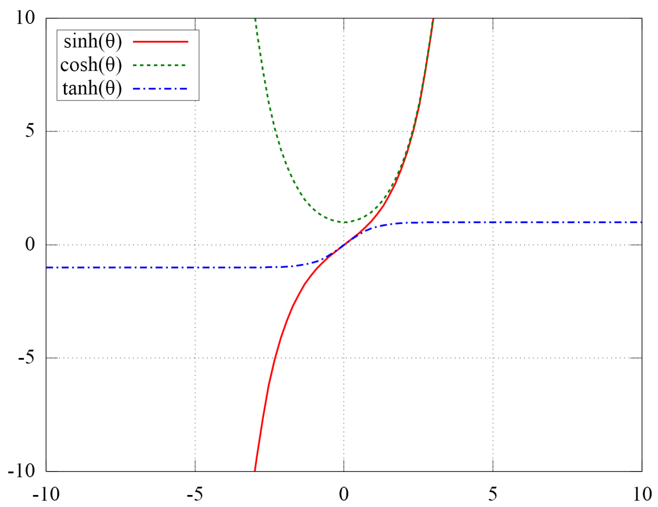
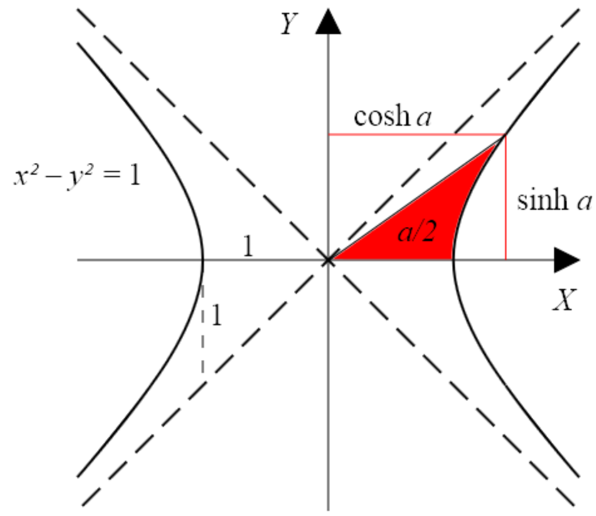
Definitions of Hyperbolic Functions





$$\sinh\alpha = \frac{1}{2}(e^{\alpha} - e^{-\alpha})$$

$$\cosh\alpha = \frac{1}{2}(e^{\alpha} + e^{-\alpha})$$

$$\tanh\alpha = \frac{(e^{\alpha} - e^{-\alpha})}{(e^{\alpha} + e^{-\alpha})}$$

$$x^2 - y^2 = 1$$

$$\cosh^2\alpha - \sinh^2\alpha = 1$$

$$\frac{1}{4}(e^{\alpha} + e^{-\alpha})^2 - \frac{1}{4}(e^{\alpha} - e^{-\alpha})^2 = 1$$

$(\cosh\alpha,\ \sinh\alpha)$

$\downarrow \quad \downarrow$

$x \quad y$

Euler Formula

$$e^{+ix} = \cos x + i \sin x$$

$$e^{-ix} = \cos x - i \sin x$$

$$e^{+ix} = \mathbf{cosh}\, ix + \mathbf{sinh}\, ix$$

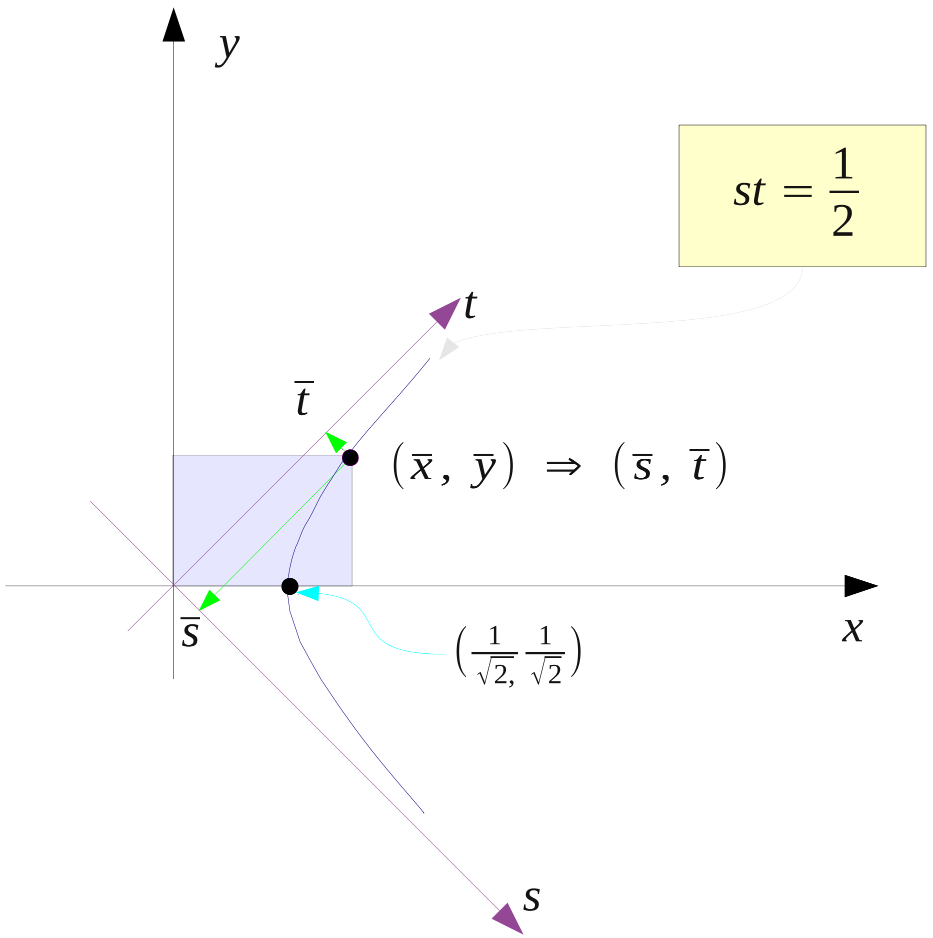
$$e^{-ix} = \mathbf{cosh}\, ix - \mathbf{sinh}\, ix$$

$$\cos x = \frac{1}{2}\left(e^{+ix} + e^{-ix}\right) \Rightarrow \mathbf{cosh}\, ix$$

$$i \sin x = \frac{1}{2}\left(e^{+ix} - e^{-ix}\right) \Rightarrow i\, \mathbf{sinh}\, ix$$

$$i \frac{\sin x}{\cos x} = \frac{\left(e^{+ix} - e^{-ix}\right)}{\left(e^{+ix} + e^{-ix}\right)} \Rightarrow i\, \mathbf{tanh}\, ix$$

Coordinates Changes



$$\begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} \cos\left(\frac{\pi}{4}\right) & -\sin\left(\frac{\pi}{4}\right) \\ \sin\left(\frac{\pi}{4}\right) & \cos\left(\frac{\pi}{4}\right) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

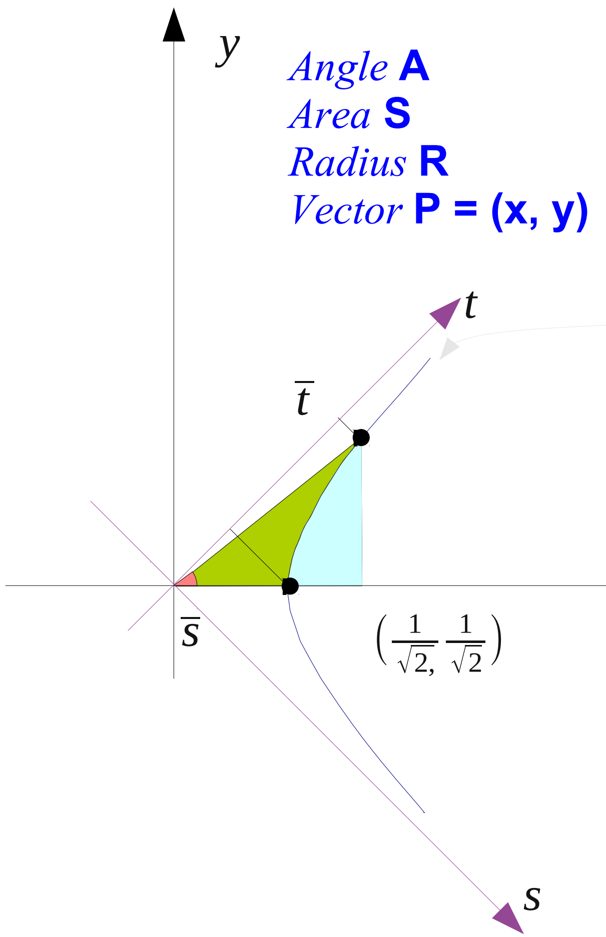
$$x - y = s\sqrt{2}$$
$$x + y = t\sqrt{2}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\left(\frac{-\pi}{4}\right) & -\sin\left(\frac{-\pi}{4}\right) \\ \sin\left(\frac{-\pi}{4}\right) & \cos\left(\frac{-\pi}{4}\right) \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$

$$+s + t = x\sqrt{2}$$
$$-s + t = y\sqrt{2}$$

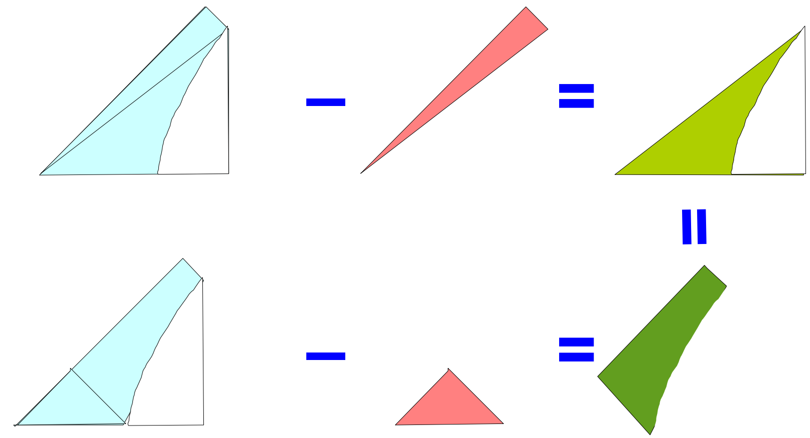
Area:S Angle:A → 2S = A (1)



$$st = \frac{1}{2}$$

$$x - y = s\sqrt{2}$$

$$x + y = t\sqrt{2}$$



$$S = \int_{\frac{1}{\sqrt{2}}}^{t} \frac{1}{2s}\, ds = \int_{\frac{1}{\sqrt{2}}}^{\frac{(x+y)}{\sqrt{2}}} \frac{1}{2s}\, ds$$

$$2S = 2\int_{\frac{1}{\sqrt{2}}}^{\frac{(x+y)}{\sqrt{2}}} \frac{ds}{2s}$$

$$= \ln(x+y) = \ln\left(x \pm \sqrt{x^2 - 1}\right)$$

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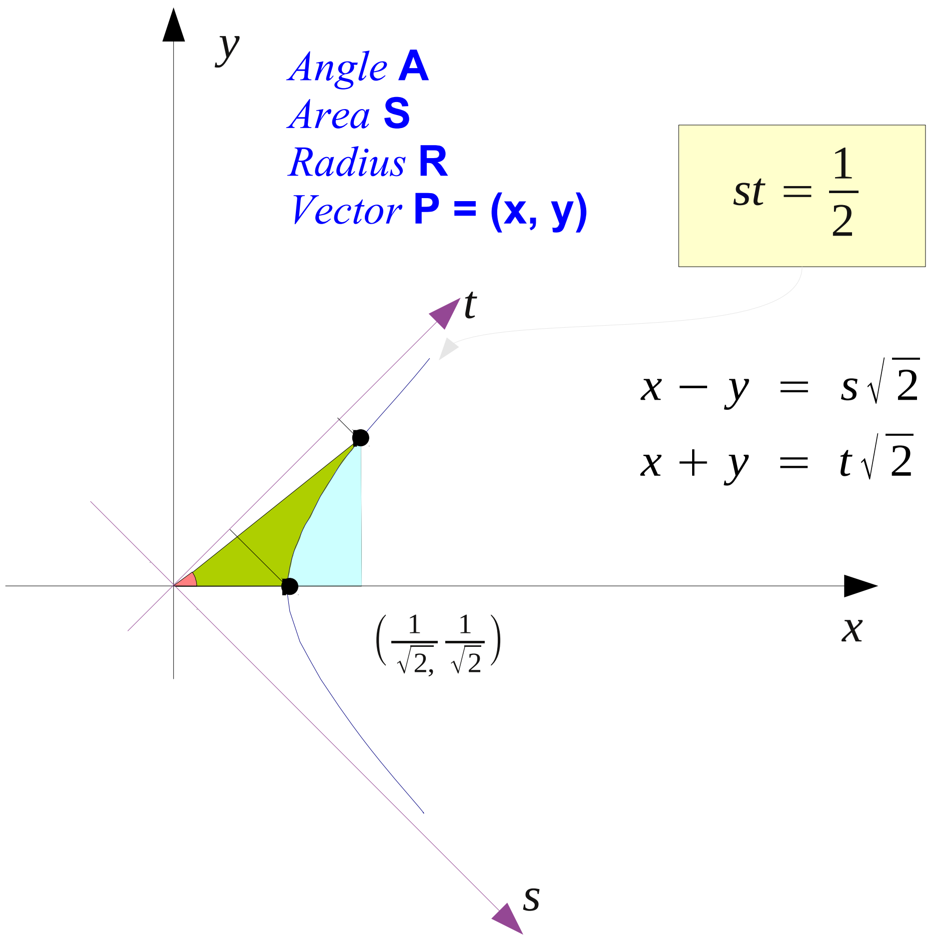
Area:S Angle:A → 2S = A (2)



$$2S = \ln(x+y) = \ln\left(x \pm \sqrt{x^2-1}\right)$$

$$\cosh A = x$$

$$A = \cosh^{-1} x$$

$$\cosh A = \frac{1}{2}\left(e^{A} + e^{-A}\right) = x$$

$$2x = u + \frac{1}{u} \qquad \left(u = e^{A}\right)$$

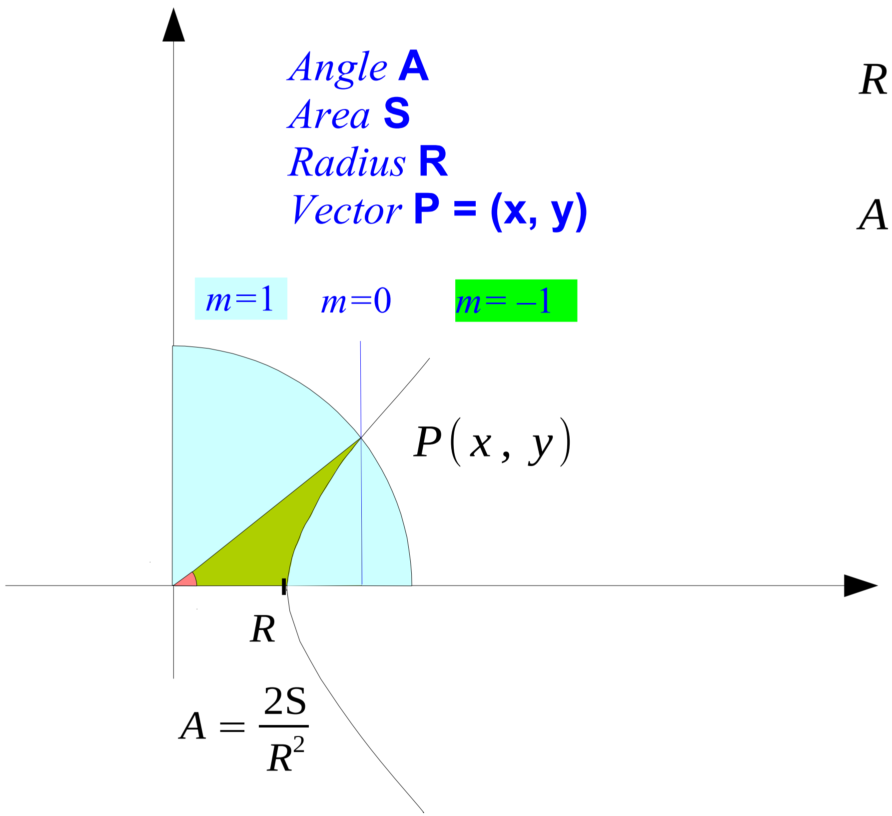
$$u^2 - 2xu + 1 = 0$$

$$u = x \pm \sqrt{x^2 - 1} \Rightarrow e^{A}$$

$$A = \ln\left(x \pm \sqrt{x^2 - 1}\right)$$

$$2S = A = \ln\left(x \pm \sqrt{x^2 - 1}\right)$$

Area:S Angle:A → 2S = A (3)



$$R = (x^2 + y^2)^{1/2}$$

$$A = \tan^{-1}\left(\frac{y}{x}\right)$$

$$R = (x^2 - y^2)^{1/2}$$

$$A = -\tan^{-1}\left(-\frac{y}{x}\right)$$

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References

[1] http://en.wikipedia.org/
[2] J. S. Walther, A Unified Algorithm for Elementary Functions
[3] J. Calvert, http://mysite.du.edu/~jcalvert/math/hyperb.htm