

Hyperbolic Functions (1A)

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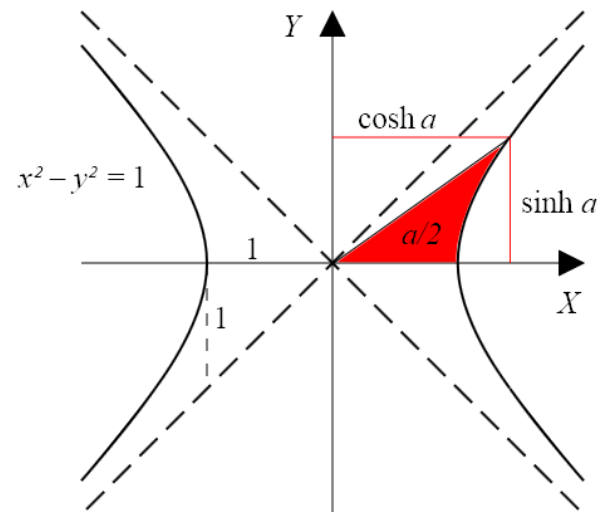
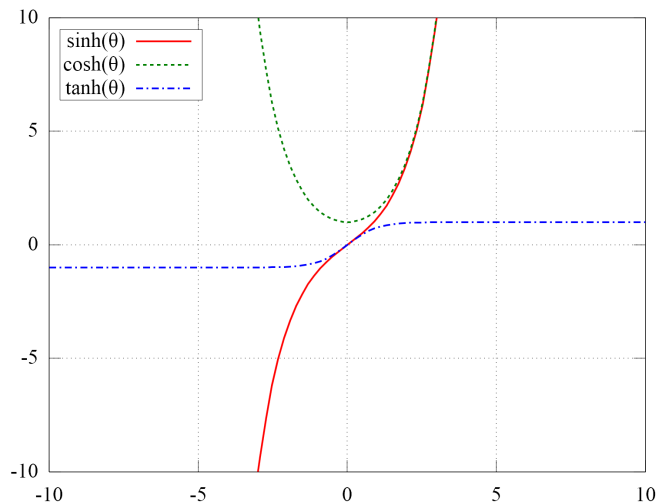
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Definitions of Hyperbolic Functions



$$\sinh \alpha = \frac{1}{2}(e^{\alpha} - e^{-\alpha})$$

$$\cosh \alpha = \frac{1}{2}(e^{\alpha} + e^{-\alpha})$$

$$\tanh \alpha = \frac{(e^{\alpha} - e^{-\alpha})}{(e^{\alpha} + e^{-\alpha})}$$

$$x^2 - y^2 = 1$$

$$\cosh^2 \alpha - \sinh^2 \alpha = 1$$

$$\frac{1}{4}(e^{\alpha} + e^{-\alpha})^2 - \frac{1}{4}(e^{\alpha} - e^{-\alpha})^2 = 1$$

(cosh α , sinh α)

↓
x

↓
y

Hyperbolic Functions of Complex Numbers

Euler Formula

$$e^{+ix} = \cos x + i \sin x$$

$$e^{-ix} = \cos x - i \sin x$$

$$e^{+ix} = \mathbf{\cosh ix + \sinh ix}$$

$$e^{-ix} = \mathbf{\cosh ix - \sinh ix}$$

$$\cos x = \frac{1}{2}(e^{+ix} + e^{-ix})$$

$$i \sin x = \frac{1}{2}(e^{+ix} - e^{-ix})$$

$$i \frac{\sin x}{\cos x} = \frac{(e^{+ix} - e^{-ix})}{(e^{+ix} + e^{-ix})}$$

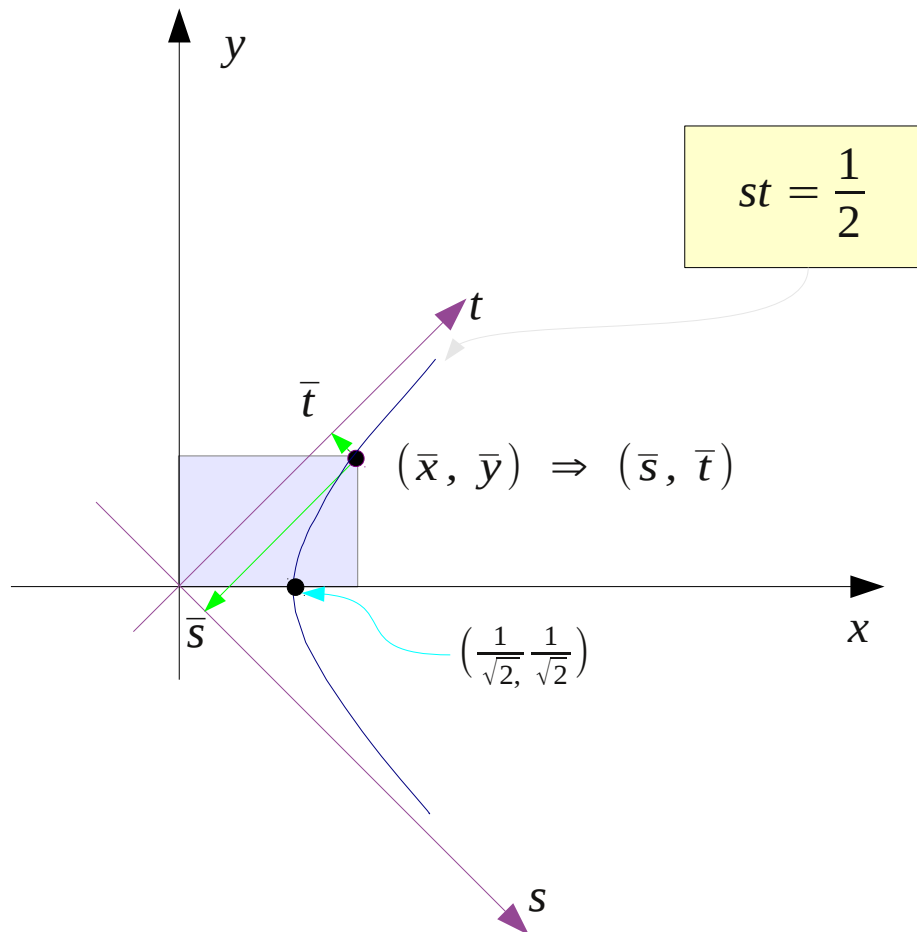
$$\Rightarrow \mathbf{\cosh ix}$$

$$\Rightarrow \mathbf{i \sinh ix}$$

$$\Rightarrow \mathbf{i \tanh ix}$$



Coordinates Changes



$$\begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} \cos(\frac{\pi}{4}) & -\sin(\frac{\pi}{4}) \\ \sin(\frac{\pi}{4}) & \cos(\frac{\pi}{4}) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$x - y = s\sqrt{2}$$

$$x + y = t\sqrt{2}$$

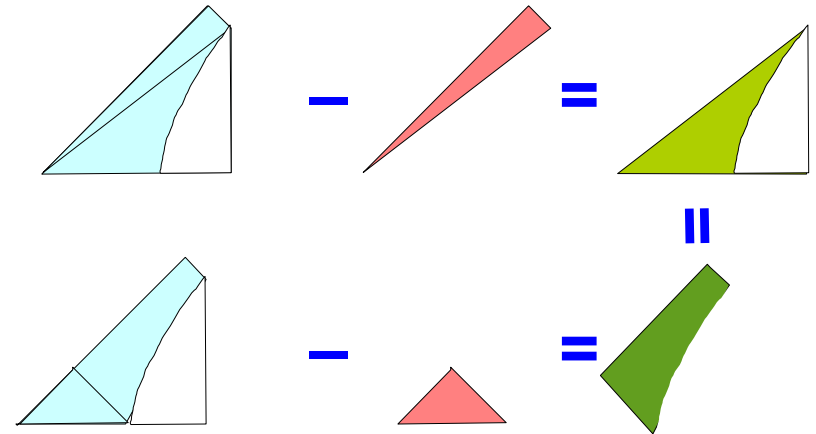
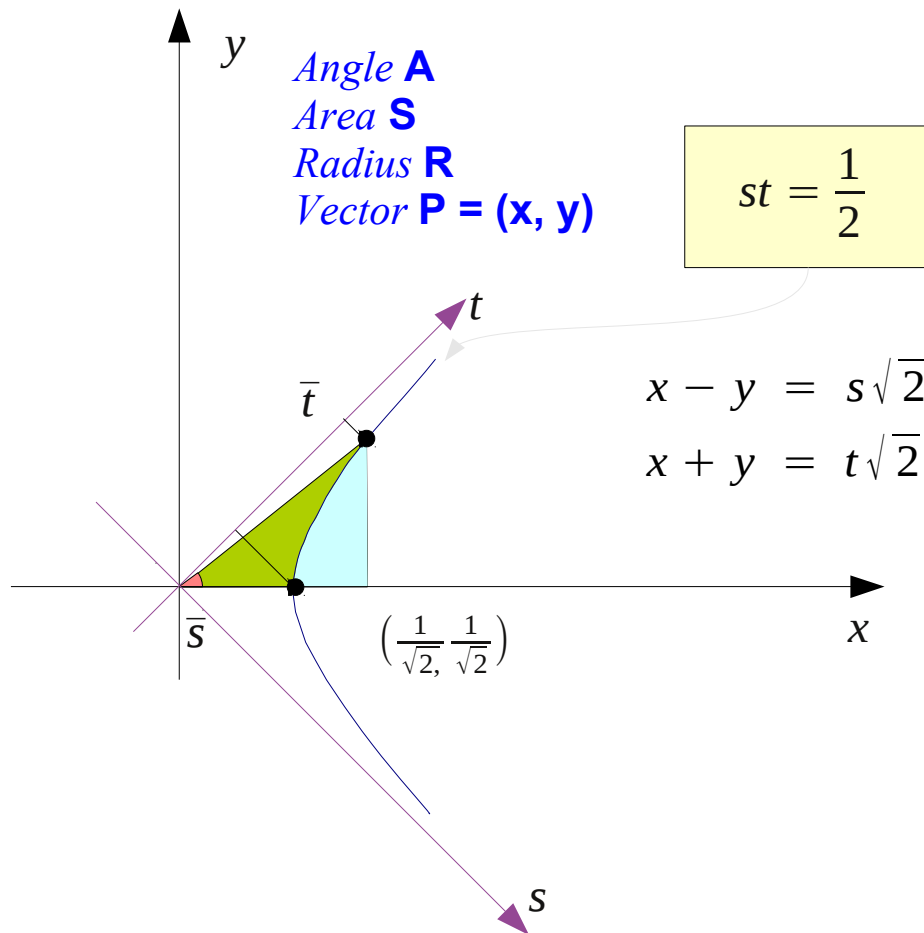
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(\frac{-\pi}{4}) & -\sin(\frac{-\pi}{4}) \\ \sin(\frac{-\pi}{4}) & \cos(\frac{-\pi}{4}) \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$

$$+s + t = x\sqrt{2}$$

$$-s + t = y\sqrt{2}$$

Area: S Angle: $A \rightarrow 2S = A \quad (1)$

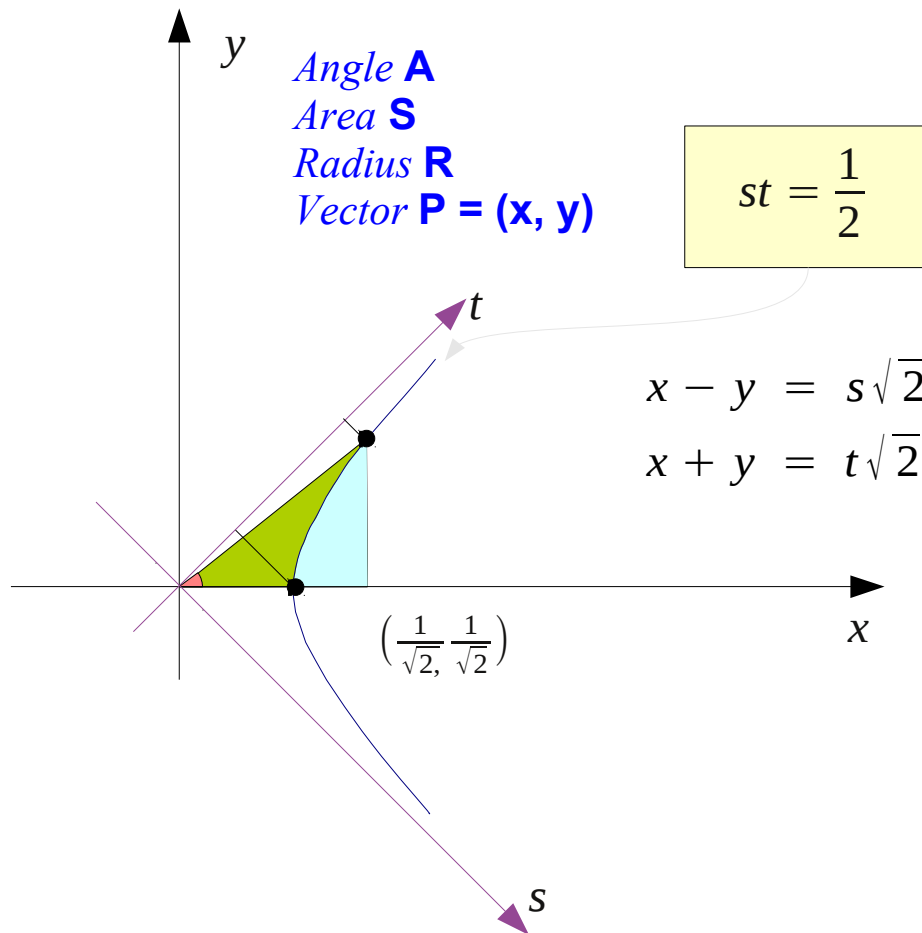


$$S = \int_{\frac{1}{\sqrt{2}}}^t \frac{1}{2s} ds = \int_{\frac{1}{\sqrt{2}}}^{\frac{(x+y)}{\sqrt{2}}} \frac{1}{2s} ds$$

$$2S = 2 \int_{\frac{1}{\sqrt{2}}}^{\frac{(x+y)}{\sqrt{2}}} \frac{ds}{2s}$$

$$= \ln(x+y) = \ln(x \pm \sqrt{x^2 - 1})$$

Area: S Angle: $A \rightarrow 2S = A$ (2)



$$2S = \ln(x+y) = \ln(x \pm \sqrt{x^2-1})$$

$$\cosh A = x$$

$$A = \cosh^{-1} x$$

$$\cosh A = \frac{1}{2}(e^A + e^{-A}) = x$$

$$2x = u + \frac{1}{u} \quad (u = e^A)$$

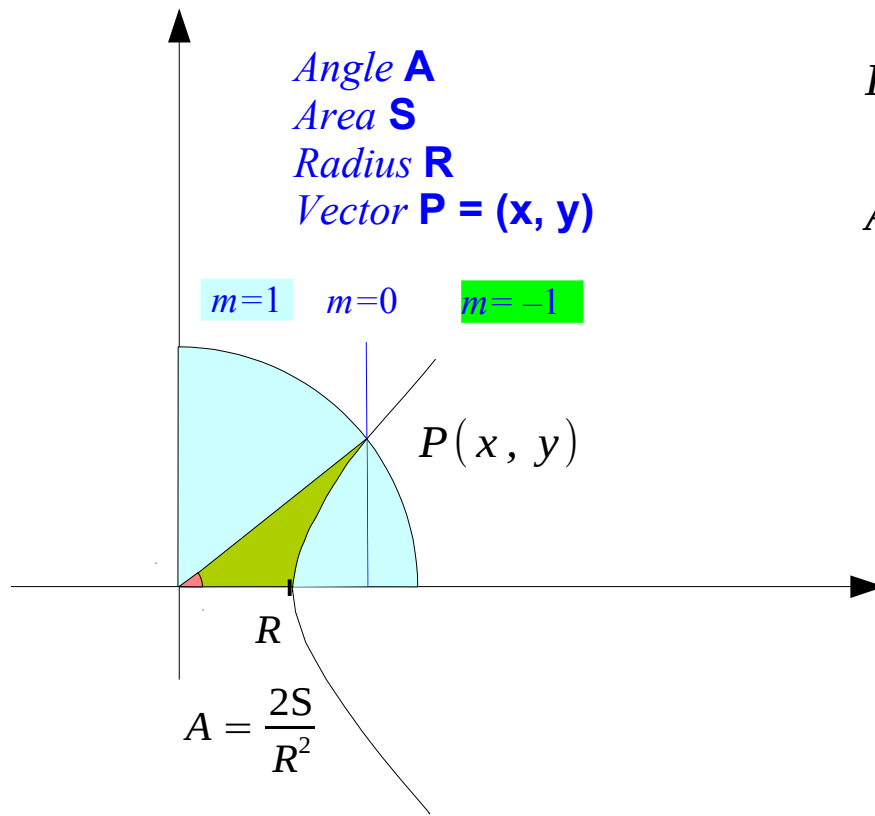
$$u^2 - 2xu + 1 = 0$$

$$u = x \pm \sqrt{x^2 - 1} \Rightarrow e^A$$

$$A = \ln(x \pm \sqrt{x^2 - 1})$$

$$2S = A = \ln(x \pm \sqrt{x^2 - 1})$$

Area: S Angle: $A \rightarrow 2S = A \quad (3)$



$$R = (x^2 + y^2)^{1/2}$$

$$A = \tan^{-1}\left(\frac{y}{x}\right)$$

$$R = (x^2 - y^2)^{1/2}$$

$$A = -\tan^{-1}\left(-\frac{y}{x}\right)$$

References

- [1] <http://en.wikipedia.org/>
- [2] J. S. Walther, A Unified Algorithm for Elementary Functions
- [3] J. Calvert, <http://mysite.du.edu/~jcalvert/math/hyperb.htm>