EGM 6321 - Team 6 - Homework 1

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1 Problem 1

1.1 Problem Statement

Derive the first and second derivative of $f(s,t)_{s=Y^1(t)}$ with respect to t and improve on the existing solutions[*].

1.2 Solution

With f a function of $s(t)$ and t, the chain rule must be invoked for the total derivative with respect to t.

$$\frac{d}{dt}f(s,t) = \frac{\partial f}{\partial s}\frac{\partial s}{\partial t} + \frac{\partial f}{\partial t}\cancelto{1}{\frac{\partial t}{\partial t}}$$

Substituting for f evaluated at s, $f(s,t) = f(Y^1(t),t)$, $s = Y^1(t)$ and using the notation $\frac{\partial}{\partial t}Y^1(t) = \dot{Y}^1(t)$.

$$\frac{d}{dt}f(Y^1(t),t) = \frac{\partial}{\partial s}f(Y^1(t),t)\dot{Y}^1(t) + \frac{\partial}{\partial t}f(Y^1(t),t)$$

For the second derivative define

$$g(s,t) := \frac{d}{dt}f(s,t)$$

Then again by chain rule

$$\frac{d^2}{dt^2}f(s,t) = \frac{d}{dt}g(s,t) = \frac{\partial g}{\partial s}\frac{\partial s}{\partial t} + \frac{\partial g}{\partial t}\cancelto{1}{\frac{\partial t}{\partial t}}$$

$$= \frac{\partial}{\partial s}\left(\frac{\partial f}{\partial s}\frac{\partial s}{\partial t} + \frac{\partial f}{\partial t}\right)\frac{\partial s}{\partial t} + \frac{\partial}{\partial t}\left(\frac{\partial f}{\partial s}\frac{\partial s}{\partial t} + \frac{\partial f}{\partial t}\right)$$

$$= \frac{\partial^2 f}{\partial s^2}\left(\frac{\partial s}{\partial t}\right)^2 + \frac{\partial f}{\partial s}\frac{\partial s}{\partial t}\cancelto{0}{\frac{\partial s}{\partial s}} + \frac{\partial^2 f}{\partial s \partial t}\frac{\partial s}{\partial t} + \frac{\partial^2 f}{\partial t \partial s}\frac{\partial s}{\partial t} + \frac{\partial f}{\partial s}\frac{\partial^2 s}{\partial t^2} + \frac{\partial^2 f}{\partial t^2}$$

Assuming real and continuous functions so that the order of the partial derivatives may be changed (Schawrz's theorem[†])

$$= \frac{\partial^2 f}{\partial s^2}\left(\frac{\partial s}{\partial t}\right)^2 + 2\frac{\partial^2 f}{\partial s \partial t}\frac{\partial s}{\partial t} + \frac{\partial f}{\partial s}\frac{\partial^2 s}{\partial t^2} + \frac{\partial^2 f}{\partial t^2}$$

Substituting for f evaluated at s, $f(s,t) = f(Y^1(t),t)$, $s = Y^1(t)$ and using the notation $\frac{\partial}{\partial t}Y^1(t) = \dot{Y}^1(t)$.

$$\frac{d^2}{dt^2}f\left(Y^1(t),t\right) = \frac{\partial f}{\partial s}(Y^1,t)\ddot{Y}^1 + \frac{\partial^2 f}{\partial s^2}(\dot{Y}^1)^2 + 2\frac{\partial^2 f}{\partial s \partial t}\dot{Y}^1 + \frac{\partial^2 f}{\partial t^2}$$

[*]As assigned in Lecture 2
[†]Schwarz's theorem

2 Problem 2

2.1 Problem Statement

Perform a dimensional analysis of[‡].

$$c_o\left(Y^1,t\right) = -F^1\left[1-\overline{R}U^2_{,ss}\left(Y^1,t\right)\right] - F^2u^2_{,s} - \frac{T}{R} + M\left[\left[1-\overline{R}u^2_{,ss}\right]\left[u^1_{,tt}-\overline{R}u^2_{,stt}\right] + u^2_{,s}u^2_{,tt}\right]$$

2.2 Solution

[‡] As assigned in Lecture 2

3 Problem 3

3.1 Problem Statement

Show $c_3(Y^1, t)\ddot{Y}^1$ is nonlinear with respect to Y^1.[§]

3.2 Solution

[§]As assigned in Lecture 4

4 Problem 4

Boundary value problem, $y(a) = \alpha$, $y(b) = \beta$, find c,d in terms of α, β[¶].

4.1 Problem Statement

4.2 Solution

[¶]As assigned in Lecture 5

5 Problem 5

Verify $L_2(y_H^1) = L_2(y_H^2) = 0$[‖].

5.1 Problem Statement

5.2 Solution

‖As assigned in Lecture 6