Anti-Image Postfilter (7B)

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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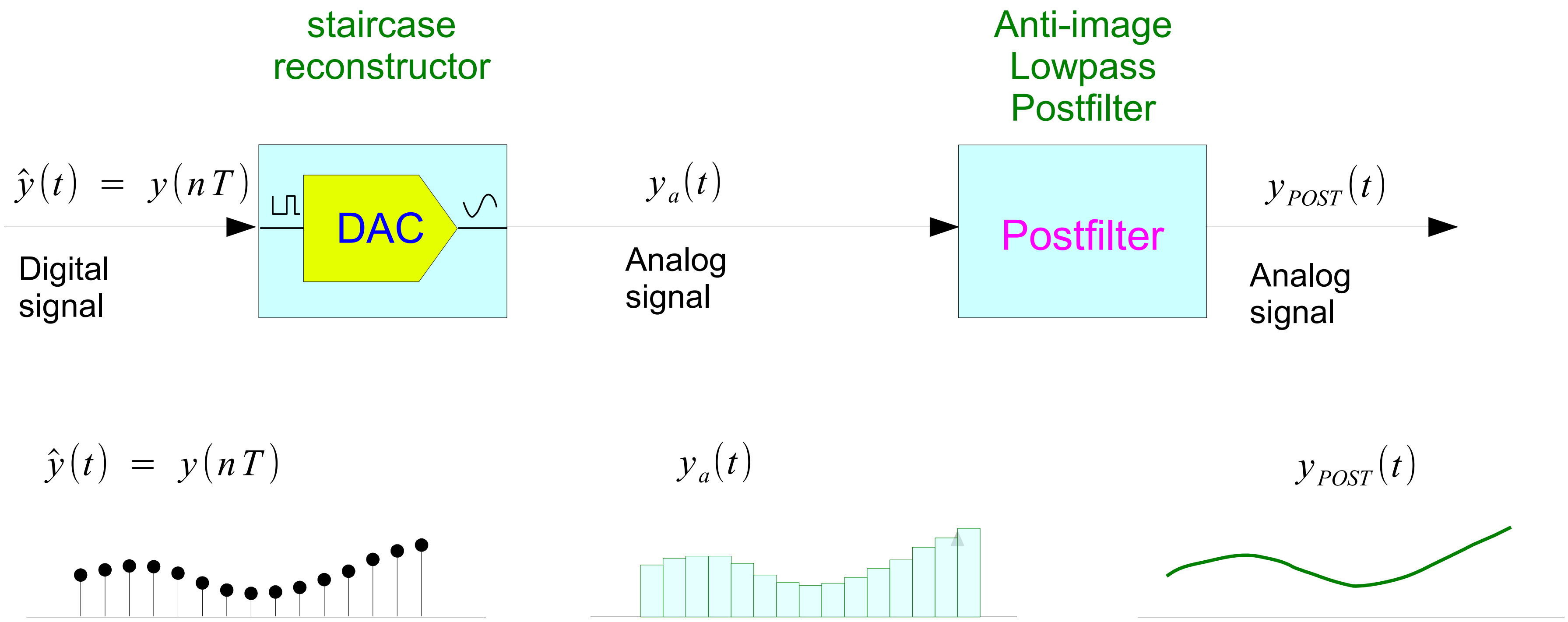
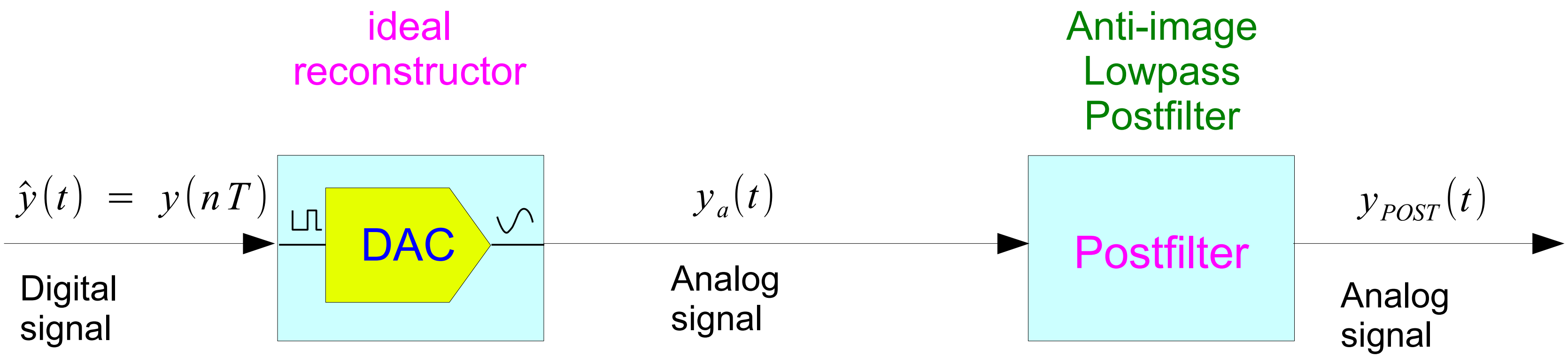
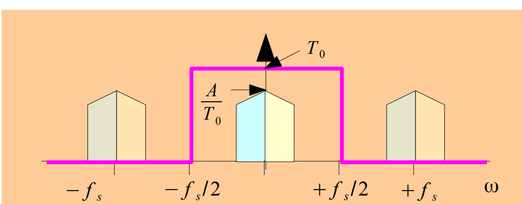
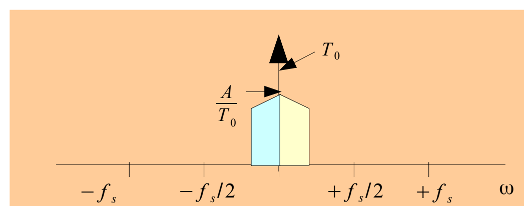

staircase
reconstructor
Anti-image
Lowpass
Postfilter
$\hat{y}(t) = y(nT)$
DAC
$y_a(t)$
Postfilter
$y_{POST}(t)$
Digital
signal
Analog
signal
Analog
signal
$\hat{y}(t) = y(nT)$
$y_a(t)$
$y_{POST}(t)$


ideal
reconstructor
Anti-image
Lowpass
Postfilter
$\hat{y}(t) = y(nT)$
DAC
$y_a(t)$
Postfilter
$y_{POST}(t)$
Digital
signal
Analog
signal
Analog
signal


T_0
$\frac{A}{T_0}$
$-f_s$
$-f_s/2$
$+f_s/2$
$+f_s$
ω


T_0
$\frac{A}{T_0}$
$-f_s$
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$+f_s/2$
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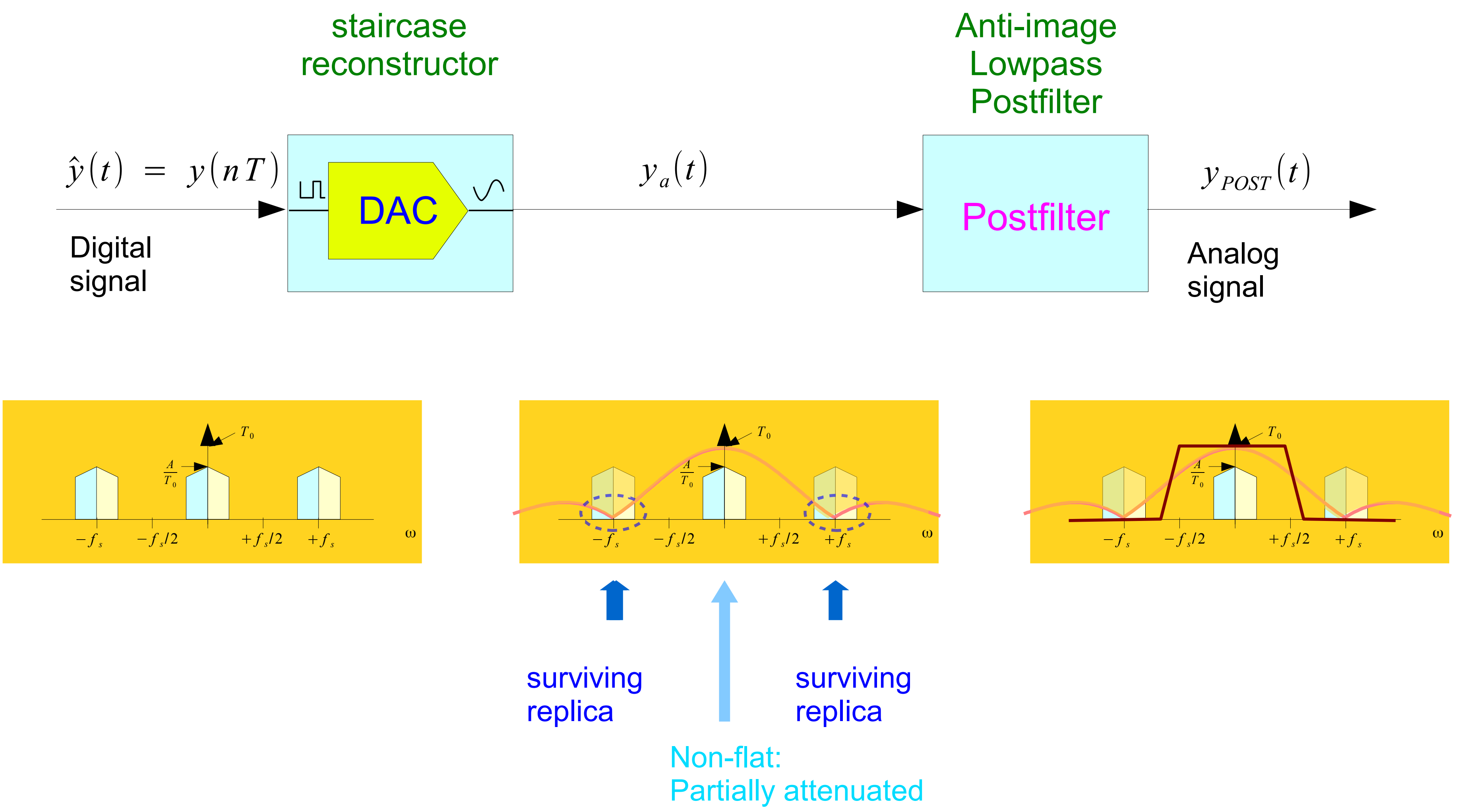
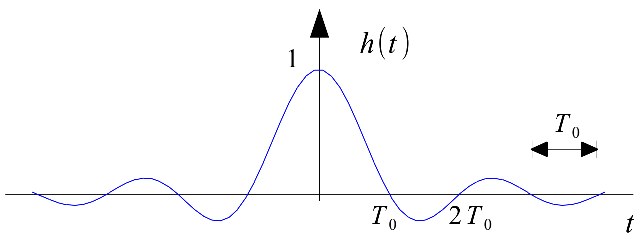


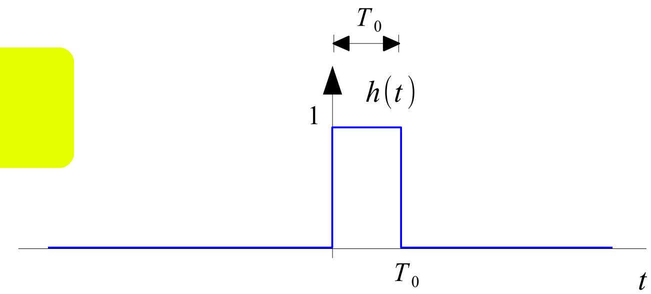
staircase reconstructor
Anti-image Lowpass Postfilter
$\hat{y}(t) = y(nT)$
DAC
$y_a(t)$
Postfilter
$y_{POST}(t)$
Digital signal
Analog signal
surviving replica
surviving replica
Non-flat:
Partially attenuated

CTFT of Reconstructors (1)

$t = \pm T_0, \pm 2T_0, \pm 3T_0, \cdots \Rightarrow h(t) = 0$



$$\frac{1}{T_0} \equiv f_s$$

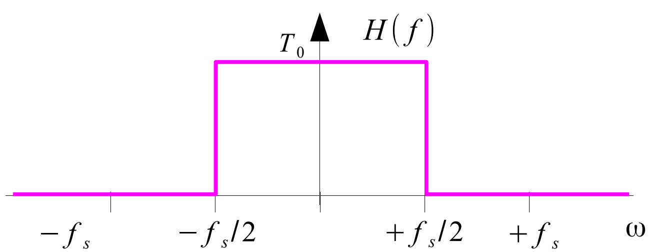


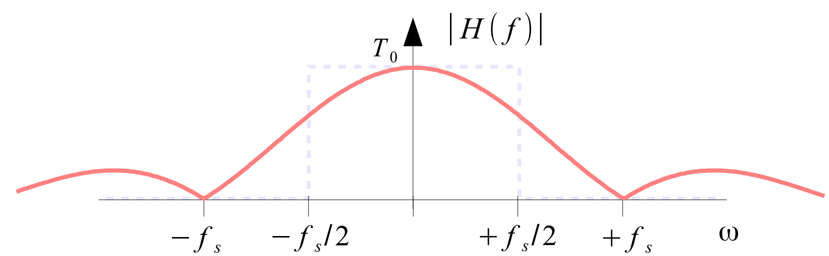
$$h(t) = \frac{\sin(\pi t / T_0)}{\pi t / T_0} = \frac{\sin(\pi f_s t)}{\pi f_s t}$$

$$h(t) = u(t) - u(t - T_0) = \begin{cases} 1, & 0 \le t \le T_0 \\ 0, & otherwise \end{cases}$$

CTFT

CTFT





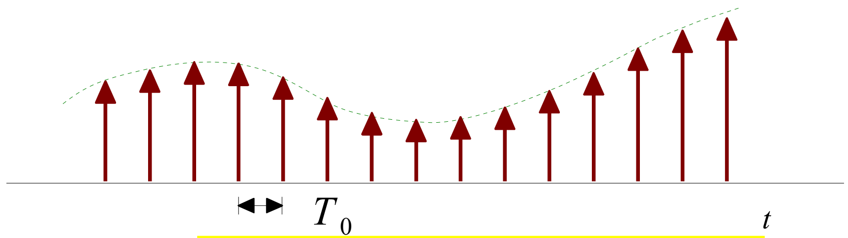
$$H(f) = \begin{cases} T_0, & |f| \le f_s/2 \\ 0, & otherwise \end{cases}$$

$$H(f) = T_0 \cdot \frac{\sin(\pi f T_0)}{\pi f T_0} e^{-j\pi f T_0}$$

$t = \pm\frac{1}{T_0}, \pm\frac{2}{T_0}, \pm\frac{3}{T_0}, \cdots \Rightarrow H(f) = 0$

Reconstruct via Convolution

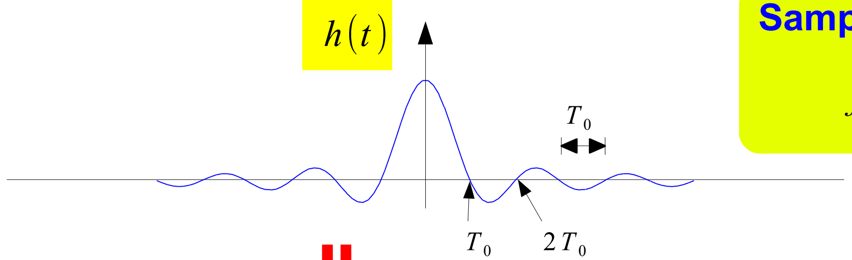
Ideal Reconstructor



$$\hat{y}(t) = \sum_{n=-\infty}^{\infty} y(nT_0)\delta(t - nT_0)$$

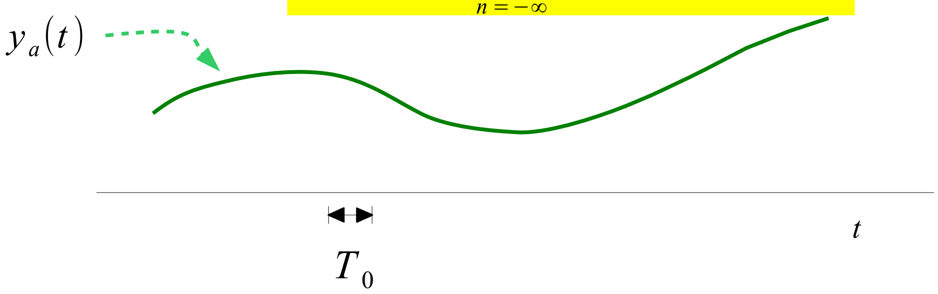
*

$h(t)$



=

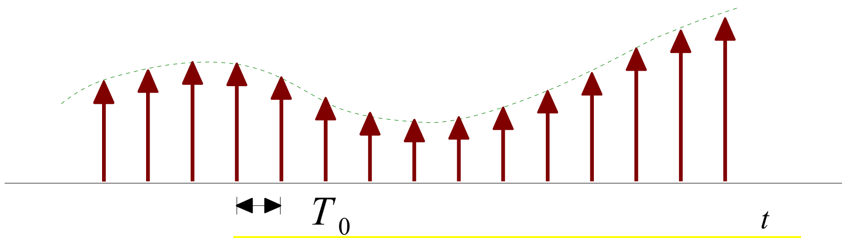
$$y_a(t) = \sum_{n=-\infty}^{\infty} y(nT_0)\, h(t-nT_0)$$



Sampling frequency

$$f_s = \frac{1}{T_0}$$

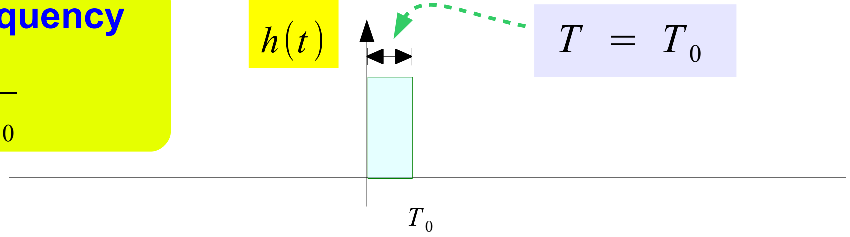
Practical Reconstructor



$$\hat{y}(t) = \sum_{n=-\infty}^{\infty} y(nT_0)\delta(t - nT_0)$$

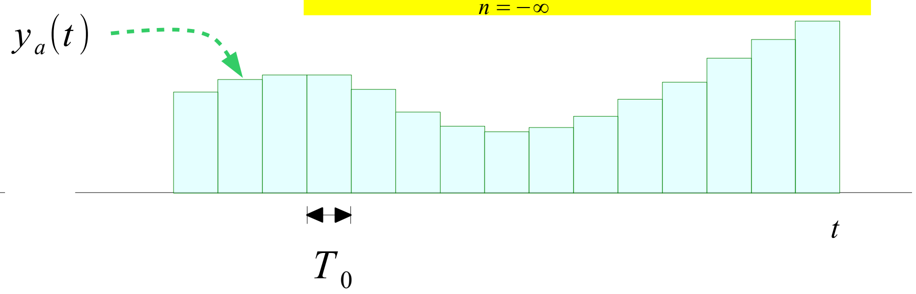
*

$h(t)$



=

$$y_a(t) = \sum_{n=-\infty}^{\infty} y(nT_0)\, h(t-nT_0)$$



Surviving spectral replicas
Can be removed by
An additional lowpass filter

Anit-image Postfilter

Cutoff Frequency $f_{max} \leq \frac{f_s}{2}$

Time domain

Effect of rounding off the corners
Of staircase output making smoother

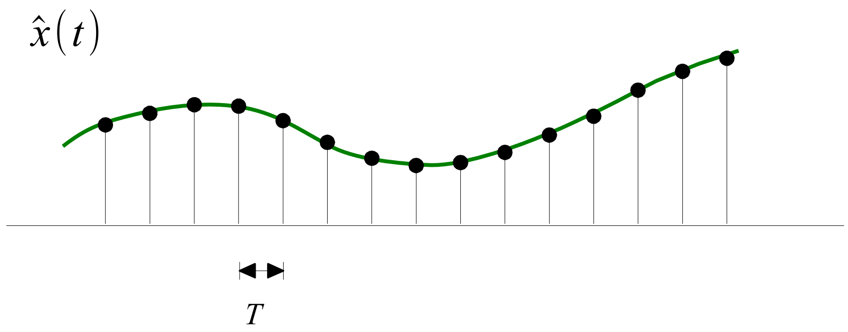
Freq domain

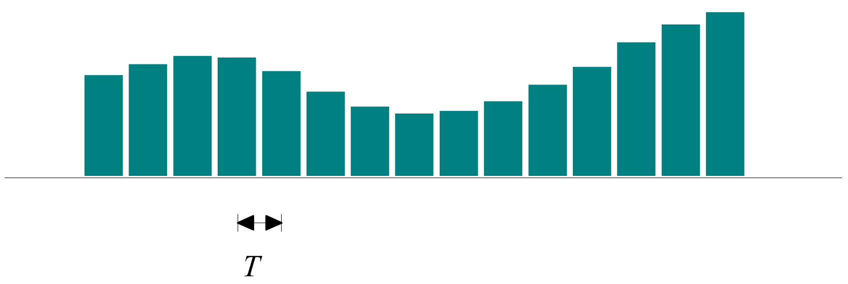
(reconstructor + postfilter) to remove
The spectral replicas as much as possible

Emulate the ideal reconstructor

Two stage → simplicity of implementation of reconstructor …
DAC – generating an analog output that remains constant during T

Analog Reconstructor





$$\hat{y}(t) = \sum_{n=-\infty}^{+\infty} y(nT)\,\delta(t-nT)$$

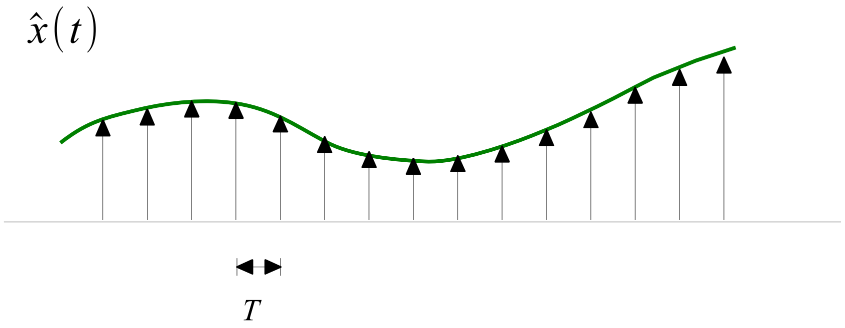
$$Y_a(f) = H(f)\,\hat{Y}(f)$$

$$y_a(t) = \int_{-\infty}^{+\infty} h(t-t')\,\hat{y}(t')\,dt'$$

$$\hat{Y}_a(f) = \frac{1}{T}\sum_{m=-\infty}^{+\infty} Y(f-m f_s)$$

$$y_a(t) = \sum_{n=-\infty}^{+\infty} y(nT)\,h(t-nT)$$

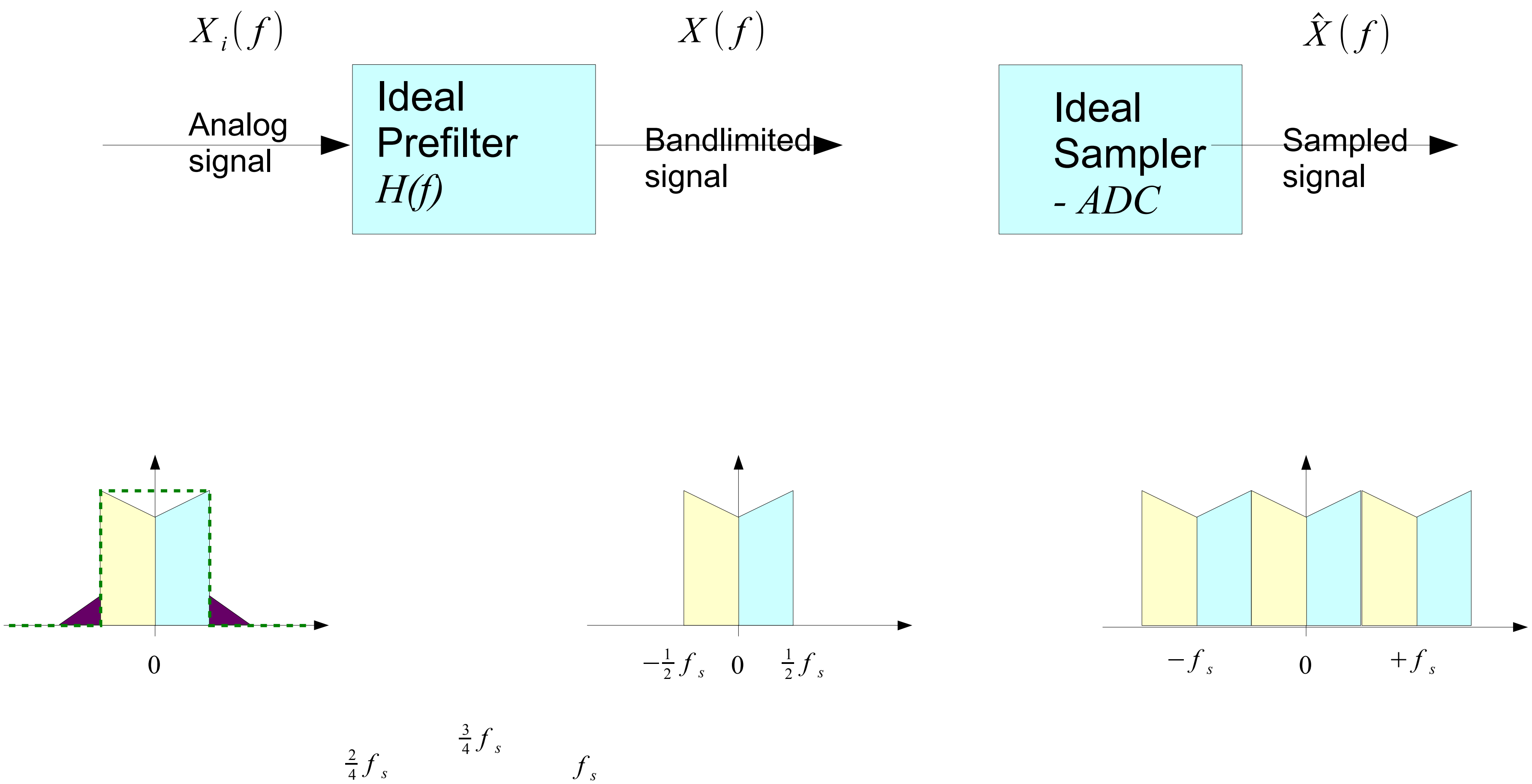
Impulse Response of Ideal Reconstructor



$$\hat{Y}(f) = \frac{1}{T} Y(f) \qquad -\frac{f_s}{2} \le f \le +\frac{f_s}{2}$$

$$y(t) = \sum_{n=-\infty}^{+\infty} y(nT)\, h(t-nT)$$

$$h(t) = \frac{\sin(\pi t/T)}{\pi t/T} = \frac{\sin(\pi f_s t)}{\pi f_s t}$$



$X_i(f)$
Analog signal
Ideal Prefilter $H(f)$
$X(f)$
Bandlimited signal
Ideal Sampler - ADC
$\hat{X}(f)$
Sampled signal
0
$-\frac{1}{2}f_s$ 0 $\frac{1}{2}f_s$
$-f_s$ 0 $+f_s$
$\frac{2}{4}f_s$
$\frac{3}{4}f_s$
f_s

References

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www.ece.rutgers.edu/~orfanidi/intro2sp